Stochastic Pulse Arrival-Time Pick-Off Resolution

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Summary – Problem of pulse arrival-time pick-off appears in electronic measurements of time-intervals as well as in measurements of physical quantities by means of time-interval determination (distance, particle time-of-flight, profilometry). Accuracy of the measurement depends on characteristics of START and STOP pulse signals and on the method of measuring time interval. The ultimate time-resolution is limited, however, by the accuracy of pulse time pick-of. Theoretical and experimental aspects of this problem were analyzed and pulse timing-errors are defined.

I. INTRODUCTION

Problem of pulse arrival-time pick-off, or pulse timing appears in many electronic measurements, especially in time-interval measurements as well as in measurements of physical quantities by means of the time interval determinations (distance, particle time-of-flight, profilometry). These measurements are of significant importance in many scientific and technical measurement areas, such as laser and radar ranging and sensing, optoelectronic distance and dimension measurements, characterization of optical communications systems, nuclear and fluorescence time spectroscopy etc. [1,2,3,4].

Ultimate accuracy of the time interval measurements depends on the precision of arrival-time determination of START and STOP pulses which determine the beginning and the end of time interval. In the real measurements the START pulses are well defined, stable, and its energy is sufficient to drive accurately the timing circuits. In the general case we assume that the STOP pulses are stochastic in nature. Strictly speaking, the exact measurement of the pulse arrival-time is physically impossible, because we need some, even small, amount of pulse energy to identify its appearance. This means that the pulse time pick-off technique, uses pulse discrimination i.e. detection of the moment when the pulse reaches defined discrimination level. This inherent time-difference in the measurements represents systematic error and could be compensated. There are two main factors that influence the pulse timing resolution: pulse parameters and timing method choused. Assuming the stochastic character of the pulse signals, the associated measurements are necessarily statistical, i.e. we have to measure the probability density distribution of the pulse arrival-time (time-delay spectrum) and then to calculate its mean value and standard deviation (timing error).

In this paper the more detailed theoretical analysis of the dependence of pulse timing-error on stochastic pulse parameters will be given. The obtained expressions can be used as a basis for the optimizations of different methods and circuits to obtain minimum timing-error in measurements. This process includes optimum pulse processing (optimum filtering) as well as the optimization of timing discrimination methods.

II. THEORY OF PULSE TIMING RESOLUTION

The analysis of the stochastic pulse timing-resolution (expressed by the timing error) was based on the following presumptions [1]:

(1) The pulses are random in time and are characterized by their stochastic parameters, amplitude, rise-time, duration and accompanied noise (Fig.1).

(2) Measurement of pulse arrival-time (time pick-off) uses time discrimination i.e. detection of the moment when the pulse reaches defined discrimination level.

(3) Stochastic measurement process is characterized by “dead time”. During this time we are processing the data on specific pulse arrival-time and, consequently, the system does not accept the succeeded pulse. This fact could introduce some statistical error, however in most practical cases the error due to “dead time” effect is low.

(4) We consider in the analysis the series of stochastic pulses as a stationary and ergodic process. This enables us to rely the theoretical calculations, considering the process in time, with experiments doing with ensembles of samples taken from the original process in time. As closer this assumption is to the reality, the better is agreement of theoretical calculations with experimental results.

(5) Due to random appearance of pulse signals, we have to take into consideration the “pile-up” effect (pulse overlapping) in some measurements (nuclear and atomic spectroscopy).

(6) Optimum pulse processing for minimum timing error was done by linear time-invariant filters.

A. Statistical parameters of stochastic pulse signals

The analysis was done using sample of one pulse signal (Fig.2) taken from the series of pulses presented on Fig.1. Generally we can express analytically the series of stochastic pulse signals as

$$y(t) = \sum_i A_i g(t - t_i, T_{ri}) + n_y(t) \qquad (1)$$

where; A_i is the stochastic amplitude of the ith pulse, t_i is the arrival-time, T_{ri} is the pulse rise-time and n_y represent the additive noise.

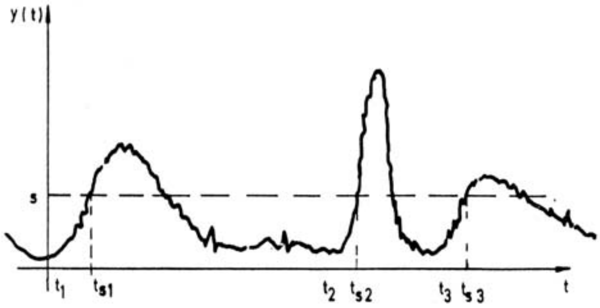

Fig.1 Series of random pulses in time

Ith pulse, which appearance we are going to measure, could be expressed analytically be the following expression:

$$y(t) = y_g(t) + y_p(t) \tag{2}$$

where: $y_g(t)$ represents analytically real pulse signal and y_p is the two component signal of disturbance caused by pulses pile-up effect and additive noise, so we can write

$$y(t)_p = y_m(t) + n_y(t) \tag{3}$$

and

$$y_g(t) = A\cdot g(t), \tag{4}$$

where A is the pulse amplitude and g(t) describes its shape.
Discrimination time t_s is given by

$$y(t_s)=s \tag{5}$$

where *s* is the pulse discrimination level.
Combining expressions (2) and (4) we get

$$g_s = g(t_s) = (s-y_p)/A \tag{6}$$

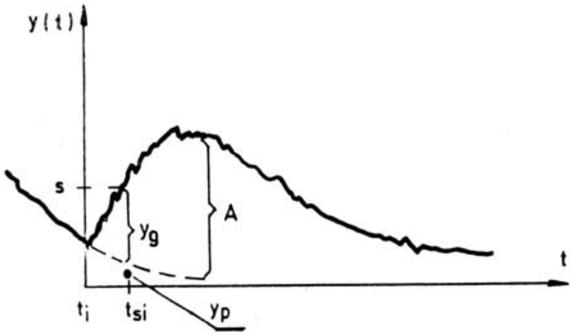

Fig.2 Complete presentation of the ith pulse signal

Considering the function *g* as a stochastic variable we can find their probabilistic distribution function (pdf) and, consequently, the mean value and the variance of pulse arrival time, after finding inverse function of g, t = φ(g).

After introducing a new variable, u=s−y and supposing that variables u and A are statistically independent, pdf of variable g is

$$p_{gs}(g_s) = \int_{A_1}^{A_2} |A|\, p_u(g_s A)\, p_A(A)\, dA \tag{7}$$

Further step is to normalize pdf of both variables u and A, which gives us the following equalities:

$$\int_{u_1}^{u_2} p_u(u)\,du = \int_{A_1}^{A_2} p_A(A)\,dA = 1 \tag{8}$$

The mean value of the variable g_s is

$$\overline{g}_s = (s - \overline{y}_p)(\overline{\tfrac{1}{A}}) \tag{9}$$

and its variance

$$\sigma^2_{g_s} = (\overline{\tfrac{1}{A}})^2\, \sigma^2_{yp} + \overline{g}_s^{\,2}\, \frac{\sigma^2_{1/A}}{(\overline{\tfrac{1}{A}})^2} \tag{10}$$

If the pdf of the variable g_s is given, and the functions g(t) as well as its inverse t=φ(g) are known, the pdf of the pulse arrival picking-of time (time discrimination point) is

$$p_{ts}(t) = p_{gs}\{g(t_s)\}\cdot|dg(t)/dt|_{t=ts} \tag{11}$$

After normalization of pdf of the variable t , and using well known relations we can express the mean value and the variance of the pulse pick-of time t_s.

$$\overline{t_s} = \int_{g_1}^{g_2} \varphi(g_s)\, p_{g_s}(g_s)\, dg_s \tag{12}$$

$$\sigma^2_{ts} = \int_{g_1}^{g_2} \left\{\varphi(g_s) - \overline{t_s}\right\}^2 p_{g_s}(g_s)\, dg_s \tag{13}$$

For the calculations of the last two integrals it is necessary to develop the function φ(g) into the Taylor series in the vicinity of the point g = g_s. The results of the calculations are given as

$$\overline{t_s} = \varphi(\overline{g_s}) + \sum_{k}^{n} \varphi^{(k)}(\overline{g_s})/k!\ \mathrm{M}_{k,g_s} \tag{14}$$

and

$$\sigma^2_{ts} = \sum_{k}^{n}\sum_{r}^{n} \frac{\varphi^{(r)}(\overline{g_s})\,\varphi^{(k-r)}(\overline{g_s})}{r!(k-r)!}\, \mathrm{M}_{k,\overline{g_s}}\, \mathrm{M}_{r,g_s}\, \mathrm{M}_{k-r,g_s} \tag{15}$$

where $\varphi^{(k)}$ and $\varphi^{(k-r)}$ are rth and (k-r)th derivatives of the function φ(g), while M represent central statistical moments of stochastic variable g_s.

This analysis is applicably only, if the noise frequency spectrum lies in the lower frequency region than the spectrum of the pulse signal, i.e. the noise is much “slower” in time than the pulse signal.

Relation (15) represent the result for the general case problem of stochastic pulse timing, and could be, under some specific circumstances, used for the optimization of pulse processing circuits to obtain minimum timing-error for “ideal” timing discriminators. Optimization of the real discriminator circuits is the further procedure, which is usually done separately.

B. Parameters of timing discriminator

The role of the pulse timing discriminator is to produce a very short, logic pulse, accurately positioned in time with minimum timing error. Important parameters that characterize discriminator are: time-walk and time-walk-error, long term timing-error and timing jitter.

Time-walk is caused by the input signal amplitude and rise-time variations as well as by the discriminator triggering inertia (energy-sensitivity). These effects are slow in time.

Long-term timing-error is also slow phenomenon. It is a result of long-time changes of discriminator circuit’s parameters, due to ageing and environmental effects variations (temperature, humidity).

Timing-jitter represents fast statistic variations of output pulse time-position. It is predominantly caused by the noise influence on pulse time pick-off.

III. METHODS AND TECHNIQUES FOR MINIMUM TIMING ERROR

A. Optimum pulse processing for minimum timing-error

Optimum pulse processing is the procedure to find optimum processing circuit (filter), which will transform the signal in such a way that the overall timing-error due to stochastic nature of the pulse will be minimal.

There are several criteria [5, 14] for this optimization (pulse arrival-time estimation):(1) Maximum likelihood estimation (MLE) criterion was widely applied in stochastic signal amplitude and time-delay estimation [3]. The idea was to use MLE criterion to find out the impulse-response function $h(\tau)$ of the optimum filter (optimum estimator), which minimizes pulse timing-error; (2) Minimum error-rate approach [6] attempts to find out the filter that minimizes the mean-number of zero-crossings of the noise component of the stochastic signal. This method could be interesting in the case when the “spike” noise is predominant; (3)Minimum square-error criterion leads to maximization of signal-to-noise ratio, which in the timing case means minimization of signal variation (noise)-to-signal slope ratio [1,2]. This criterion is more appropriate approach if the signal variations are much slower than the signal rise-time. The analysis results in optimum (matched) filter that minimizes pulse timing-error against signal variations for assumed ideal timing discriminator circuit. To find optimum impulse response function $h_0(\tau)$ the following expression should be minimized.

$$\sigma_t^2 = \frac{\sigma_s^2(t)}{\left[\frac{d}{dt}\bar{S}(t)\right]^2_{t=T_m}} \tag{16}$$

where T_m is a timing point and σ_s^2 is the input signal variance which includes effects of statistical processes of signal generation. The denominator in (16) represents squared average signal slope. It can be seen that the relation (16) is the special case of the solution given by (15) and is valid for higher signal-to-noise ratio. From expression (16) a functional G = G [$h(\tau)$] could be made and minimized, which gives us $h_0(\tau)$ of the optimum timing filter. More general solution has the form:

$$h_0(\tau) = \left[\frac{d\bar{S}(t-\tau)}{d\tau}\cdot\frac{1}{\bar{S}(t-\tau)+\alpha}\right]_{t=T_m} \tag{17}$$

where, α is a factor indicating relative noise at the system input. Overall optimization therefore includes minimization of the influence of variable pulse shape and noise by proper signal filtering and choice of discriminator circuits.

B. Choice and optimization of timing discriminators

In the literature many pulse timing methods and associated timing discriminators are described [7, 8, 9]. Most of them were used in nuclear and atomic time-spectroscopy as well.

(1) Leading-edge discriminator (LED) is simple fast comparator circuit, which produces an output pulse when the input signal crosses the fixed threshold. Disadvantage of this technique is the strong influence of the signal amplitude and rise-time variations to overall timing-error. It shows, however, good resistance to noise.

(2) Constant-fraction discriminator (CFD) represents the combination of signal branching circuit and zero-crossing comparator. The input signal is delayed and a fraction of the un-delayed pulse is subtracted from it. Zero crossing of this bipolar pulse produces timing pulse. Two versions of CFD are in use: true CFD and amplitude-and-rise-time-compensated (ARC) CFD. In the true CFD zero-crossing time occurs while the attenuated input signal reaches its maximum. In ARC timing, the beginning of input signal is used for pulse time pick-off. If we suppose linear rise of the signal in this region, the timing is not affected by pulse amplitude nor rise-time variations. This method is one of the most used for timing in optical distance measurements, nuclear time-spectroscopy, robotics and in other scientific and technical applications.

(3) Charge-balanced discriminator (CBD) is based on step-recovery diode property that is charge-controlled ultra fast OFF/ONN switching, when the total injected charge in the diode becomes zero. The input pulse injects the charge into unbiased diode and the diode starts conducting. The same pulse, inverted and amplified by factor k, starts to remove the stored charge after well defined time-delay. At the moment when the total stored charge becomes zero,

diode turns off in the less than 100 picoseconds. It was shown [1,8] that diode acts as an almost perfect integrator and timing discriminator. CBD does not exhibit dependence of timing-error on amplitude nor rise-time variations. As it acts as a non-regenerative comparator, there is also no effect of triggering inertia (energy sensitivity). Influence of the noise is also lowered due to effect of signal integration. CBD contents in itself a unique combination of signal processing circuits and discriminator. With pulse preamplifier circuit included, it represents a very good approximation of the theoretical optimum timing filter with ideal discriminator [10, 13].

(4) Double-threshold discriminator (DTD) uses the passage of the pulse through two discrimination levels. We distinguish two versions of this method: true and extrapolated-leading-edge timing (ELET). True DTD uses low threshold to give the timing information (minimum time-walk) and the high threshold to identify pulse hit (noise rejection). The ELET technique assumes linear rise of the signal and, using this double-level discrimination, extrapolates the beginning of the input signal by additional circuits. The method is very promising, especially for integrated circuit technology, and deserves thorough theoretical analysis [14].

(5) Combination of high-pass and low-pass RC pulse-shaping circuits, with different discrimination methods; leading-edge, zero-crossing and constant-fraction was used as well. Electronic circuits are also suitable for integrated technology. Some of circuit combinations have shown good timing performances in a system of pulsed time-of-flight laser radar [9]. It is necessary to perform, however, deeper theoretical analysis of their timing-resolution to be able to compare it with other timing methods.

All described timing methods use, in the principle, discrimination on the pulse leading edge. Circuits working on trailing edge were reported also [14], but the method did not find wider applications.

C. Comparison of theoretical and experimental results

Theoretical analysis of three types of timing discriminators was done, assuming linear-rise of pulse leading-edge model. The selection was based on the following facts: (1) LED was the first technique used in pulse timing measurements and is the basis for many other types of timing discriminators. (2) CFD is widely used in measurements assuring excellent timing resolution. (3) CBD is an old method, applied in nuclear spectroscopy and is very useful for slower pulse timing.

Theoretical analysis was based on the assumptions that the linear part of the input pulse was used for timing. This is close to reality, because the pulse shapes at the discriminator input have exponential components and could be approximated by linear function at the beginning of the signal. Comparison was made for the following parameters: (1) noise influence, (2) amplitude-variation effect, (3) pulse rise-time effect and (4) discriminator triggering charge-sensitivity influence on timing error.

By means of the pulse linear-rise model, simple relations are obtained for calculations the contribution of described disturbing factors to timing error, for three selected types of timing discriminators [11,12,13]. Using the accessible or estimated data on pulse signal, as well as discriminator parameters, relative timing-errors (normalized to pulse rise-time) have been calculated.

TABLE I

ESTIMATED RELATIVE TIMING ERROR

(f = 0.2, k=5, m=0.1, d/A=0.1)

σ_{T_m}/T_R	$a = \frac{\sigma_A}{A} = \frac{\sigma_{T_R}}{T_R}$ (10^{-2}-10^{-1})	$e = \frac{\sigma_E}{E}$ (10^{-3}-10^{-2})	$n = \frac{\sigma_n}{A}$ (10^{-2}-10^{-1})
LED	$1.4 \cdot 10^{-3}$-$1.4 \cdot 10^{-2}$	$3.2 \cdot 10^{-2}$-10^{-1}	10^{-2}- 10^{-1}
CFD	0	$3.2 \cdot 10^{-2}$-10^{-1}	$1.3 \cdot 10^{-2}$-$1.3 \cdot 10^{-1}$
CBD	0	0	$1.3 \cdot 10^{-3}$-$1.3 \cdot 10^{-2}$

Table I shows results of calculations with following input parameters: f=0,2 (attenuation factor for CFD); k=5 (amplification factor for CBD); d/A=0,1 (normalized discrimination threshold for LED). Other parameters: a, e and n represent normalized values of pulse amplitude and rise-time variations, triggering energy variation, and noise level, respectively.

In accordance with obtained results, it is evident that the pulse amplitude variations affect only LED timing. Both LED and CFD are equally sensitive on triggering energy effect, but this is not the case with CBD. The best result, regarding the noise influence on timing-error, was obtained by CBD. Although both CFD and CBD use similar signal bifurcation-circuits, which give similar noise level at the discriminator input, the noise and signal integration by step-recovery diode makes the CBD better.

Two groups of measurements were performed, using the experimental set-up presented on Fig.3 [12]. Assuming stochastic character of both Start and Stop pulse signals, we have to measure pdf, the probability density function (distribution) of the pulse time pick-off, and then calculate the timing error. By means of pulse gliding generator and noise generator we could simulate real pulse signal, that appears in time-interval measurements.

First group of timing-error measurements by LED-LED and CFD-CFD structures was performed with constant pulse amplitude and inserted additional noise in Stop channel. Full-width at half-maximum (FWHM) of time distribution function is connected (assuming Gaussian shape of time-spectrum) with standard deviation of timing-error with relation:

$$\sigma_T = \frac{FWHM}{2.35} \tag{18}$$

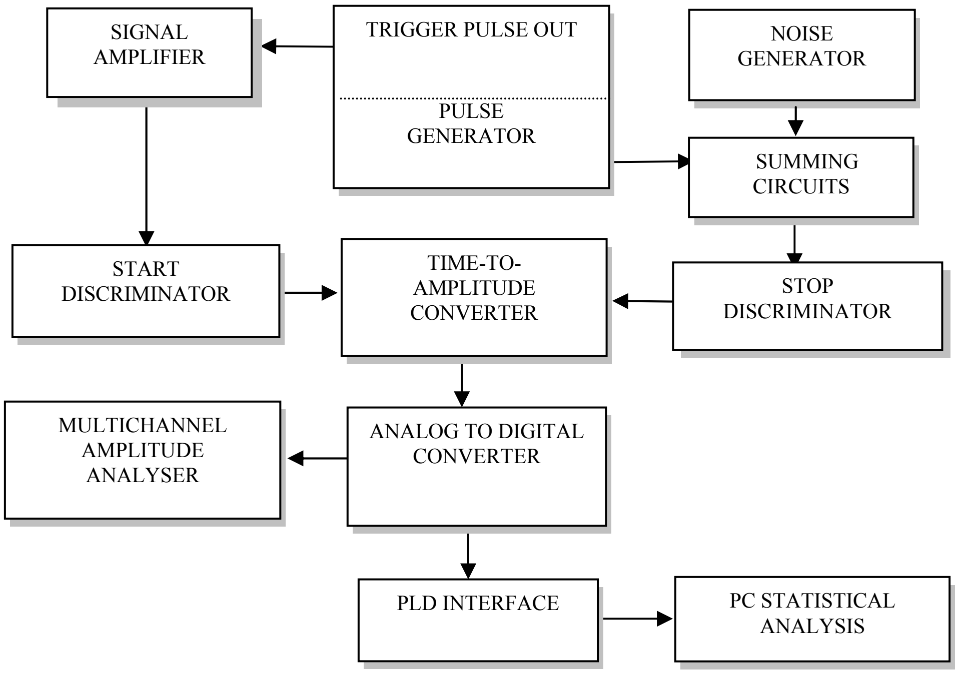

Fig.3 Statistical measurement set-up

Influence of the internal statistical fluctuation in measurement system in the absence of external noise was taken into account, so the corrected value of σ_T is:

$$\sigma_{T,corr} = \sqrt{\sigma_T^2(ext.noise) - \sigma_T^2(noext.noise)} \tag{19}$$

and the standard deviation of timing error of considered discriminators in Start and Stop channels is:

$$\sigma_{T_1} = \sigma_{T_2} = \frac{\sigma_{T,corr}}{\sqrt{2}} \tag{20}$$

Results of the measurements done with LED and CFD structures are in very good agreement with those given by simplified theory (Table II).

Second group of measurements with same set-up comprised the timing characteristics of CBD discriminator in Stop channel. In the Start channel a pulse generator is used, producing standard disturbance-free short pulse signal. In the Stop channel pulses having rise-times of 100ns, 200ns and 500ns, with amplitude dynamics of 15:1 were used for measurements. For this case obtained standard deviation of the timing-error was: 120ps, 220ps and 350ps, respectively.

Calculated relative (normalized) timing error of 10^{-3} is in a very good accordance with results presented in Table I.

TABLE II
EXPERIMENTAL RESULTS

A = 0.5V Tr =30ns	EXPERIMENT				THEORY
	$\sigma_n(RMS)$	σ_T	$\sigma_{T,C}$	$\sigma_{T_2} = \frac{1}{\sqrt{2}}\sigma_{T,C}$	σ_{T_2}
LED - LED	0	0.28 ns	/	/	/
	10 mV	0.82 ns	0.77 ns	0.55 ns	0.6 ns
CFD - CFD	0	1.04 ns	/	/	/
	10 mV	1.46 ns	1.02 ns	0.72 ns	0.82 ns (f=0.25)

IV. CONCLUSION

The problem of the resolution of stochastic pulse arrival-time pick-off has been treated, both theoretically and experimentally, with very good agreement under some necessary assumptions:(1) Use of linear-rise of pulse leading-edge model, (2) Gaussian white noise presence, and (3) Use of timing-error minimization criterion based on minimum noise-to-signal-slope ratio.

Timing-error minimization process should include optimal pulse signal processing and choice of timing discrimination methods with lowest sensitivity on pulse shape or pulse parameters variations as well as on triggering-energy effect.

Experimental results of timing measurements with leading-edge, constant-fraction and charge-balanced discriminators have shown very good agreements with applied simplified theory of pulse timing, based on linear-rise of pulse leading-edge model as well as on the criterion of minimization of noise-to-signal-slope ratio.

Charge-balanced timing discriminator appears very suitable for precise timing of slow pulse signals. The method assures picosecond time-resolution even with wider pulse signals of several hundreds of nanoseconds, exhibiting significant timing insensitivity on noise as well as on pulse amplitude and rise-time variations.

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