ATTRACTOR OF SERIES RESONANT DC/DC CONVERTER

16th Int. Conference on
Electrical Drives
and
Power Electronics
Slovakia
September 24 – 26, 2007

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Abstract. The state trajectory from starting to the stationary state is simulated and in stationary state analytically calculated. Natural state variables for this circuit are inductor current and capacitor voltage. It was shown that the normalized trajectories in stationary state form one closed loop, so called attractor, composed of four different circular segments. After verifying simulation model, the influence of the quality factor of the resonant circuit on the duration of starting process is examined. By introducing small perturbations in the stationary state, the inherent stability of power circuit is demonstrated. The analysis results can be used for control circuit design.

Keywords: analysis, power electronics, resonant DC/DC converter, simulation, stability

1. INTRODUCTION

Series resonant DC/DC converters link two DC systems. The DC output from a resonant converter is obtained by inserting a bridge rectifier. Due to the action of the bridge rectifier an analytic analysis of power circuit properties is rather complicated and does not provide appropriate insight into the dynamic behaviour of power circuit.

Awareness of dynamic behaviour is essential for understanding operation and properties of the converter and right choice of components for the oscillating circuit. The system dynamics can be analyzed on the basis of a graphic representation of system response in dependence of time. This is the most frequent method used in the analysis of circuit response [1].

The second method of analysis is based on graphic representation of system response in state-space models where state variables are coordinate axes. The system response from the initial point in space moves along state trajectory. Time is an implicit variable along state trajectory. In case of the second-order systems, the state-space deforms into a plane, which makes it easy to represent trajectories graphically [2].

In all its topological states a resonant DC/DC converter is basically a second-order system and can be used as a good example for analysis of dynamic behavior based on state-plane representations. The paper analyzes influence if circuit elements changes: inductance L, resistance R, load V_2 and initial conditions for response dynamics, and time needed to reach periodic steady state in which trajectory would be a closed curve which is called attractor.

2. EQUIVALENT CIRCUIT DIAGRAM AND ANALYTICAL CALCULATION OF STEADY STATE

The circuit diagram in Fig. 1 represents a power circuit of a series resonant DC/DC converter. The input DC source and the switching circuit that modulates the voltage of the DC input source are not shown in the picture.

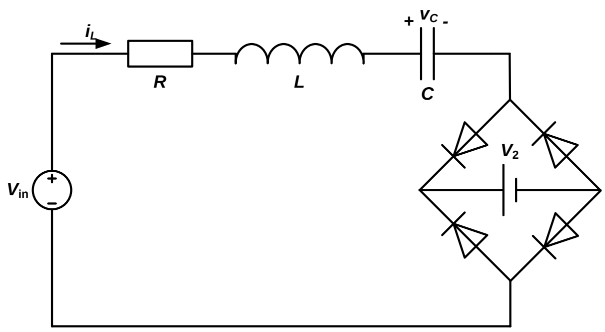

Fig. 1. Circuit diagram of a series resonant DC/DC converter

The source v_{in} supplies the square-wave voltage of fixed amplitude V_1 and frequency $f_s = 1/T$, which is controllable as shown in Fig. 2.

After the source voltage is applied to the converter, a nearly sinusoidal current i_L flows through the inductor L. The current is rectified by the diode bridge rectifier and passed to the load. The voltage on the capacitor C is changed due to the current i_L.

The current amplitude of the resonant circuit i.e. the average current of the load depends on the difference of source frequency f_s and resonant frequency f_o. By controlling the frequency of the square wave voltage f_s we control the average current delivered to the load. The load is represented by a DC voltage source of value V_2.

It is important to point out that the circuit goes through four configurations, which depend on source voltage polarity and source current polarity. When the source voltage is positive, and current flows from the source ($i_L > 0$) the load voltage is subtracted from the source voltage. When the current i_L flows into the source ($i_L < 0$), the load voltage is added to the source voltage. The same is true for the negative source voltage. Accordingly, the voltage applied to the resonant *RLC*-circuit changes its value four times during each cycle and equals to $\pm V_1 \pm V_2$, as shown in Fig. 2.

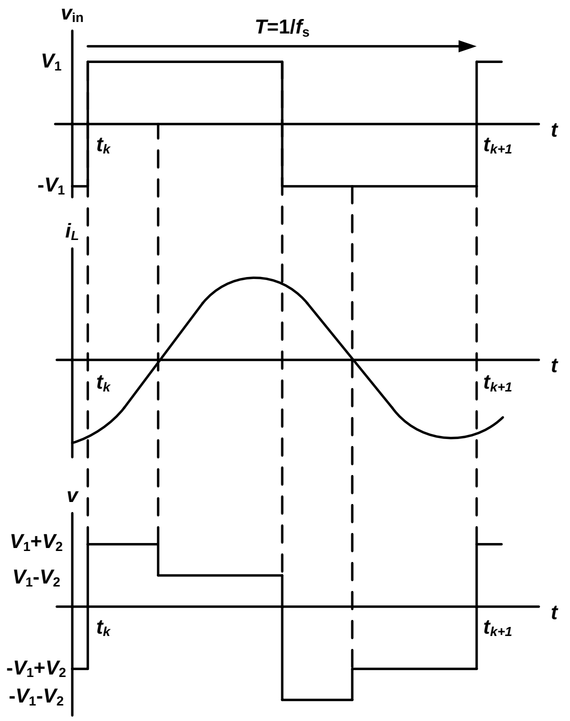

Fig. 2. Voltage waveforms v_{in} and V and current waveform i_L

An analytical state analysis of the resonant DC/DC converter requires a mathematical model of the circuit diagram.

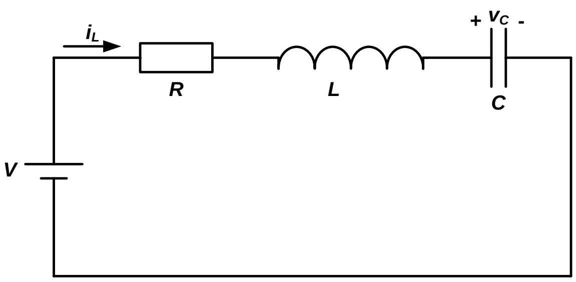

Fig. 3. Series *RLC*-circuit

For the series *RLC*-circuit shown in Fig. 3, it applies:

$$V = Ri_L(t) + L\frac{di_L(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i_L(t)dt \quad (1)$$

Second-order differential equation gives analytical solutions of the system:

$$\begin{aligned} i_L(t) = &\left[\frac{V - v_{C0}}{L\omega_o} - i_{L0}\frac{R}{2L\omega_o}\right] e^{-\frac{R}{2L}t}\sin(\omega_o t) + \\ &+ i_{L0}\, e^{-\frac{R}{2L}t}\cos(\omega_o t) \end{aligned} \quad (2)$$

$$\begin{aligned} v_C(t) = &V - \left[(V - v_{C0})\frac{R}{2L\omega_o} - i_{L0}\left(\frac{R^2}{2L\omega_o} + L\omega_o\right)\right] e^{-\frac{R}{2L}t}\sin(\omega_o t) - \\ &-(V - v_{C0})e^{-\frac{R}{2L}t}\cos(\omega_o t) \end{aligned} \quad (3)$$

where $\omega_o = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$ is the circuit resonant frequency.

When $R = 0$, the calculation is simplified and the following solutions are obtained:

$$i_L(t) = \frac{V - v_{C0}}{L\omega_o}\sin(\omega_o t) + i_{L0}\cos(\omega_o t) \quad (4)$$

$$v_C(t) = V(1 - \cos(\omega_o t)) + v_{C0}\cos(\omega_o t) + i_{L0}L\omega_o\sin(\omega_o t) \quad (5)$$

where $\omega_o = \sqrt{\frac{1}{LC}}$ is the circuit resonant frequency.

Analytical expressions are valid for each topological state, therefore, the end of each cycle is the start of the next cycle. Thus initial conditions v_{C0} and i_{L0} are determined per each cycle.

3. STATE VARIABLES TRAJECTORY ON THE STATE PLANE

Representation on the state plane provides a better insight in dynamic behaviour of the second-order circuits, thus for further analysis of the circuit behaviour, the circuit responses are represented by the state variables trajectories on the state plane.

The coordinates in a representation of state variables trajectories on the state plane are the state variables – inductor current and capacitor voltage. The sinusoidal inductor current waveforms i_L and capacitor voltage v_C for the resonant converter become piecewise elliptical trajectories on the state plane. If instead of state variables the scaled quantities $x = \sqrt{L}i_L$ and $y = \sqrt{C}v_C$ are used as state variables, the elliptical trajectories become piecewise circular arcs.

This can be easily mathematically verified, e.g. for an *LC*-circuit we have:

$$\sqrt{L}i_L(t) = \sqrt{L}\frac{V - v_{C0}}{L\omega_o}\sin(\omega_o t) + \sqrt{L}i_{L0}\cos(\omega_o t) \quad (6)$$

$$\begin{aligned} \sqrt{C}v_C(t) = &\sqrt{C}V(1 - \cos(\omega_o t)) + \sqrt{C}v_{C0}\cos(\omega_o t) \\ &+ \sqrt{C}i_{L0}L\omega_o\sin(\omega_o t) \end{aligned} \quad (7)$$

A simple mathematical analysis where $\omega_o = \sqrt{\frac{1}{LC}}$, shows that:

$$\left[\sqrt{L}i_L(t)\right]^2 + \left[\sqrt{C}(V - v_C(t))\right]^2 = \left[\sqrt{C}(V - v_{C0})\right]^2 + \left[\sqrt{L}i_{L0}\right]^2 \quad (8)$$

The radiuses of circular arcs are determined by the voltage applied to the *RLC*-circuit and values of i_{L0} and v_{C0} at the beginning of each cycle.

The circular arcs are centered at points the coordinates of which are determined by the mean values of the sinusoids i_L i v_C during the intervals corresponding to these arcs. Accordingly, the center of each arc has an x coordinate of 0 and its y coordinate takes one of the four values $\sqrt{C}(\pm V_1 \pm V_2)$.

Trajectory in Fig. 4 represents the behaviour of the series resonant DC/DC converter in Fig. 1, during the first cycles of operation. The circuit response is simulated with values in Tab. 1:

Tab. 1. Given circuit elements values

L, mH	C, μF	R, Ω	T, ms	i_{L0}, A	v_C, V	V_1, V	V_2, V
25	127	0	10	0	0	12	4

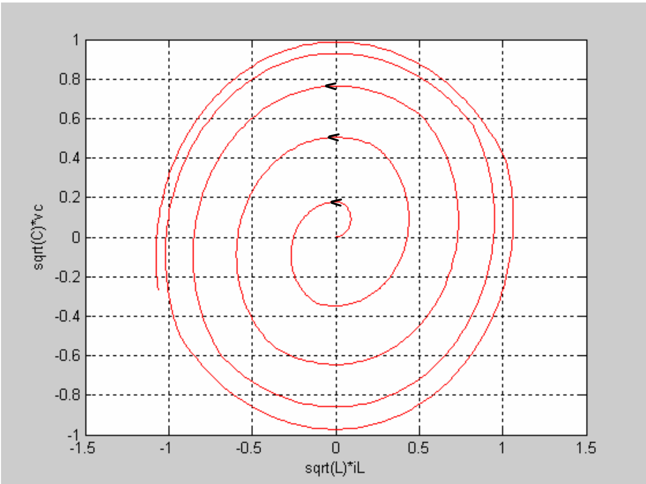

Fig. 4. State variables trajectory on the state plane – the first 5 cycles

Since the voltage v_C increases when the current i_L is positive, the arcs are traversed counter clockwise with angular velocity $\omega_o = 2\pi f_o$. This trajectory is not a closed loop, because the state at the end of the cycle does not equal the state at the beginning of the cycle, i.e. the analyzed circuit does not obtain periodic steady state at given time.

After transition state, the duration of which depends on the circuit elements, the system turns into periodic steady state.

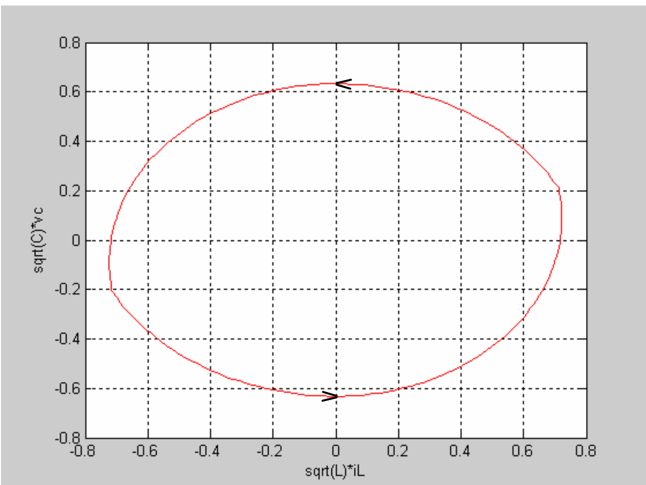

Fig. 5. Attractor of series resonant DC/DC converter

In the nominal periodic steady state, the trajectory is a closed loop, because the state at the end of the cycle equals the state at the beginning. Such a closed trajectory is called attractor and it is shown in Fig. 5. With the given circuit elements, the periodic steady state is obtained during 40 cycles.

4. INFLUENCE OF INDUCTANCE, RESISTANCE AND LOAD CHANGES ON CIRCUIT FEATURES

The features of DC/DC resonant converter when the resonant frequency f_o is lower than the source frequency f_s are analyzed.

<u>Change of inductance L</u>

In the system with no losses (R = 0), the resonant frequency of the resonant circuit is decreased, i.e. it is 'distanced' from the source frequency due to the increase of the inductance L. Thus the circuit impedance increases while the inductor current and capacitor voltage decrease.

Tab. 2 and Fig. 6 show the influence of inductance change on the circuit response for the following elements: C = 127 μF, R = 0 Ω, T = 10 ms, V_1 = 12 V, V_2 = 4 V. Column 'No. of cycles' represents number of cycles required for the system to turn into periodic steady state.

Tab. 2. Influence of inductance change

L, mH	ω_o, rad/s	I_{max}, A	$V_{C\,max}$, V	No. of cycles
25	561,21	4,55	55,95	41
30	512,32	2,33	27,85	21

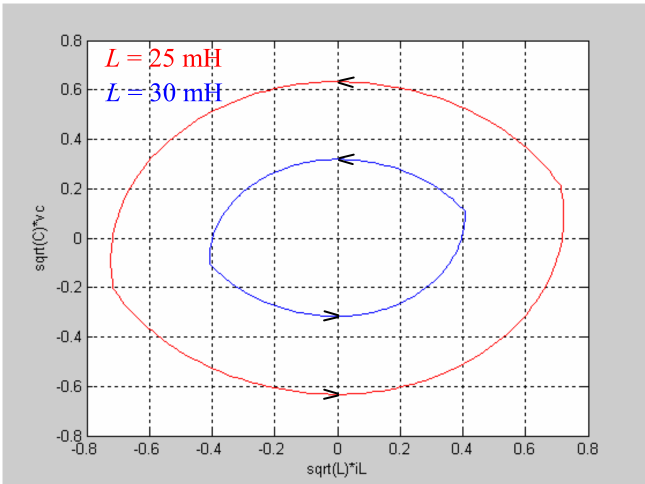

Fig. 6. Influence of inductance change L on the circuit behaviour

<u>Change of resistance R</u>

In the system where $R \neq 0$, the resistance increase causes the decrease of the resonant frequency f_o, faster suppression of resonance as well as the decrease of the inductor current and the capacitor voltage.

Tab. 3 shows the influence of the resistance change on the circuit response for the following elements: $L = 30$ mH, $C = 127$ µF, $T = 10$ ms, $V_1 = 12$ V, $V_2 = 4$ V.

Tab. 3. Influence of the resistance change

R, Ω	ω_o, rad/s	I_{max}, A	$V_{C\,max}$, V	No. of cycles
3	558,00	2,48	32,78	7
10	524,37	0,94	12,96	3

Fig. 7 shows the influence of the resistance change R on the circuit behavior during the first 3 cycles, and Fig. 8 shows the circuit behavior in the periodic steady state for different resistances.

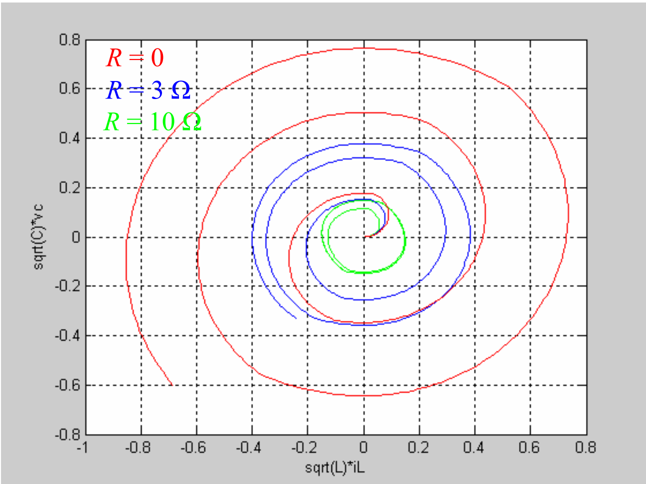

Fig. 7. Circuit behavior during the first three cyles after connection to the supplies for different resistances R

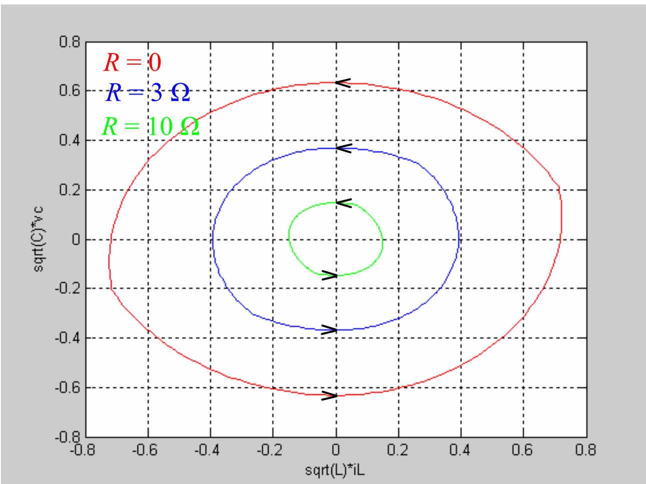

Fig. 8. Influence of the resistance change R on the circuit behavior

Change of load V_2

The load in the circuit has the same role as a dumping element. The load with a higher mean power has a stronger suppression effect in the circuit. Consequently, even the circuit without resistance achieves the periodic steady state. The circuit without both the load and the resistance would never achieve the steady state.

The Tab. 4 shows the influence of the load change on the circuit response for the following elements: $L = 25$ mH, $C = 127$ µF, $R = 0$ Ω, $T = 10$ ms, $V_1 = 12$ V.

Tab. 4. Influence of the load change

V_2, V	ω_o, rad/s	I_{max}, A	$V_{C\,max}$, V	No. of cycles
7	561,21	3,69	46,78	21
10	561,21	5,22	29,09	11

Fig. 9 shows the influence of the load change V_2 on the circuit behaviour during the first three cycles after the series resonant DC/DC converter is connected to the supplies, and Fig. 10 shows one cycle in the periodic steady state for different loads V_2.

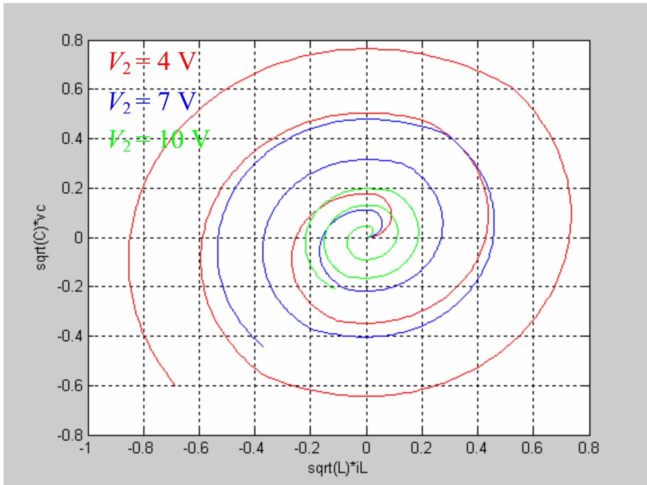

Fig. 9. Circuit behaviour during the first 3 cycles after connection to the supplies for different loads V_2

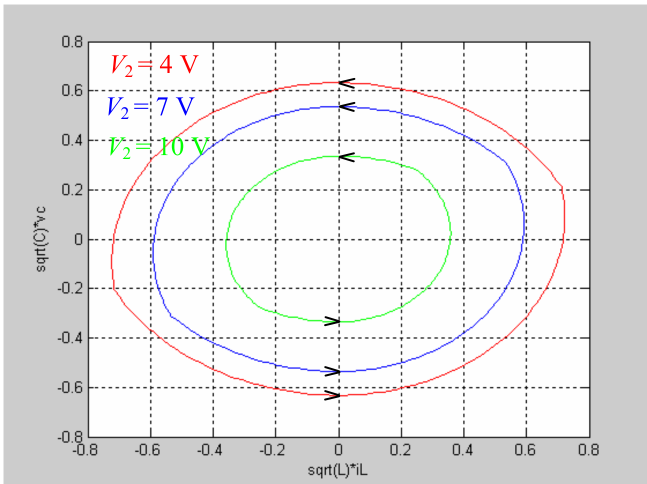

Fig. 10. Influence of load change V_2 on the circuit behaviour

5. INFLUENCE OF INITIAL CONDITIONS AND THE DISTURBANCES IN THE CIRCUIT ON THE CIRCUIT BEHAVIOR

Influence of initial conditions

The analysis of the behaviour of the DC/DC resonant converter can be divided into an initial cycle when the system goes from the initial state to the periodic steady state and the cycle of operation known as the periodic steady state. During the initial cycle, right after the converter is connected to the supply source, the inductor current and the capacitor voltage depend on the source voltage and the energy in the circuit at the moment of connection, i.e. at the beginning of observation.

If the beginning of each cycle of the source voltage is considered to be the beginning of observation, the initial state is actually the state at the end of the previous cycle. If the state at the end of the cycle is different from the state at the beginning of the cycle, the system is not in the periodic steady state. It can be said that the system is in the transient state.

For the circuit with given elements, if initial conditions are zero, it takes approximately 40 cycles of the voltage source to obtain the periodic steady state. During the transient state, the maximum capacitor voltage and the maximum inductor current exceed the voltages and currents in the periodic steady state.

This is best seen in Fig. 11. and Fig. 12. For clarity; only the first ten cycles on the state plane are represented in Fig. 11, and in Fig. 12 the current values are scaled by 10.

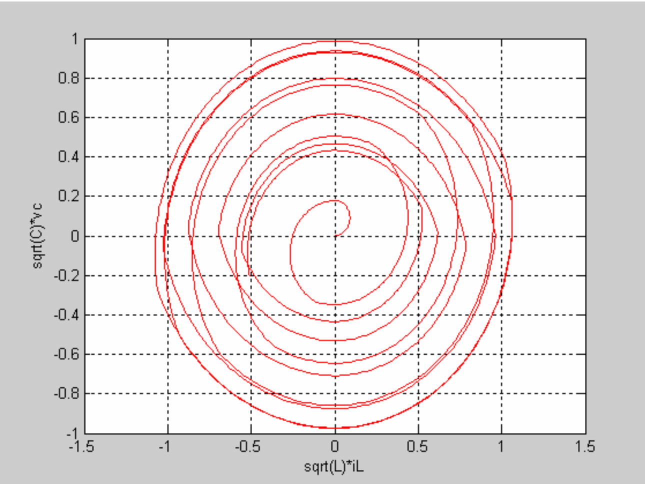

Fig. 11. State variables trajectory on the state plane – the first 10 cycles

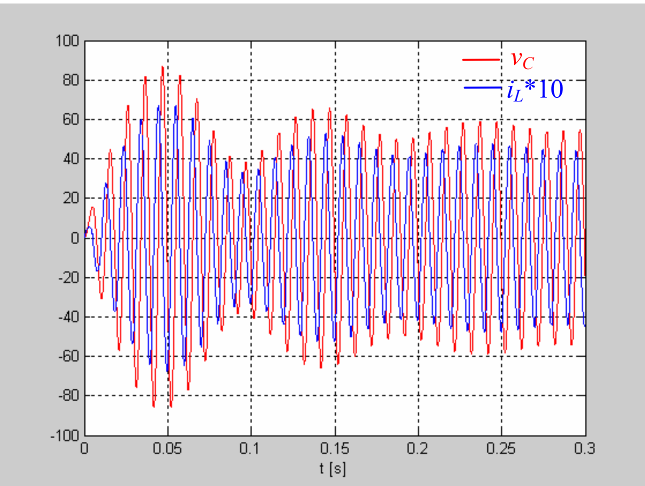

Fig. 12. Time intervals of inductor current i_L and capacitor voltage v_C – the first 30 cycles

If the initial conditions are approximately equal to the conditions at the beginning of the cycle in the periodic steady state, the circuit changes instantaneously into the periodic steady state, as it is shown in Fig. 13.

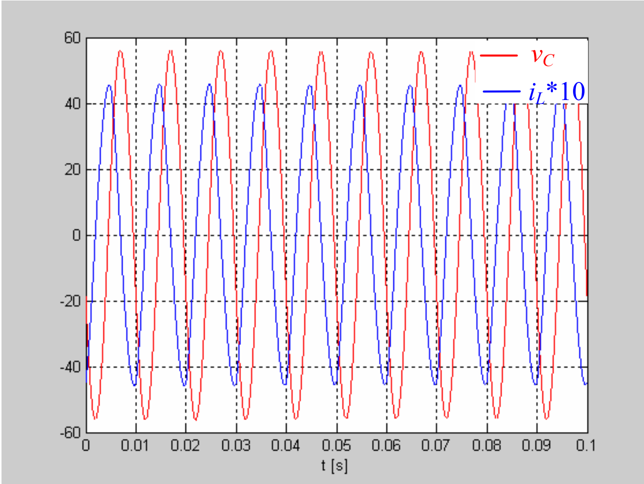

Fig. 13. Circuit response given the initial conditions equal those obtained at the beginning of the cycle in the periodic steady state: i_{L0} = – 4,49, v_{C0} = – 18,77 – the first 10 cycles

If the given initial conditions deviate from the final conditions less at the beginning, the transient state is shorter. The simulations of different initial conditions prove this.

<u>Disturbances in the periodic steady state</u>

If there are any disturbances in the circuit in the periodic steady state, e.g. due to the capacitor voltage perturbation, the system resonates and after a certain time it goes back to the periodic steady state.

The time to reassume the periodic steady state depends on intensity of disturbances and suppressions in the circuit. Fig. 14 shows the change of the trajectory, which happens when a certain energy is given or taken from the steady state capacitor voltage. If the energy of the capacitor is increased by 30% at the beginning of the cycle, the system goes from point A to point B. If the energy of the capacitor is decreased by 30%, the system goes from A to C. Fig. 14 shows that the mean trajectory is an attractor in the periodic steady state. The external trajectory represents the first cycle after the energy is increased, and the internal trajectory represents the first cycle after the energy is decreased.

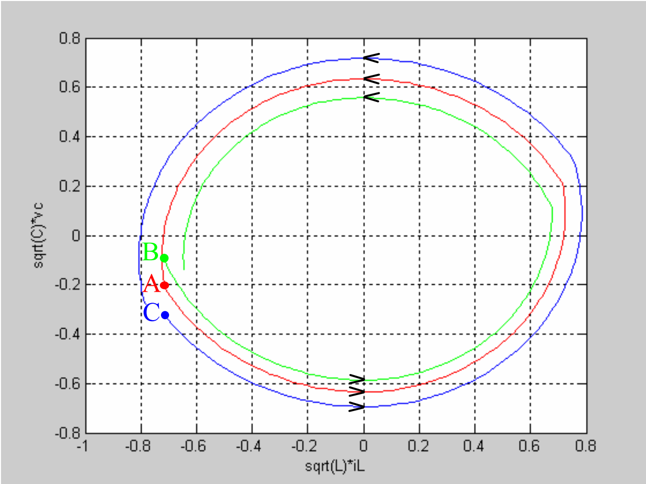

Fig. 14. State trajectories of the series resonant DC/DC converter during the first cycle after the capacitor voltage perturbation

6. CONCLUSION

Dynamic behavior of a series resonant DC/DC converter is simulated by the programme package MATLAB and the state variables trajectories on the state plane are analyzed.
It is shown that after a transient state whose duration depends on the circuit elements, the system goes to the periodic steady state. In the nominal periodic steady state the trajectory is a closed loop, because the state at the end of the cycle equals the state at the beginning of the cycle. Such a closed trajectory is an attractor that consists of four circular arcs, if the state variables are scaled.
Changes of circuit elements: values of inductance L, resistance R, and load V_2 are analyzed. It is shown that the increase of these elements induces the decrease of the inductor current and the capacitor voltage, faster suppression of resonance and coming up to the periodic steady state, e.g. for the $L = 30$ mH, $C = 127$ µF, $R = 3$ Ω, $T = 10$ ms, $V_1 = 12$ V, $V_2 = 4$ V it is required 7 cycles for the system to turn into periodic steady state.
The dynamics response is influenced by initial conditions. If the given initial conditions deviate from the final conditions less at the beginning, i.e. at the end of the cycle in the periodic steady state, the transition state is shorter. The disturbance appearing in the periodic steady state switches temporarily off the system from the periodic steady state, and the recovery time depends on the strength of disturbances.
The system of a resonant DC/DC converter is inherently stable (as a DC motor). Upon switching on it assumes the steady state. If there are any disturbances in the circuit in the periodic steady state, the system resonates and after a few cycles it goes back to the periodic steady state.
It can be concluded that the representation of the state by means of trajectories provides right answers to many questions and enables analysis of the converter operation at different conditions and right choice of the components for the converter.
Next step is to introduce the non-linearity of inductance and explore the chaotic behavior and to compute transfer functions.

7. REFERENCES

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