ROBUST BLIND SEPARATION OF STATISTICALLY DEPENDENT SOURCES USING DUAL TREE WAVELETS

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ABSTRACT

Blind source separation (BSS) problem is commonly solved by means of independent component analysis (ICA) assuming statistically independent and non-Gaussian sources. The strict independence assumption can be relaxed to existence of subbands where signals are less dependent. In this paper, we use dual tree complex wavelets for the subband decomposition of observed signals and small cummulant based approximation of mutual information for finding the most independent subband(s). We compare the proposed method to previously reported shift invariant and decimated wavelet packet based approach, as well as to innovations based approach. We found proposed dual tree wavelets scheme as an efficient and robust solution of the BSS problem of statistically dependent sources. One important application of the proposed method is related to unsupervised segmentation of medical and remotely sensed multispectral images.

Index Terms **—** blind source separation, statistically dependent sources, dual tree wavelets, independent component analysis, mutual information.

1. INTRODUCTION

The BSS problem is formulated as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \qquad (1)$$

where $\mathbf{x} \in \mathbb{R}^{N}$ represents vector of measured signals, $\mathbf{A} \in \mathbb{R}^{N \times M}$ is an unknown mixing matrix and $\mathbf{s} \in \mathbb{R}^{M}$ represents unknown vector of the source signals. The problem is usually solved using ICA techniques assuming statistically independent non-Gaussian sources [1][2]. In this paper, we shall assume number of sources M and number of sensors N to be equal.

Although there are many cases in which statistical independence assumption holds, in practical applications this assumption is not always fulfilled. One example is EEG signals [3]. The other illustrative example is related to low-dimensional multispectral imaging where spectrally similar classes can not be considered completely independent. We illustrate this on an artificial example of the mixtures of human face images [4]. In these cases significant degree of statistical dependence between source signals exists, and separation using direct application of the ICA gives poor results.

There are several reported methods that deal with the statistical dependence problem.

One approach, which is also exploited in this paper, is based on the assumption that wideband source signals are dependent, but there exist subband(s) where they are less dependent. It leads to subband decomposition ICA [3][5][6][7]. The basic problem is finding the subband with least dependent components. Some authors use *a priori* knowledge on signal statistics [5], or assume that high-pass bands are less dependent. Very useful method is based on observations that innovations are usually more independent and more non-Gaussian then original signals [4]. Another approach is based on an adaptive filter that minimizes mutual information between the filter outputs [7].

In our previous reports [8], we have used wavelet packets for subband decomposition, followed by a computationally efficient small cumulants based approximation of mutual information [9][8] in order to find the subband with least dependent components. We have shown that the proposed method results with good separation performance and is more robust with respect to additive Gaussian noise than the competing techniques.

However, by choosing between shift invariant and decimated wavelet packet filter banks we were trading off between the computational complexity and separation efficiency.

Dual tree complex wavelet filter banks were proposed recently [10][11] [12]. They have good analytic properties and approximate shift invariance, with lower redundancy and lower numerical complexity than non-decimated wavelets. Therefore, we propose use of the dual tree complex wavelet filter banks as an efficient, robust and computationally low cost approach for solution of the BSS problem of statistically dependent sources.

2. PROBLEM FORMULATION

We base our approach on the assumption introduced by Cichocki et. al [5], that wide-band source signals are dependent, but some of their subband components are independent. The vector of source signals is represented as a sum of L subband components:

$$\mathbf{s}(t) = \sum_{i=1}^{L} \mathbf{s}_i(t) \tag{2}$$

According to the assumption, there exist a set of independent components $\mathbf{s}_k$, $1 \le k \le L$. Hence, standard ICA can be applied to the set, resulting in demixing matrix $\mathbf{W}$ such that

$$\mathbf{y}_k(t) = \mathbf{W}\mathbf{x}_k(t) \tag{3}$$

where $\mathbf{y}_k$ are recovered subband components of source signals.

We introduce linear operator T_k that extracts a set of independent subband components $\mathbf{s}_k$:

$$\mathbf{s}_k(t) = T_k\left[\mathbf{s}(t)\right] \tag{5}$$

We apply T_k on (1) and obtain

$$\mathbf{x}_k(t) = T_k\left[\mathbf{A}\mathbf{s}(t)\right] = \mathbf{A}T_k\left[\mathbf{s}(t)\right] = \mathbf{A}\mathbf{s}_k(t) \tag{6}$$

Hence, demixing matrix $\mathbf{W} \triangleq \mathbf{A}^{-1}$ obtained by the ICA on $\mathbf{x}_k$ (3) is also the separation matrix for the BSS problem of statistically dependent sources (1).

The challenge is how to find the appropriate linear operator T_k. In the subband ICA approach, the operator T_k represents a prefilter applied to all observed signals. There are several ways for the prefilter design.

A fixed filter bank (FB) approach is used in [6], where existence of at least two sets of independent components is assumed. We emphasize that method proposed in this paper does not require any additional assumption or previous knowledge of signal statistics. The high-pass filtering can be also regarded as a simplified and computationally efficient FB scheme that obviously fails if the dependent components are located in a high-frequency subband.

An adaptive filter was used in [7], where the iterative filter design was based on minimization of mutual information between the filter outputs. The approach is computationally demanding.

Very efficient competing approach is application of the ICA on innovations rather then observations [4]. The linear predictive filter is trained on a set of observed signals. The innovations (prediction errors) have shown to be more independent then the observed signals. This approach fails if the dependent components are non-predictable, like in the case of wide-band interferer. The presence of wideband non-predictable noise present in all sensors is not rare in practice.

3. PROPOSED METHOD

Our motivation is to develop a computationally efficient and robust method for blind separation of statistically dependent source signals that circumvent the encountered problems.

In this work we propose dual tree complex wavelet transform for the realization of the operator T. We denote individual subbands as T_i.

In the preceding work [8], we have used real wavelet packets (WP) due to its computationally efficient implementations, good analytic properties and due to the possibility of selection of the arbitrary narrow subband by progressing the WP analysis in each point to the desired number of decomposition levels. Non-decimated WPs have shown better performance then decimated due to shift variance and aliasing problems, but the price was high computational complexity.

The dual tree complex wavelet transform (DTCWT) employs two real DWTs giving the real and the imaginary part of the transform. Each set of filters satisfies perfect reconstruction conditions, and they are designed so that the transform is approximately analytic. Denoting the complex wavelet as $\psi(t)$, we obtain:

$$\begin{aligned} \psi(t) &= \psi_h(t) + j\psi_g(t) \\ \psi_g(t) &\approx \mathcal{H}\{\psi_h(t)\} \end{aligned} \tag{7}$$

where $\mathcal{H}$ denotes the Hilbert transform. We summarize the proposed procedure in the following four steps.

The first step is decomposition of observed signals using DTCWT. If the signals were multidimensional, the appropriate transform of the same dimensionality is used.

The second step is search for the subband(s) that contains least dependent components. We used a measure of mutual information (MI) between the filter outputs $\mathbf{y}$ at each subband. One approximation of the MI is entropy based:

$$I(\mathbf{y}) = \sum_{n=1}^{N} H(y_n) - H(\mathbf{y}) \tag{8}$$

where H denotes the entropy. Direct application of (8) is computationally demanding, so we employ an approximation based on second, third and fourth order cumulants [9]:

$$\hat{I}_c\left(y_1, y_2, ..., y_N\right) \approx \frac{1}{4} \sum_{\substack{1 \le i < j \le N \\ i \ne j}} cum^2(y_i, y_j) + \frac{1}{12} \sum_{\substack{1 \le i < j \le N \\ i \ne j}} \left(cum^2(y_i, y_i, y_j) + cum^2(y_i, y_j, y_j) \right)$$
$$+ \frac{1}{48} \sum_{\substack{1 \le i < j \le N \\ i \ne j}} \left(cum^2(y_i, y_i, y_i, y_j) + cum^2(y_i, y_i, y_j, y_j) + cum^2(y_i, y_j, y_j, y_j) \right) \quad (9)$$

We apply the MI measure on the real part of the decomposition points and choose the subband with the least mutual information. The alternative is to choose a set of subbands with low MI.

The third step is partial reconstruction from the selected subband(s). The reconstruction uses both real and imaginary parts of the DTCWT. The result is interpreted as an application of operator T_k in (6).

The fourth step is application of the standard ICA algorithm to partially reconstructed subband signals $\mathbf{x}_k$ in order learn demixing matrix $\mathbf{W}$. Demixing matrix is then applied to the observed signals $\mathbf{x}$, Eq.(1), in order to restore the unknown statistically dependent sources $\mathbf{s}$.

4. EXPERIMENTAL RESULTS

We demonstrate the capability of the proposed scheme to successfully separate four images of human faces and compare its performance to competing methods. Human faces are known to be highly dependent. For the implementation of two-dimensional DTCWT we employed [13] library.

Four source images are shown in Figure 1(A). External background Gaussian noise was added to the sources as an additional wide-band dependent component. Due to average S/N ratio of ~30dB, the noise is hardly visible, but it influences significantly the separation performance. Figure 1(B) shows observed images obtained by mixing original face images with a 4x4 random matrix. Figure 1(C) shows separated images obtained by direct application of the JADE ICA algorithm [15][16]. We employ Amari's error P_{err} to measure the separation performance:

$$P_{err} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \left\{ \left(\sum_{j=1}^{N} \frac{|q_{ij}|}{\max_k |q_{ik}|} - 1 \right) + \left(\sum_{j=1}^{N} \frac{|q_{ji}|}{\max_k |q_{ki}|} - 1 \right) \right\} \quad (10)$$

where $\mathbf{Q}=\mathbf{WA}$, $q_{ij}=[\mathbf{Q}]_{ij}$ and $0 \le P_{err} \le 2$.

The quality of separation in Figure 3(C) is visibly poor, which is confirmed by the Amari's error P_{err}= 1,06.

Figure 1(D) shows the results of the top competing method based on innovations. The Amari's error is now P_{err}= 0.28. It is fair to say that the results would be significantly better in absence of the wide-band interferer, whose presence is, on the other hand, not rare in the real-world applications.

Finally, Figure 1(E) shows the results of the proposed method. The Amari's error is P_{err}= 0.02, which is the best result of all methods under consideration.

Table 1 shows normalized mutual information in all decomposition subbands. MI values are normalized by its maximum value. The subbands that correspond to low MI are marked with asterisk(s). The minimum MI subband (**) is chosen for the reconstruction. The decision on the maximum number of decomposition levels can be made by observing trends of MI (in this example it is 4).

Level		Normalized MI
1	V	0.0087
	H	0.0126
	D	0.0107
2	V	0.0083
	H	0.0110
	D	0.0093
3	V	0.0079**
	H	0.0132
	D	0.0093
4	V	0.0080*
	H	0.0155
	D	0.0111
5	V	0.3939
	H	1.0000
	D	0.0116

Table 1: Normalized mutual information between corresponding nodes of the DTCWT of the observed signals. MI is calculated for the real part of decomposition points only.

The separation was quite successful, which also stands for the real WP approach: use of decimated WPs resulted in P_{err}= 0.036, while in case of non-decimated WPs the Amari's error was P_{err}= 0.033. In absence of the wide-band interferer, non-decimated WPs significantly outperformed decimated WP approach, but with higher computational cost. The MATAB code can be downloaded from [17].

5. CONCLUSION

A novel method for blind separation of statistically dependent sources based on dual tree complex wavelet transform is proposed. The method is built on assumption that wideband sources are dependent, but there exists sub-band(s) where they are less dependent. No previous knowledge on the source signals is needed. The selection of subband(s) is based on a computationally efficient small cumulant based approximation of mutual information.

When compared to competing methods, the method is shown to be computationally efficient and robust with respect to the additive noise. By performance, dual tree complex wavelet based separation method matches the shift-invariant wavelet packet approach, but with significantly lower computational cost.

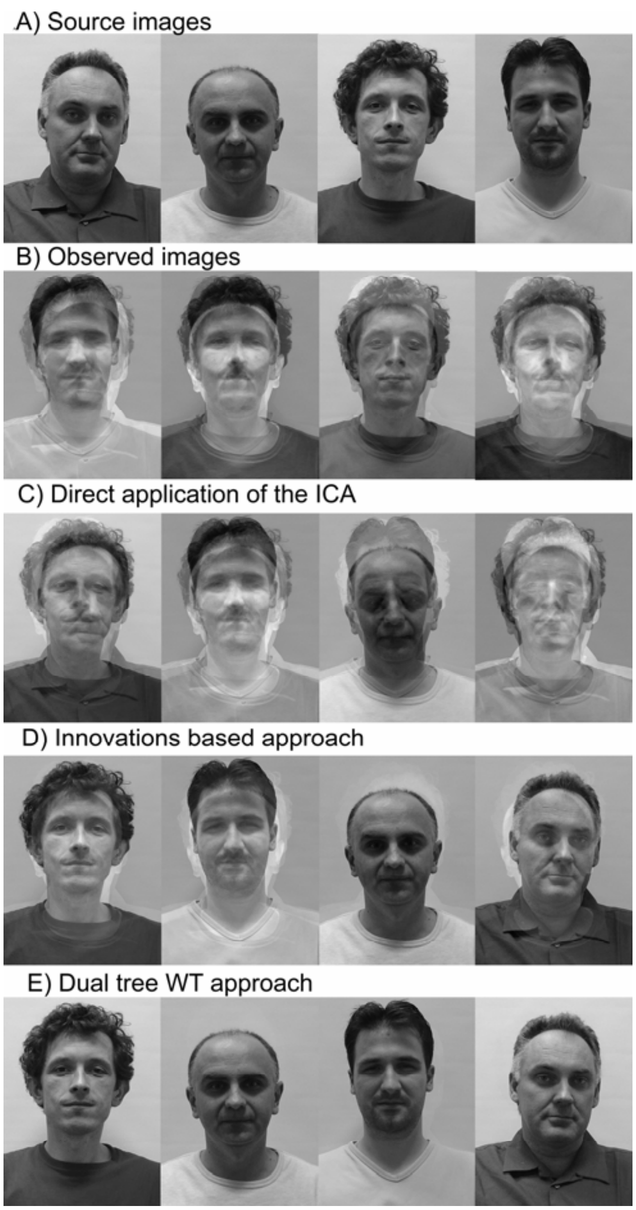

Figure 1: Blind separation of mixtures of human face images.

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