Abstract
This paper establishes technical analysis of stock prices based on average trading prices. Analysis procedure begins with defining average prices on daily basis which are involved in stock market investment decisions for the first time in financial theory as well as in practice. Namely, all theoretical statements are confirmed by movements of Podravka stocks, as component of CROBEX index on Zagreb Stock Exchange. Using exponential smoothing methodology difference between short-term and long-term investment strategy is defined according to bull and bear signals.

Keywords: average trading prices, technical analysis, exponential weighted moving average method, rolling standard deviation, Bollinger's range, bull and bear signals

1. INTRODUCTION

The approaches used to analyze stocks and make investment decisions are divided into two categories: fundamental analysis and technical analysis. Fundamental analysis involves analyzing the characteristics of a company in order to estimate its "value". Technical analysis takes a completely different approach; it doesn't care about the "value" of a company. Technicians, sometimes called chartists, are only interested in the price movements in the market.

Technical analysis studies supply and demand in a market in an attempt to determine what direction, or trend, will continue in the future. Technical analysis is a method of evaluating stocks by analyzing the statistics of the past prices movements and volume. Therefore, it uses charts and other tools such as indicators and oscillators to identify patterns that can suggest future movements. Technical analysis relies on three basic assumptions:

• at any given time, a stock's price reflects everything that has or could affect the company - including fundamental factors;
• the repetitive nature of price movements is attributed to market psychology, i.e. market participants tend to provide a consistent reaction to similar market situations. It means that history tends to repeat itself;
• price movements are believed to follow trends.

2. PRICE MOVEMENTS IN TRENDS

In technical analysis it has been shown that after a trend of price movements has been established, the future price movement is more likely to be in the same direction. Most technical trading strategies are based on this assumption.

Empirical researches discover two types of trend distinguishing according to:

• time structure and
• general direction.

According to time structure there are long-term trends, intermediate trends or short-term trends. These are connected with investment strategies. Namely, there is a significant difference between an investor and a trader. It means that an investor expects profit only in long-term period, while traders prefer to profit in short-term period. So, it can be defined that
long-term investment strategy is associated within time frame of 50 trading days; intermediate strategy is adapted with 20 trading days, while short-term investment strategy is associated within 10 trading days. Furthermore, these ranges are suggested by John Bollinger.

According to general direction prices could trend up, trend down, or trend sideways. In financial literature synonym for up trend market is bull market. A bull market tends to be associated with increasing investor confidence, motivating investors to buy in anticipation of further capital earnings.

Technical term for down trend market is bear market. A bear market tends to be accompanied by widespread pessimism. Investors anticipating further losses are motivated to sell.

3. SMOOTHING TECHNIQUES AND ROLLING ESTIMATES

Smoothing techniques are very often used in financial literature in general. In this contest it will be used primarily as an indicator of "bullish" or "bearish" signs in the stock market. Precisely speaking, in this paper smoothing techniques will be used to decline stochastic variations. The simplest method of smoothing time series is simple moving average (SMA) method, which is given by:

$$\text{SMA}_1(k) = \frac{1}{k} (P_t + P_{t-1} + P_{t-2} + \ldots + P_{t-k+1}),$$

where $k$ is number of previous period for which prices are observed.

In other words, forecast value for a one period ahead is the simple average of current stock price and previous $(t-(k-1))$ prices. Choice of period $k$ depends on the particular purpose of the research. Namely, it has to be noticed that the larger the choice of $k$, the smoother the series will be.

The main disadvantage of SMA is all observations are equally weighted. To mitigate the effects of extreme observations on moving average estimates can be weighted differently. Therefore, a common procedure that puts more weight on the most recent observations is based on exponentially declining weights and the resulting weighted moving average is called exponential weighted moving average (EWMA).

According to exponential smoothing method forecast values could be calculated recursively:

$$\hat{P}_t = (1 - \lambda) \cdot P_t + \lambda \cdot \hat{P}_{t-1}$$

where $\hat{P}_t$ is present period forecast and $\hat{P}_{t-1}$ is previous period forecast.

By continuous substitution equation in (2) becomes:

$$\hat{P}_t = (1 - \lambda) \cdot P_t + \lambda(1 - \lambda) \cdot P_{t-1} + \lambda^2(1 - \lambda) \cdot P_{t-2} + \ldots
+ \lambda^i(1 - \lambda) \cdot P_{t-i} + \ldots$$

$$i = 0, 1, 2, \ldots, k-1.$$ (3)

In equation (3) parameter lambda $0 < \lambda < 1$ is called smoothing constant. When $k$ converges to infinity, relation (3) can be noticed as:

$$\hat{P}_t(\lambda) = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i P_{t-i}.$$ (4)

From relation (4) follows that $w_i \rightarrow 0$ when $\lambda^i \rightarrow 0$, according to:

$$\hat{P}_t(k) = \sum_{i=0}^{k-1} w_i P_{t-i}, \quad w_i = \frac{\lambda^{i-1}}{\sum_{i=0}^{k-1} \lambda^{i-1}}.$$ (5)
From equations (2), (3) and (4) it follows that the closer lambda is to unity the more weight is put on the previous period's estimate relative to the current period's observation. Therefore weights are decreasing in exponential manner. However, weights can decrease slowly or faster.

Classical approach of forecasting time series suggests lambda with minimal sum of squared errors. Because the interest of this paper is to describe pattern of a time series, this suggestion will not be strongly considered. Namely, for the smoothing purpose, parameter lambda can be estimated according to chosen period \( k \). This is in a close connection with time structures of investment strategies.

That's why parameter lambda can be estimated in follow way:

\[
\hat{\lambda} = \frac{k - 1}{k + 1}.
\]  

According to formula (6) lambda is real number from interval \((0, 1)\). The largest is period \( k \) the closer lambda is to unity. Therefore, weights are decreasing slowly and time series is smoother.

4. BOLLINGER BANDS

Bollinger Bands were created by John Bollinger. Bollinger Bands are plotted at standard deviation levels above and below a moving average (EWMA), where the standard deviation is a measure of volatility. In this paper the rolling standard deviation of relevant prices is used:

\[
SD_t = \hat{\sigma} \sqrt{\frac{k}{k - 1}}, \quad t = k, ..., n,
\]  

where \( k \) is number of periods within the rolling standard deviations are computed. In equation (7) factor \( \frac{k}{\sqrt{k - 1}} \) ensures unbiased estimation.

During periods of extreme price changes (i.e., high volatility), the bands indicate to divergence. During periods of stagnant pricing (i.e., low volatility), the bands narrow to contain prices. The longer prices remain within the narrow bands the more likely a price breakout. They are one of the most powerful concepts available to the technically based investor, but they do not, as is commonly believed, give absolute buy and sell signals. What they do is answer the question of whether prices are high or low on a relative basis. Using this information, an investor can make buy and sell decisions, confirming price action. Closing prices are most often used to compute Bollinger Bands, while other variations can also be used. For example the typical price (TP) is:

\[
TP = \frac{\text{high} + \text{low} + \text{close}}{3}.
\]  

The weighted price (WP) is defined:

\[
WP = \frac{\text{high} + \text{low} + 2 \cdot \text{close}}{4}.
\]  

In this paper for the first time we suggest so called the real weighted price (RWP):
\[ RWP_t = \frac{\sum_{i=1}^{m} p_i q_i}{\sum_{i=1}^{m} q_i} = \frac{\text{turnover}}{\text{volume}}, \quad \forall t, \quad (10) \]

where \( m \) is number of transactions in current trading day \( t \), while \( p_i \) is executive price of \( i \)-th transaction, and \( q_i \) is trading quantity. According to (8) and (9) equations the advantage of the real weighted price is in the fact that executive prices are weighted by trading quantity, i.e. executive price which is traded more has greater weight and vice versa.

Bollinger recommends using a 20-day simple moving average for the centre band and 2 standard deviations for the outer bands. The length of the moving average and number of deviations can be adjusted. In this paper the lengths of the moving average are 10 and 50 respectively and numbers of deviations are 1.5 and 2.5 according to comparison of the short-term and long-term investment strategies.

It can be concluded that Bollinger Bands serve two primary functions:
- to identify periods of high and low volatility, and
- to identify periods when prices are at extreme levels.

Even so, a security can become overbought or oversold for an extended period of time. Knowing whether or not prices are high or low on a relative basis can enhance the interpretation of other indicators and oscillators. Therefore, the relative strength index (RSI) is used. The RSI is a technical analysis oscillator showing price strength by comparing upward and downward movements.

\[
RSI = 100 - \left( \frac{100}{1 + \frac{U}{D}} \right), \quad (11)
\]

where \( U \) is an absolute value of the moving average of upward executive price change, and \( D \) is an absolute value of the moving average of downward executive price change. The RSI is oscillator that ranges between 0 and 100. In situation when RSI reach the 70% a security is considered to be overbought, or oversold at the level of 30%. Generally, if the RSI rises above 30% it is considered bullish for the underlying stock. Conversely, if the RSI falls below 70%, it is a bearish signal. The centreline for RSI is 50%. Levels of 80% and 20% are also used.

5. TECHNICAL ANALYSIS IN CROATIA

The complete procedure of presented technical analysis is established using observations of Podravka stocks as the most frequently traded stock from CROBEX index at Zagreb Stock Exchange.

Figure 1 shows by the one line rolling standard deviation movements from the point of investor's view (long-term trading periods) and the other line are the movements from the point of trader's view (short-term trading periods). Representative for long term periods is rolling standard deviation for time frame of 50 trading days and representative for short term periods is rolling standard deviation of 10 trading days.

Estimated according to real weighted prices short term Bollinger Bands are presented at Figure 2.
Figure 1.
Rolling standard deviation estimates for short-term and long-term trading periods

![Rolling standard deviation estimates for short-term and long-term trading periods](image)

Source: According to data on www.zse.hr

Figure 2.
Short-term Bollinger Bands estimates according to real weighted prices

![Short-term Bollinger Bands estimates according to real weighted prices](image)

Source: According to data on www.zse.hr

Long-term Bollinger Bands estimated according to real weighted prices are illustrated at Figure 3.

Figure 3.
Long-term Bollinger Bands estimates according to real weighted prices

![Long-term Bollinger Bands estimates according to real weighted prices](image)

Source: According to data on www.zse.hr

Apart from various indicators technical analysis requires measurements using oscillators.
An example of oscillator relative strength index is used in this analysis. Figure 4.

Table 1.

<table>
<thead>
<tr>
<th>Short-term strategy</th>
<th>Long-term strategy</th>
<th>Trading days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>buy signal</td>
<td>no signal</td>
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At the end the final decision to buy or to sell depends on the concrete investor (trader) preference to risk more in order to earn more. Apart from technical analysis results for such a decision additional capital market information will be used.

REFERENCES

3. The Zagreb Stock Exchange: http://www.zse.hr