Modelling of on-chip spiral inductors

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Abstract – This paper presents modelling of on-chip spiral inductors. At lower frequencies the inductors are relatively well modelled by two-element and three-element equivalent circuits. As the operating frequency of modern integrated circuits reached GHz frequency range, accurate on-chip inductor models are necessary in the RF design process. Data for inductor modelling is obtained by measuring S-parameters of on-chip inductors. Several models have been evaluated. The accuracy is evaluated by comparing simulation results with measurements. Firstly, simple π-network is used capable of modelling the inductor up to approximately 1 GHz. This model is not able to predict skin-effect and substrate losses at higher frequencies. The second model includes the skin-effect by using resistance-capacitor-inductance network. The inductor model parameters are additionally optimised by a MATLAB optimisation tool.

I. INTRODUCTION

The operating frequency of modern integrated circuits (ICs) has reached GHz frequency range and integration of inductors on a chip has become economically acceptable. When speaking of on-chip passive components, inductors are the most complex to model. However, accurate models are a must for successful IC design. It is especially important to take into account the frequency dependence of the model parameters caused by skin effect and substrate losses at higher frequencies [1-3].

The modelling procedure involves determination of a network of frequency-independent components. At lower frequencies inductors are relatively well described by two-element or three-element equivalent schemes shown in Figs. 1, 2 and 3.

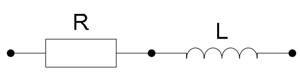

Fig. 1. Two-element resistance-inductance series network.

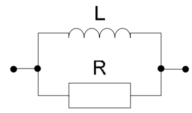

Fig. 2. Two-element resistance-inductance parallel network.

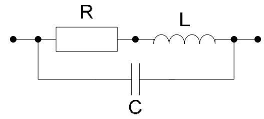

Fig. 3. Three-element equivalent circuit.

When focusing on GHz frequency range these models are no longer valid. By measuring S-parameters of on-chip inductors, data for building equivalent circuit models at higher frequencies is obtained [5-7].

Section II briefly introduces S-parameters. Section III describes the model extraction procedure. Section IV discusses results and conclusions are presented in Section V.

II. SCATTERING PARAMETERS (S-PARAMETERS)

Scattering parameters (S-parameters) enable accurate description one-port, two-port and multi-port networks up to 100 GHz and beyond [4]. Similarly to *y*- or *z*-parameters, they describe the performance of a two port network completely. In contrast to *y*- and *z*-parameters, they can be measured in the GHz frequency range. S-parameters are important in microwave design because they are easier to measure, they are conceptually simple, analytically convenient and capable of providing detailed insight into measurements and modelling issues. Similarly to all other two port parameters, S-parameters are linear by definition.

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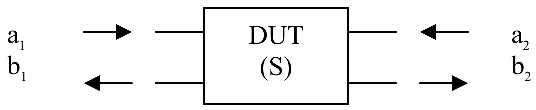

$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0}$$
$$S_{12} = \frac{b_1}{a_2}\bigg|_{a_1=0}$$
$$S_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0}$$
$$S_{22} = \frac{b_2}{a_2}\bigg|_{a_1=0}$$

Fig. 4. Definitions of S-parameters.

S_{11} and S_{21} are determined by measuring the magnitude and phase of the incident, reflected and transmitted signals when the output is terminated in a perfect *Zo* load. This condition guarantees that a_2 is zero. S_{11} is equivalent to the input complex reflection coefficient or impedance of the DUT (Device Under Test), and S_{21} is the forward complex transmission coefficient.

By placing the source at Port2 and terminating Port1 in a perfect load (making a_1 zero), S_{22} and S_{12} measurements can be made. S_{22} is equivalent to the output complex reflection coefficient or output impedance of the DUT, and S_{12} is the reverse complex transmission coefficient.

The accuracy of S-parameter measurements depends greatly on the measurement procedure. When the DUT is connected to the test ports of a vector network analyzer, it is necessary to perform calibration and de-embedding.

III. Model Extraction Algorithm

S-parameter tables typically consist of 9 columns of data acquired by measurements with a VNA (Vector Network Analyzer) using 50 Ω characteristic impedance in a broad frequency range, e.g. from 50 MHz to 10 GHz, as shown in Table I. The first column represents the frequency while the rest of the columns represent real and imaginary parts of S-parameters.

TABLE I
STRUCTURE OF MEASURED S-PARAMETERS TABLE

f	*S11* real	*S11* imag	*S12* real	*S12* imag	*S21* real	*S21* imag	*S22* real	S22 *imag*
50 MHz	. .	. .	. .	. .	. .	. .	. .	. .
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
10 GHz								

Model extraction procedure starts with the simplest equivalent circuit which contains one ideal inductor, as shown in Fig. 5.

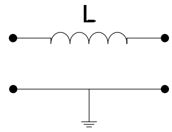

Fig. 5. Initial equivalent circuit.

Taking into account parasitic components between each port and the ground and additional parasitics between two ports, a π-network is assumed, as shown in Fig. 6. In this network there are three admittances: *Y1*, *Y2* and *Y3*. These should not be confused with the *y*-parameters of a two-port network.

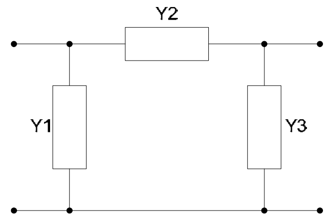

Fig. 6. π-network.

These admittances are calculated from the measured S-parameters. For this purpose a MATLAB code is written. The code takes the measured S-parameters as input and translates them firstly into *y*-parameters and then into the admittances Y_1, Y_2 and Y_3.

The algorithm for the conversion of the measured S-parameters into Y_1, Y_2 and Y_3 is given in Fig. *7*.

$$S_{meas} \rightarrow y_{meas} \rightarrow (Y_1, Y_2, Y_3)_{\mathrm{meas}}$$

Fig. 7. Conversion algorithm.

The admittance Y_1 represents the input port parasitics towards ground. Similarly, the admittance Y_3 represents the output port parasitics towards ground. The admittance Y_2 represents the core of the on-chip inductor.

The admittances Y_1, Y_2 and Y_3 obtained from measurements are noisy. In order to extract the parameters for the equivalent circuit, the results are usually averaged in the given frequency range and, then, constant *R*, *L* and *C* values are extracted for various equivalent circuits.

Finally, S-parameters are calculated based on known equivalent circuit parameters and compared with measured S-parameters. The results are graphically presented for easier comparison.

IV. Results

Fig. *8* shows the model suitable for modelling electrical behaviour of on-chip inductors up to the frequency one decade below the first resonant frequency.

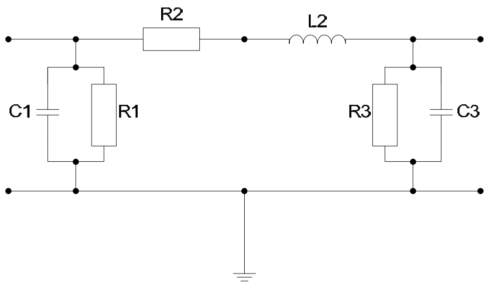

Fig. 8. Initial model of on-chip spiral inductor.

The admittances Y_1 and Y_2 consist of a parallel resistance-capacitance network, while Y_3 is represented by a series connection of a resistance and an inductance. The equivalent circuit elements are calculated in the frequency range from 50 MHz to 10 GHz by using the MATLAB algorithm described earlier and are shown in Figs. 8-10.

Fig. 10 clearly shows how the resistance increases due to skin effect and substrate losses from approximately 1 do 10 GHz. While the inductance is well modelled by a constant inductance, it is obvious that a more elaborate equivalent circuit should be used in the GHz frequency range. The results shown in Figs. 8 and 9 show that the parallel R-C network, that represents the parasitics connected between the input/output port and ground, can be reasonably well approximated, although the measured data are noisy. The measured and calculated S-parameters are shown in Figs. 11-14. Although the S-parameters seem reasonably well fitted, Fig. 10 shows that the model for the admittance Y_3, especially its real part, has to be improved, both at low and at high frequencies.

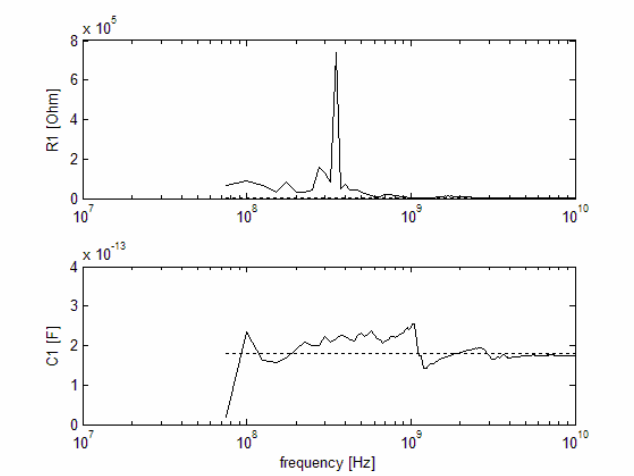

Fig. 8. R_1 and C_1 values extracted from the initial model (full line: measured data, dashed line: model).

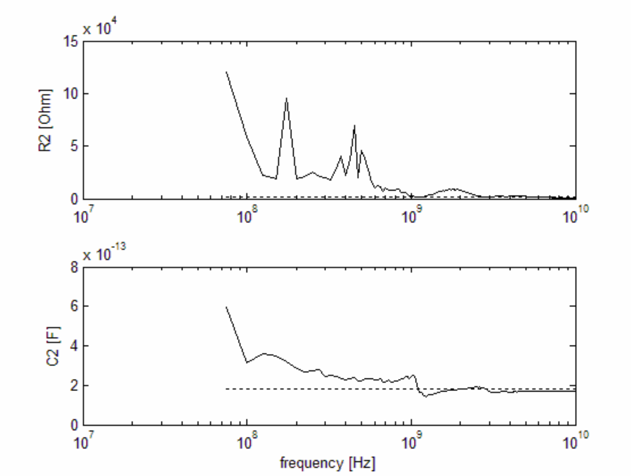

Fig. 9. R_2 and C_2 values extracted from the initial model (full line: measured data, dashed line: model).

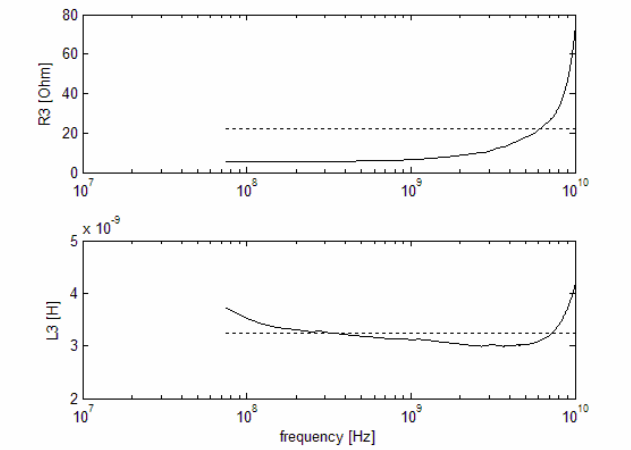

Fig. 10. R_3 and L_3 values extracted from the initial model (full line: measured data, dashed line: model).

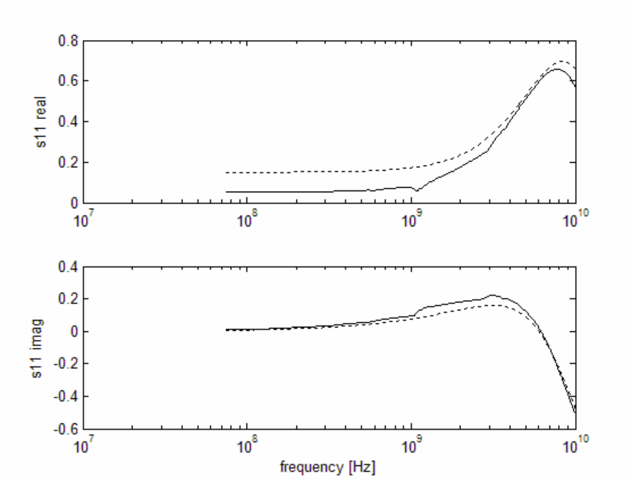

Fig. 11. Measured and calculated S_{11} of the initial model (full line: measured data, dashed line: model).

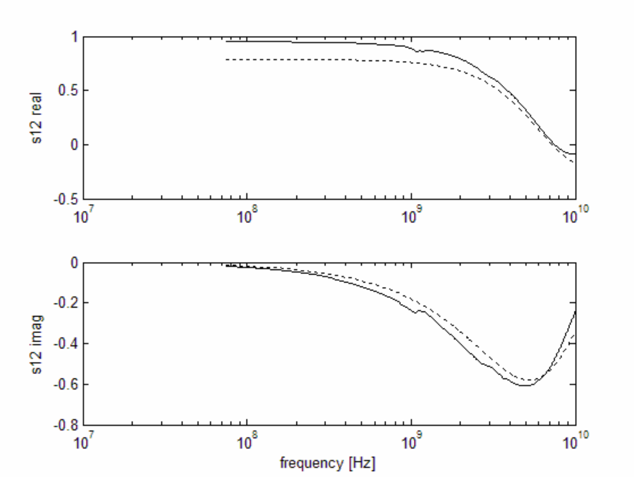

Fig. 12. Measured and approximated S_{12} of the initial model (full line: measured data, dashed line: model).

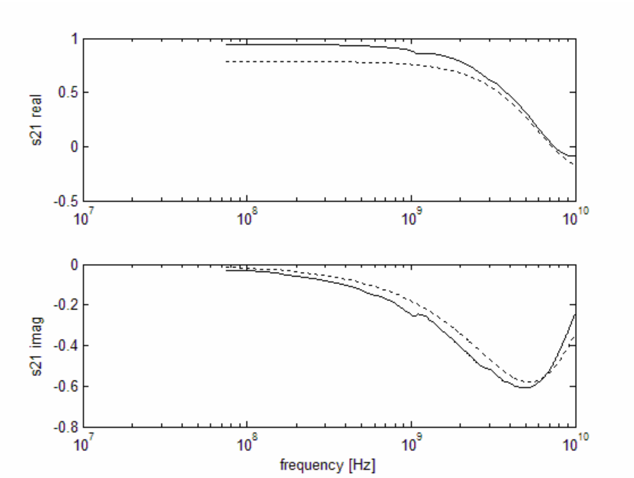

Fig. 13. Measured and approximated S_{21} of the initial model (full line: measured data, dashed line: model).

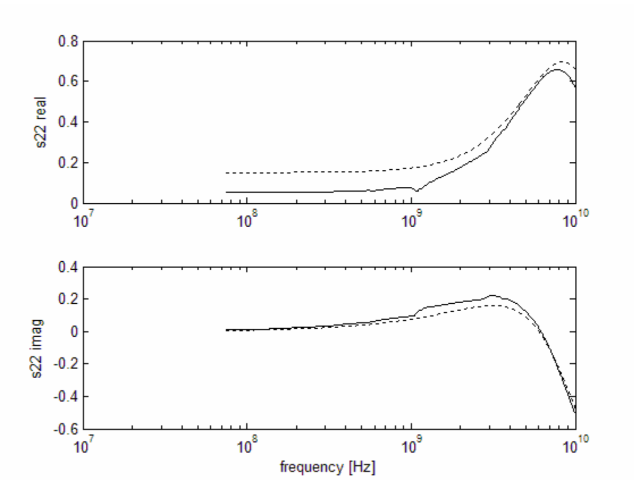

Fig. 14. Measured and approximated S_{22} of the initial model (full line: measured data, dashed line: model).

Changing the model by transforming the central admittance of the π-network (Y_3) into circuit shown in Fig. 15, the skin effect and substrate losses can be modelled more accurately. MATLAB optimisation is used to improve the accuracy of the extracted components of Y_3 (R_{31}, L_3, R_{32} and C_2). The values of the equivalent circuit elements for the initial and final model are given in Tables II and III and the comparison of the measured and modelled S-parameters is presented in Figs. 17-20. Fig. 16 shows that the new model is capable of reproducing the skin effect/substrate losses in the GHz frequency range. Both the real part and imaginary part of Y_3 are modelled correctly at high frequencies. The modelled S-parameters are in good agreement with measured data up to 10 GHz.

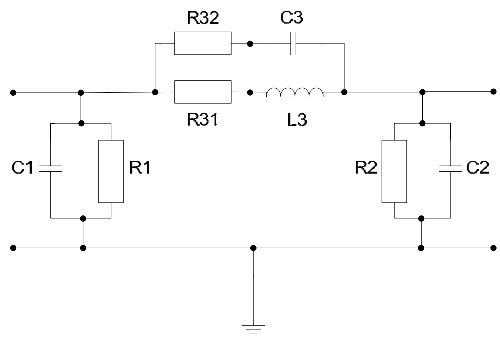

Fig. 15. Improved model.

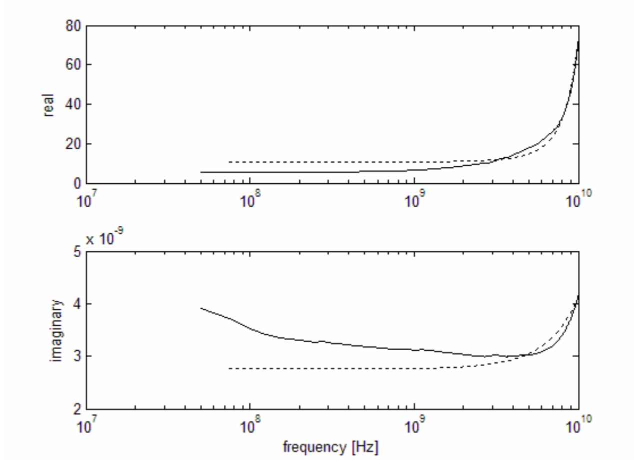

Fig. 16. Y3 real and imaginary values of the improved model (full line: measured data, dashed line: model).

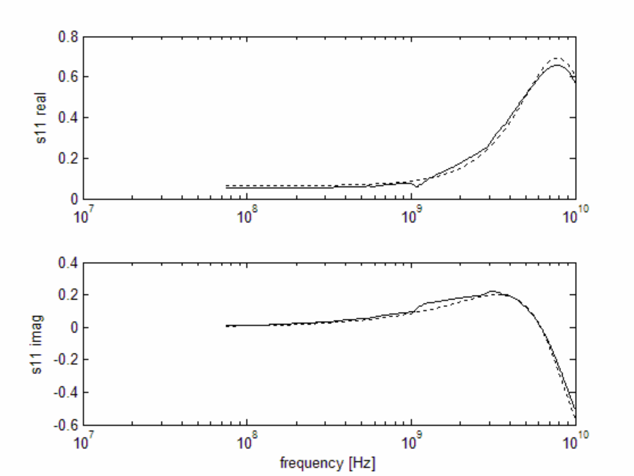

Fig. 17. Measured and modelled S_{11} of the improved model (full line: measured data, dashed line: model).

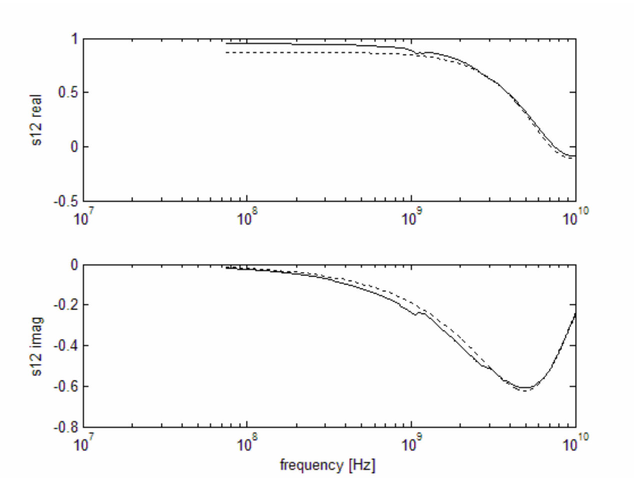

Fig. 18. Measured and modelled S_{12} of the improved model (full line: measured data, dashed line: model).

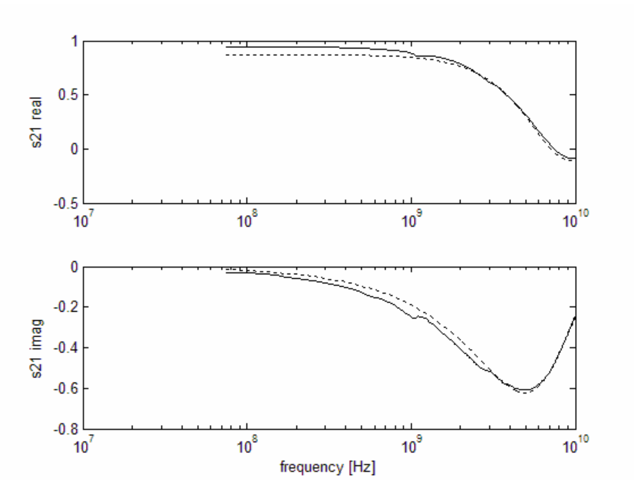

Fig. 19. Measured and modelled S_{21} of the improved model (full line: measured data, dashed line: model).

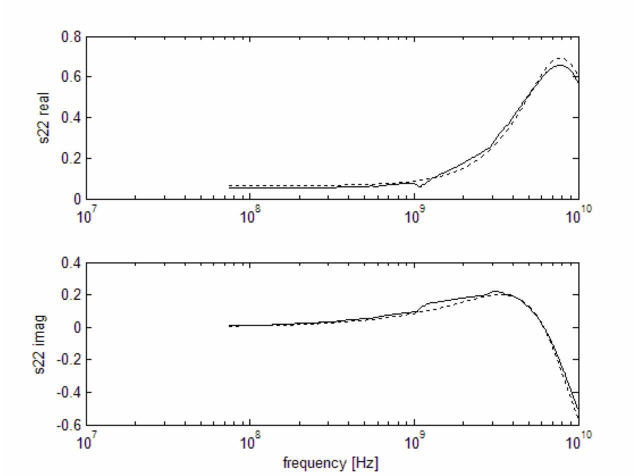

Fig. 20. Measured and modelled S_{22} of the improved model (full line: measured data, dashed line: model).

TABLE II
PARAMETER VALUES FOR THE INITIAL MODEL

Initial model	
R1	17.0 kΩ
C1	0.21 pF
R2	11.9 kΩ
C2	0.25 pF
R3	22.4 Ω
L3	3.22 nH

TABLE III
PARAMETER VALUES FOR THE IMPROVED MODEL

Improved model	
R1	17.0 kΩ
C1	0.21 pF
R2	11.9 kΩ
C2	0.25 pF
R31	10.67 Ω
L3	2.76 nH
R32	128 Ω
C3	37.6 fF

V. CONCLUSION

A MATLAB code that allows extraction of equivalent circuit parameters of on-chip inductors is developed. Two equivalent circuit models are investigated. The simpler one does not include the skin effect and substrate losses, and it shows reasonable agreement with measured data up to 1 GHz. Modelling of the skin effect and substrate losses is taken into account in the second model allowing it to be used up to 10 GHz. As part of the extraction procedure additional optimization of model parameters is performed in MATLAB.

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