Downstream Bit Rate Calculation for ADSL2+ Loops Limited with Far-end Crosstalk

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Abstract: In this paper an efficient method for downstream bit rate calculation on ADSL2+ local loops limited by crosstalk is presented. Crosstalk is one of the most limiting factors in broadband cable communications. In order to calculate bit rate of ADSL2+ loops we have done an in-depth analysis of a local telecom operator's cable infrastructure in terms of crosstalk.

On the basis of measurements carried out on twisted quad cables in a frequency range up to 2.2 MHz, we have derived theoretical models of far-end crosstalk (FEXT) and insertion gain. The measurements were performed on cables that are part of an operating infrastructure, not on cables on a reel, thus providing a true insight into the situation telecoms worldwide are facing today.

The results indicate that number and assignment of ADSL2+ to loops in a cable binder have a great impact on a bit rate. Strategy of allocating ADSL2+ to loops inside the cable binder reduces level of interference between users, allowing, in this case, traffic prediction and dynamic access to spectrum available in a cable. Presented bit rate calculation method and crosstalk models are crucial foundation for spectrum management used in a process of local loop unbundling, which is lawful obligation of numerous incumbent telecom operators.

1. INTRODUCTION

DSL (Digital Subscriber Line) systems are widespread transmission systems in cooper access networks. Today ADSL2+ (Asymmetric DSL) transmission systems guarantee the highest bit rates in downstream direction among DSL systems that support transmission up to 2.2 MHz. Also, there are a lot of barriers which limit higher bit rates of these systems. The major barrier for ADSL2+ systems using FDD (Frequency Division Duplex) is far-end crosstalk (FEXT) [1] [5] [6].

In this paper we describe a method of downstream bit rate calculation applicable to ADSL2+ systems limited by FEXT. In an ongoing project with a local telecom operator we have obtained insertion gain and crosstalk measurements that were performed on a deployed loop plant. The measurements were performed on loops between 300 m and 1700 m in length, on frequencies in a range from 20 kHz up to 2.2 MHz. Based on measurements, we have presented and analyzed insertion

gain and crosstalk 1% worst-case models which are important for transmission system design and service provisioning. The models can be used in any access network based on twisted quad cables. Finally, the bit rate calculation has been done taking into account several parameters; for instance, assignment of the transmission systems to loops inside the cable binder, bit loading table, and crosstalk.

There are three main contributions of this paper. First, we have presented a method for an empirical ADSL2+ local loop bit rate calculation. Second, crosstalk models used in this paper were developed on the basis of measurements performed by a local telecom operator, thus providing an insight into the real situation in a cable infrastructure used by numerous telecoms worldwide. Finally, the methods for bit rate calculation could be used as input for spectrum management that is necessary in a local loop unbundling environment, where several operators use the same cable infrastructure.

The paper is organized as follows. Section 2 describes some calculation prerequisites, such as cable description, cable filling strategies, and 1% worst case model. Section 3 explains how to normalize crosstalk to a specified loop length. In section 4 we describe the insertion gain and the crosstalk models for the reference loop length, based on measurements. Proposed bit rate calculation method is described in section 5, while results of downstream bit rate calculation are elaborated in section 0. Finally, some concluding remarks are given in section 0.

2. CALCULATION PREREQUISITES

2.1 Cable description

Measurements used as a basis for local loop analyses in this paper were performed on cables with polyethylene insulation and laminated polyethylene sheath. Such a cable consists of arbitrary number of basic groups (30, 60 or 100). Each basic group consists of five star quads, i.e. ten pairs. We do not make distinction between cables with different number of basic groups, because we do not focus on the interference between pairs in different basic groups. In Figure 1 a cross section of a basic group is shown. Pair numbering system shown in this figure will be used throughout the paper.



Figure 1 - Basic group cross section and pair numbering

2.2 Cable filling strategies

Conclusion derived from the Figure 1 is that the crosstalk from the pair 0 to the pair 1 is not the same as the crosstalk from the pair 0 to the pair 4. Consequently, it is not the same whether two ADSL2+ systems operate in the same quad, in adjacent quads or in quads that are separated by another quad. The scheme of assigning ADSL2+ to particular pairs in a cable binder is called cable filling scenario. To determine which of the possible cable filling scenarios is the best, and which on is the worst, we have proposed a method described in the following paragraph.

Table 1 - The best and the worst filling scenarios, the victim pair is pair 0

Filling ratio, [%]	Best case	Worst case
20	4	1
30	26	12
40	824	123
50	2468	8912
60	12467	89123
70	124568	891234
80	1 2 3 4 6 7 8	8912345

A victim pair is a pair on which other pairs in a basic group generate highest level of crosstalk. In our case we will assume that the victim pair is pair 0. Number of pairs in a basic group allocated to ADSL2+ system is called filling ratio and is expressed as a percentage of such pairs. For a given filling ratio, the algorithm, using simple recursion, tests all the possible combinations. As there are not many possible combinations, the method that we propose was sufficient for the task. If the algorithm finds several combinations that generate the same level of crosstalk on the victim pair, only the first one will be considered.

From the models we have assumed that the major amount of crosstalk power transfer occurs between pairs in the same quad. When interfering pairs are in the quads that are separated by another quad, the influence of one pair to another is minimal. On the basis of our method, the table showing the best and the worst basic group filling scenarios has been created (see Table 1).

2.3 1% worst-case model

Commonly used FEXT models are statistical models that correspond to 1% worst-case value. This means that in a given cable not more than 1% of its pairs will experience crosstalk that is worse than crosstalk defined by the model [5] [6].

To be able to create the model, we have to check if the measurements are distributed according to a normal distribution. Furthermore, we heave to determine for which value of x the area under a bell-shaped curve left of x equals to 99% of the total area under the curve. Using numerical calculation we have calculated this value as:

$$x = \mu + 2.33\sigma , \qquad (1)$$

where x is 1% worst-case model boundary, μ and σ are mean and standard deviation of the measurement results, respectively.

3. FAR-END CROSSTALK CALCULATION

This section shows the way we have calculated the crosstalk at a specified loop length. We have chosen that the reference loop length is 1000 m.

Far-end crosstalk (FEXT) is the major limiting factor when several ADSL2+ systems using FDD are applied in the same basic group in a cable. FEXT is very dependant on a loop length [8]. To correctly calculate crosstalk for different loop lengths we recommend the procedure based on the ETSI FEXT model, [1], i.e.:

$$\left| H_F(f,l,N) \right|^2 = N^{0.6} K_F f^2 l \left| H_c(f,l) \right|^2,$$
(2)

where N is the number of disturbers in a cable, K_F is constant (10^{-16.5} Hz⁻²km⁻¹), f is frequency in Hz, l is loop length in km, and $|H_c(f, l)|$ is magnitude of channel transfer function.

Quality of a crosstalk model greatly depends on a number of performed measurements. We have assumed that the majority of measurements is usually carried out on an average loop length, i.e. reference length. Therefore, we propose that crosstalk measured on a loop with arbitrary length l has to be converted to another value that would be measured on a loop with the reference length l_{0} , using expression:

$$\frac{\left|H_{F}(f,l,N)\right|^{2}}{\left|H_{F}(f,l_{0},N)\right|^{2}} = \frac{N^{0.6}}{N^{0.6}} \frac{K_{F}}{K_{F}} \frac{l}{l_{0}} \frac{f^{2}}{f^{2}} \frac{\left|H_{c}(f,l)\right|^{2}}{\left|H_{c}(f,l_{0})\right|^{2}}, \quad (3)$$

$$\frac{\left|H_{F}(f,l)\right|^{2}}{\left|H_{F}(f,l_{0})\right|^{2}} = \frac{l}{l_{0}} \frac{\left|H_{c}(f,l)\right|^{2}}{\left|H_{c}(f,l_{0})\right|^{2}},$$
(4)

$$\left|H_{F}(f,l)\right|^{2} = \left|H_{F}(f,l_{0})\right|^{2} \frac{l}{l_{0}} \frac{\left|H_{c}(f,l)\right|^{2}}{\left|H_{c}(f,l_{0})\right|^{2}}.$$
(5)

We have assumed that the magnitude squared of channel transfer function is [2]:

$$\left|H_{c}(f,l)\right|^{2} = e^{-2\alpha(f)l},\tag{6}$$

where *f* is frequency in Hz, *l* is loop length in km, and $\alpha(f)$ is local loop's attenuation constant in Np/km.

Applying Eq. (6) to the Eq. (5) gives following result:

$$\left|H_F(f,l)\right|^2 = \left|H_F(f,l_0)\right|^2 \frac{l}{l_0} \frac{e^{-2\alpha(f)l}}{e^{-2\alpha(f)l_0}}.$$
(7)

Evaluation of the Eq. (7) requires local loop attenuation model and FEXT models on the reference loop length (in our case it is 1000 m). These models are presented in subsections 4.1 and 4.2, respectively.

4. REFERENCE MODEL DESCRIPTION

In this section we describe the local loop attenuation and crosstalk models used to calculate maximum achievable downlink rates. We will also present the assumptions used in these calculations.

4.1 Attenuation model on the reference loop length

Applying previously described method on the data supplied by the local telecom operator, we have created graphs showing average value (*Figure 2*) and standard deviation (*Figure 3*) of local loop insertion gain on loops having reference length of 1000 meters.



Figure 2 - The average insertion gain

For frequencies lower than 0.4 MHz, fitted average value (avg) of the insertion gain can be described by

$$G(f)_{avg} = -7.58163 - 1.77301 \cdot 10^{-5} f \text{ [dB/km]}, \qquad (8)$$

where f is frequency in Hz. The appropriate model for frequencies between 0.4 MHz and 2.2 MHz is

$$G(f)_{avg} = -0.0226959\sqrt{f} \ [dB/km],$$
 (9)

where f is frequency in Hz.



Figure 3 - Standard deviation of the insertion gain

Fitted standard deviation (std) of the insertion gain can be modeled using the following equation:

$$G(f)_{std} = 0.621285 + 3.89672 \cdot 10^{-7} f \text{ [dB/km]},$$
 (10)

where f is frequency in Hz. *Figure 4* shows 1% worst-case model of the insertion gain. Fitted 1% worst-case model of the insertion gain can be expressed by the following equation:

$$G(f)_{1\%} = -0.0251356\sqrt{f} \ [\text{dB/km}],$$
 (11)

where f is frequency in Hz.



Figure 4 - The 1% worst-case model of the insertion gain

Finally, from the theory of transmission lines, the relationship between the local loop attenuation constant α and the local loop insertion gain *G* is the following [3]:

$$\alpha(f) = -G(f) \text{ [dB/km]}.$$
 (12)

It is important to notice that the Eq. (12) returns values in dB/km. This value cannot be used directly in Eq. (7), because it has to be converted to Np/km. This can be done by dividing the result from Eq. (12) by 8.686 (the amount is equal to $20\log_{10}(e)$).

4.2 Crosstalk models on the reference loop length

As it was mentioned earlier, there are three different crosstalk types in a basic cable group. To easily reference them in figures and further in the paper, we use the naming convention described in *Table 2*.

Table 2 - Crosstalk naming convention	Fabl	le	2	-	Crossta	ılk	naming	convention
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Name	Description
FEXT-I	crosstalk between pairs in the same quad
FEXT-N	crosstalk between pairs in the neighboring quads
FEXT-A	crosstalk between pairs in quads mutually separated by third quad

Figure 5 and *Figure 6* show average value and standard deviation of these three crosstalk types.





Average FEXT-I, FEXT-N and FEXT-A are fitted using the following equation:

$$H_{F-avg}(f) = A + B\log(f) \text{ [dB]}, \tag{13}$$

where *f* is frequency in Hz, and values of parameters *A* and *B* are defined in *Table 3*.

Table 5 - Consiants used in Eq. (15)				
FEXT Type	А	В		
FEXT-I	-167.175	15.4471		
FEXT-N	-153.537	13.2116		
FEXT-A	-163.827	14.0877		

Table 3 - Constants used in Eq. (13)

Standard deviation (std) of FEXT-N and FEXT-A is fitted using the following equation:

$$H_{F-std}(f) = A + B \left(\frac{f}{10^6}\right)^C \text{ [dB]},\tag{14}$$

where f is frequency in Hz, and values of parameters A, B and C are defined in *Table 4*.



Figure 6 - Standard deviation of crosstalk

Table 4 - Constants used in Eq. (14)

FEXT Type	А	В	С
FEXT-N	7.75133	2.22727	1.72396
FEXT-A	7.85878	1.3343	1.98894

Standard deviation (std) of FEXT-I is fitted using the following equation:

$$H_{F-std}(f) = A + B \left(\frac{f}{10^6}\right)^C - D \left(\frac{f}{10^6}\right)^E \text{ [dB]}, \qquad (15)$$

where values of parameters *A*, *B*, *C*, *D*, and *E* are 4.5801, 12.6787, 0.603598, 8.27186, and 0.956867, respectively.



Figure 7 - The 1% worst-case model of crosstalk

By closer examination of the *Figure 5* one can notice that the FEXT-I and FEXT-N have almost the same values. Ubiquitous FEXT models by ITU and ETSI are usually given for pairs, not for quads. The difference between our models derived from measurements and those by standardization bodies slightly differ. This leads to an important conclusion whish is: to have efficient crosstalk models it is necessary to make measurements on operating cable plant instead of making calculations on the basis of ubiquitous models by ITU or ETSI. The 1% worst-case model is shown in *Figure* 7. The equations for fitting the 1% worst-case model were obtained by summing the equations that fit the average crosstalk and the equations used for fitting standard deviation of the measurements.

5. DOWNSTREAM BIT RATE

The key point of our bit rate calculation method is according to the bit loading of the subchannels (tones). It is important to notice that downstream ADSL2+, each 4.3125 kHz wide, are located in frequency range between 276 kHz and 2.2 MHz. The subchannels are numbered from 65 to 512.

Let S_i and N_i denote signal and noise power on the subchannel *i* (65 \leq *i* \leq 512), respectively. Then, based on signal to noise ratio (SNR) on a subchannel *i* we have determined bit loading (b_i) on that subchannel using the *Table 5*, supplied by the local telecom operator. *Table 5* shows that each additional bit results in an SNR increase of 3 dB. As well, each ADSL2+ system uses maximum allowed power on each tone and complies with [4]. *Figure 8* shows the allocated power across all frequencies or subchannels. The crosstalk type is determined using the lookup table that stores all possible SNR combinations. The crosstalk is summed using the FSAN method [5].

Furthermore, knowing b_i on each subchannel ($65 \le i \le 512$) we have calculated the total downstream bit rate using the following equation

$$R = \frac{1}{T} \sum_{i=65}^{512} b_i \text{ [bit/s]}, \qquad (16)$$

where T is the ADSL2+ symbol duration (250 μ s).



Figure 8 - ADSL2+ downstream allocated power

Table 5 - SNR to bit loading convention

SNR, [dB]	bit loading
SNR < 21	0
$21 \leq \text{SNR} \leq 24$	1
$24 \leq SNR \leq 27$	2
$27 \leq \text{SNR} < 30$	3
$30 \leq \text{SNR} < 33$	4
$33 \leq \text{SNR} < 36$	5
$36 \leq SNR < 39$	6
$39 \leq \text{SNR} < 42$	7
$42 \leq SNR < 45$	8
$45 \leq SNR < 48$	9
$48 \leq \text{SNR} < 51$	10
$51 \leq \text{SNR} < 54$	11
$54 \le \text{SNR} < 57$	12
$57 \leq \text{SNR} < 60$	13
$60 \leq \text{SNR} < 63$	14
$SNR \ge 63$	15

6. **RESULTS**

We have performed the calculation of the filling ratio versus the lowest pair bit rate in a basic cable group. Furthermore, we have performed calculations of the worst achievable bit rate for different loop lengths and for different filling configurations. The worst achievable bit rate is a bit rate on the victim pair, which is defined in subsection 2.1. The results are shown in *Figure 9* and *Figure 10*.

It can be noticed that allocation of DSL systems in a basic group has the major impact on the achievable bit rates. For example, when a loop is 1 km long, and the filling ratio is 30%, the difference between the best and the worst allocation of systems causes the bit rate at the victim pair to decrease for almost 2 Mbit/s. Furthermore, by observing the achievable bit rate at the 10% filling ratio and the 20% filling ratio, it can be noticed that for the deployment of services with adequate quality of service (QoS) it is crucial to use at least static spectrum management (SSM), while dynamic spectrum management (DSM) provides much better system performance especially in an access network built up with remote terminals [9]. By using the maximum allowed powers, bit loading per tone at higher frequencies is below acceptable values. This effect is due to the crosstalk increase as a function of the frequency squared. Figure 9 and Figure 10 have been generated under the assumption that every system uses maximum allowed power.



Figure 10 - The best cable filling

7. CONCLUSION

The bit rate calculation method presented in this paper is used for copper quad cables and is based on simple parametric crosstalk models. It is a useful tool that can be used in a preparation for spectrum management of unbundled local loops. Local loop unbundling (LLU) process mandates all telecom operators using a common cable infrastructure to follow the same rules. This brings up the issue whether or not number of transmission systems and their positioning inside a cable binder has great impact on a bit rate of each loop.

The local telecom operator whose data were used in modeling presented in this paper has carried out an extensive measurements campaign in order to address this question. An important conclusion from the conveyed analysis is that the cable filling ratio and the allocation of the transmission systems inside the cable binder have a major impact on the achievable bit rate on each loop. For example, when a loop is 1000 m long, and the cable filling ratio is 20%, the difference between the best and the worst allocation of the transmission systems causes the difference in bit rates that amounts to 2.5 Mbit/s. Hence, our analysis indicates that the implementation

of capacity demanding services like IPTV necessitates spectrum management.

8. REFERENCES

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