Modeling and simulation of PEM fuel cell – power converter system

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Abstract – Parasitic capacitances of Proton Exchange Membrane (PEM) fuel cell are causing electrical effects resulting with change of dynamic behavior of fuel cell stack output voltage. This paper shows PEM fuel cell dynamic model with capability of easy integration of humidity, temperature and pressures dynamics, as well as their control. Fuel cell stack dynamic model was linked with boost converter averaged dynamic model, obtained using state space method, containing controller that keeps converter output voltage constant. A variety of step load changes was simulated on this structure, resulting effects were observed on stack current, stack voltage and converter output voltage responses. Obtained results are showing difference between corresponding responses of this model and the one which neglects effects of parasitic capacitances of PEM fuel cell.

I. INTRODUCTION

Direct electrochemical transformation with efficiency of up to 45 %, high energy density out of small dimensions (up to 2 W/cm^2), silent operation and zero gas emissions are attributes that cause large interest in PEM fuel cells and are reasons that application of this technology is considered in transport, static production of electric energy and power supply for wireless electrical devices. But for designing a good power source based on this technology, fuel cell behavior, its characteristics, connected power converter and its control, their interaction, as well as mathematical models used to describe and simulate different operation modes, have to be well known. Only then, satisfactory system behavior and energy quality can be achieved.

Today a large number of mathematical PEM fuel cell models exists, whose purpose spreads from fuel cell designing, across describing static and dynamic behavior, up to operation analysis in complex systems, where fuel cells have to meet specific work conditions. Most of dynamic models are dealing with temperature and fluid pressures, because these variables are known to have time constants that can last even few seconds, while influence of pararasitic capacitances on output cell voltage is often neglected. Models that take into account this effect [4] usually use highly evolved fuel cell equivalent electrical circuits composed of resistance-capacitance parallels connected in series. These complex structures are describing high frequency electric effects very precisely, but determination of their capacitance values demands additional electric measurement on a real fuel cell, and presents severe problem. That is the reason why simplified model, that has only one time constant, was used in this case, to describe electric dynamic of PEM. In this model, all parasitic capacitances are represented with one connected in parallel with activation and concentration resistances of fuel cell.

As far as power converters are concerned, papers can be found that focus on power converters whose characteristics and regulation techniques have been customized to suit PEM fuel cells. Some of them deal with designs that allow cheap production [5], while others, based on good knowledge of PEM characteristics, describe complex structures resulting with high energy quality, by using hybrid PEM-battery power source [6]. As these models are often customized for certain purpose (for instance automotive applications), they are not suitable for studying of fuel cell operation effects on power converter operation, neither for analyzing effects caused by different controllers. For those reasons, appropriate boost power converter dynamic model is used here, that when connected to a fuel cell model, gives stack current, stack voltage and converter output voltage responses in short simulation execution time. In that way, a system suitable for interaction between PEM fuel cell and power converter was obtained.

II. FUEL CELL MODEL

PEM fuel cell electrochemical process starts on the anode side (Fig 1.) where H_2 molecules are brought by flow plate channels. Anode catalyst divides hydrogen on protons H^+ that travel to cathode through membrane and electrons e^- that travel to cathode over external electrical circuit. At the cathode hydrogen protons H^+ and electrons e^- combine with oxygen O_2 by use of catalyst, to form water H_2O and heat. Described reactions can be expressed using equations:

$$H_2 \rightarrow 2H^+ + 2e^- \text{ (Anode)}, \tag{1}$$

$$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2 0 \text{ (Cathode)}. \tag{2}$$

Amount of chemical energy released in these reactions depends on hydrogen pressure, oxygen pressure and fuel cell temperature. Using change in Gibbs free energy, this amount can be expressed as:

$$\Delta g_f = \Delta g_f^0 - RT_{fc}\left[\ln(p_{H2}) + \frac{1}{2}\ln(p_{O2})\right], \tag{3}$$

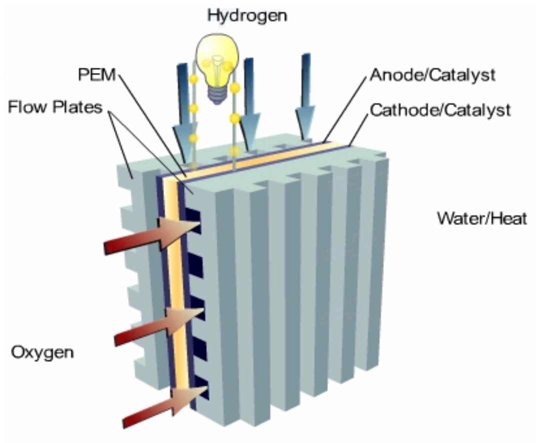

Fig. 1. Fuel cell image

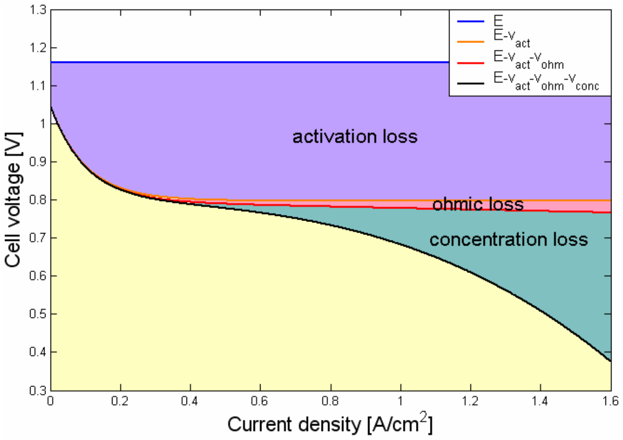

Fig. 2. Fuel cell losses

where Δg^0_f is change in Gibbs free energy at standard pressure, R universal gas constant, T_{fc} PEM temperature and p_{O2} and P_{H2} are gas pressures. Because electrical work done by fuel cell is equivalent to released chemical energy, value of open circuit fuel cell voltage E meets equation:

$$E = -\frac{\Delta g_f}{2F}, \tag{4}$$

where F is Faraday's constant.

To attain actual cell voltage (on electrical couplings) v_{fc}, voltage drops caused by activation, concentration and ohmic losses have to be deducted from open circuit voltage (Fig 2).

Cathode and anode activation losses are result of breaking and forming electron-proton chemical bonds, and parasitic electrochemical reactions caused from hydrogen proton migration through membrane at zero current. Their voltage drop was calculated using formula:

$$v_{act} = v_0 + v_a\left(1 - e^{-c_1 i}\right), \tag{5}$$

where activation voltage drop at zero current density v_0 depends on fuel cell temperature, cathode pressure and water saturation pressure $v_0 = f\left(T_{fc}, p_{ca}, p_{sat}\right)$. Voltage drop v_a inserts in (5) correlation with current density i and depends on fuel cell temperature, oxygen pressure and water saturation pressure $v_a = f\left(T_{fc}, p_{O2}, p_{sat}\right)$, and c_1 is activation voltage constant.

Concentration losses are caused by drop in reactant concentration due to dynamic flow problems between water and oxygen on cathode side, and also electro-osmotic water drag that occurs when protons travel through membrane. Voltage drop caused by these losses is described with equation:

$$v_{conc} = i \cdot \left(c_2 \frac{i}{i_{\max}}\right)^{c_3}, \tag{6}$$

where i_{max} represents current density that causes steep PEM voltage drop, parameter c_2 is function of temperature, oxygen pressure and water saturation pressure, $c_2 = f\left(T_{fc}, p_{O2}, p_{sat}\right)$, and c_3 is concentration voltage constant.

Ohmic losses are derived from membrane resistance R_{ohm} whose value depends of membrane thickness t_m, fuel cell temperature T_{fc} and membrane water content degree λ_m:

$$R_{ohm} = \frac{t_m}{\sigma_m}, \tag{7}$$

$$\sigma_m = \left(b_{11}\lambda_m - b_{12}\right)\exp\left[b_2\left(\frac{1}{303} - \frac{1}{T_{fc}}\right)\right]. \tag{8}$$

Value σ_m in (8) represents specific membrane conductivity and b_{11}, b_{12} and b_2 are membrane conductivity constants. Voltage drop of ohmic losses is expressed as:

$$v_{ohm} = R_{ohm} \cdot i. \tag{9}$$

Using calculated voltage drops and open circuit cell voltage, value v_{fc} of actual cell voltage in static condition can be attained using equation:

$$v_{fc} = E - v_{act} - v_{conc} - v_{ohm}. \tag{10}$$

Dynamic electric model was gained by implementing influence of parasitic capacitance C in previously described static model. Fuel cell equivalent electric circuit with capacitance C is shown in figure 3. On it R_{act} is resistance that corresponds to activation losses, and R_{conc} resistance that represents concentration losses:

$$R_{act} = \frac{v_{act}}{i} = \frac{v_0 + v_a\left(1 - e^{-c_1 i}\right)}{i}, \tag{11}$$

$$R_{conc} = \frac{v_{conc}}{i} = \left(c_2 \frac{i}{i_{\max}}\right)^{c_3}. \tag{12}$$

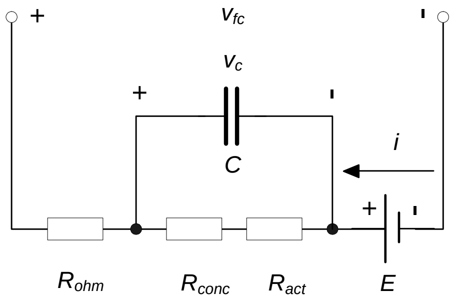

Fig. 3. Fuel cell equivalent electric circuit

Based on electrical circuit from figure 3, following equations that show current-voltage relations can be written:

$$C\frac{dv_C}{dt}+\frac{v_C}{R_{act}+R_{conc}}=i\,, \tag{13}$$

$$v_{fc}=E-v_C-iR_{ohm}\,. \tag{14}$$

Using (13), (14), and Laplace transformations, transfer function (15) was obtained, in which *s* represents Laplace operator:

$$v_{fc}=E-\left(\frac{R_{act}+R_{conc}}{sC\left(R_{act}+R_{conc}\right)+1}+R_{ohm}\right)i\,, \tag{15}$$

Using expressed equations and *MatLab Simulink*, PEM fuel cell dynamic model was created (figure 6), that allows change of membrane water content degree (*lambdam*), value of pressures (*pH2*, *pO2*, *pca*), temperature (*Tfc*) and current density (*i*). Because of this feature easy implementation of pressures, humidity and temperature dynamics as well as their regulation can be achieved using external blocks.

III. DYNAMIC BOOST CONVERTER MODEL

Boost converter, shown in figure 4, was designed for 500 W electric power, input voltage v_{in} between 12 V and 18 V and output voltage v_{out} of 48 V. Taking into account above mentioned input voltage span and *f* switching frequency of 20 kHz, inductor L_1 was designed to keep input current i_{L1} ripple under 5 % (L_1=0,405 H), and capacitor C_1 to keep output voltage v_{out} ripple under 0,5 % (C_1=1,628 mF).

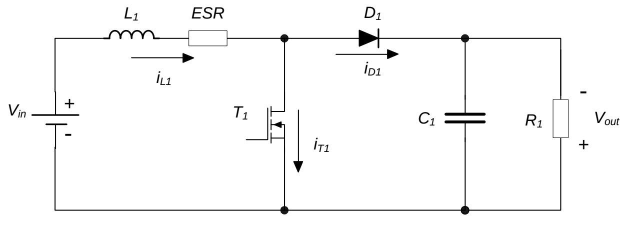

Fig. 4. Boost converter electric circuit

To describe this converter using state space method, following vectors had to be defined:

state space vector: $x=\begin{bmatrix} i_{L1} \\ v_{C1} \end{bmatrix}$,

input vector: $u=\left[v_{in}\right]$,

output vector: $y=\left[v_{out}\right]$.

Using these vector equations, converter behavior is expressed:

$$\dot{x}=Ax+Bu\,, \tag{16}$$

$$y=Ex+Fu\,. \tag{17}$$

where A, B, E and F represent data matrices. Parameters of these matrices were separately determinated for both (turn on & off) conditions of T_1 transistor switch. By merging them, averaged converter model is obtained (18, 19):

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{v}_{C1} \end{bmatrix}=\begin{bmatrix} -\frac{ESR}{L_1} & -\frac{1-D}{L_1} \\ \frac{1-D}{C_1} & -\frac{1}{R_1C_1} \end{bmatrix}\cdot\begin{bmatrix} i_{L1} \\ v_{C1} \end{bmatrix}+\begin{bmatrix} \frac{1}{L_1} \\ 0 \end{bmatrix}\cdot\left[v_{in}\right], \tag{18}$$

$$\left[v_{out}\right]=\left[0\quad 1\right]\cdot\begin{bmatrix} i_{L1} \\ v_{C1} \end{bmatrix}+\left[0\right]\cdot\left[v_{in}\right]\cdot \tag{19}$$

In these equations *ESR* represents inductor resistance, R_1 load resistance, *D* switch duty-ratio and v_{C1} capacitor voltage.

While designing PI controller, each time dependant parameter was considered as a sum of one static and one time dependant part (for example: $i_{L1}=I_{L1}+\tilde{i}_{L1}$). Using (18), (19) and Laplace transformations, transfer functions that show relations between converter output voltage and duty-ratio ($G_{vd}(s)=\tilde{V}_{out}(s)/\tilde{d}(s)$), converter output and input voltage ($G_{in}(s)=\tilde{V}_{out}(s)/\tilde{V}_{in}(s)$) and converter output voltage and Pulse Width Modulator (PWM) output ($G_{vc}(s)=\tilde{V}_{out}(s)/\tilde{v}_c(s)$) were obtained.

Using frequency analysis on an open circuit transfer function:

$$T(s)=\beta G_{vc}(s)G_{ca}(s)\,, \tag{20}$$

of a boost converter control circuit (figure 5), proportional K_p gain and integral K_I gain of PI regulator were obtained. Equation (21) shows PI regulator transfer function:

$$G_{ca}(s)=K_P+\frac{K_P\omega_{iz}}{s}=K_P+\frac{K_I}{s}\,, \tag{21}$$

With regulator parameter values K_p = 0,05 and K_I = 18,85, gain margin of 7,71 dB, and phase margin of 91.7°, was achieved, which is in accordance with recommendations (6 dB < G.M. < 20 dB, 45° < P.M.).

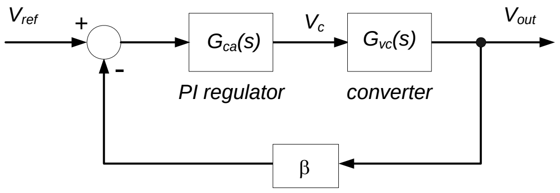

Fig. 5. Boost converter control circuit block representation

In presented averaged model of regulated boost converter, realized using *MATLAB Simulink* (figure 7), load value can be changed by input (R1), and input voltage can be changed using input (Vin), while output voltage and input (inductor) current can be observed using outputs (Vout) and (iL1) respectively.

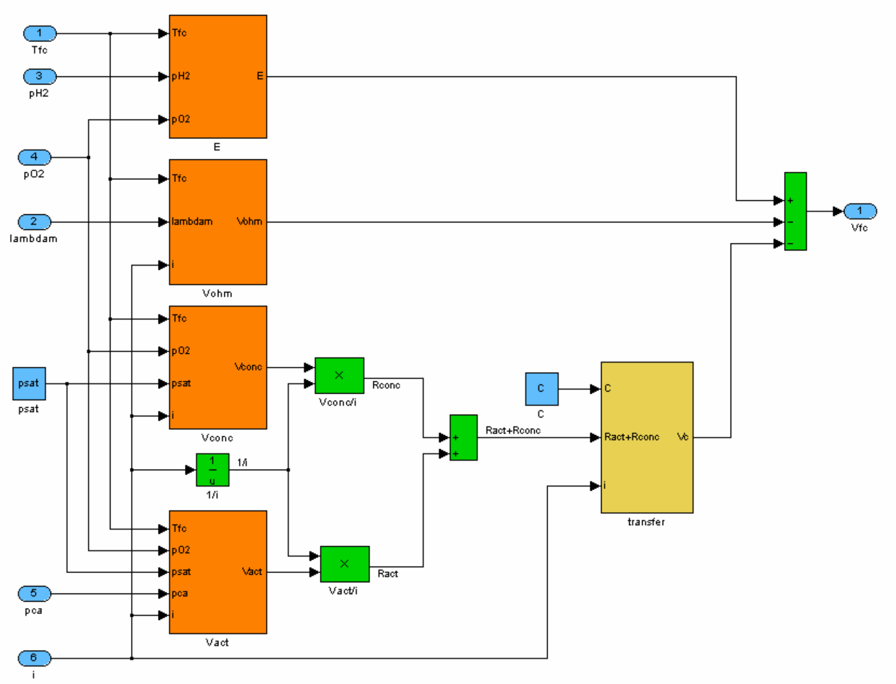

Fig. 6. MATLAB/Simulink dynamic fuel cell model

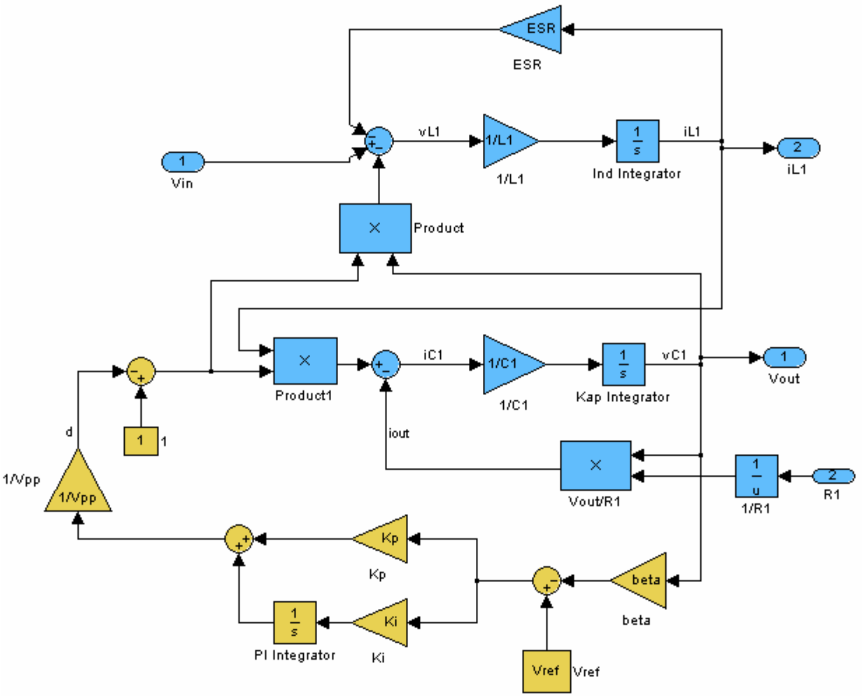

Fig. 7. MATLAB/Simulink dynamic model of controlled boost convert

VI. SYSTEM RESPONSE DUE TO INFLUENCE OF PARASITIC CAPACITANCES

With presumption of the same conditions in each cell of a fuel cell stack, fuel cell model was modified into a PEM stack model by multiplying fuel cell output voltage v_{fc} with number of cells N:

$$v_{stack} = N \cdot v_{fc} \,. \tag{22}$$

Also because stack current I_{fc} is used as an input variable of PEM stack, current density was calculated using (23), where A_{fc} represents a fuel cell active area:

$$i = \frac{I_{fc}}{A_{fc}} \,. \tag{23}$$

Controlled boost converter model was linked with described PEM model and a number of step load changes was simulated using input port (*R1*) (figure 8). Fuel cell stack output voltage, fuel cell stack current and boost converter output voltage responses (shown in figures 9,10 and 11) were observed for cases of included and excluded PEM dynamic caused by parasitic capacitance. They show that effects caused by parasitic capacitances decrease system stability, resulting with the increase of transient time duration and related oscillations. These effects are in accordance with responses of a real PEM fuel stack experimentally obtained in [7]. It is interesting to note that in case of fuel cell stack output voltage V_{fc}, parasitic effects are diminishing the size of transient response peak value. That means that PEM fuel cell model with included capacitances gives more realistic results than common simple PEM fuel cell model.

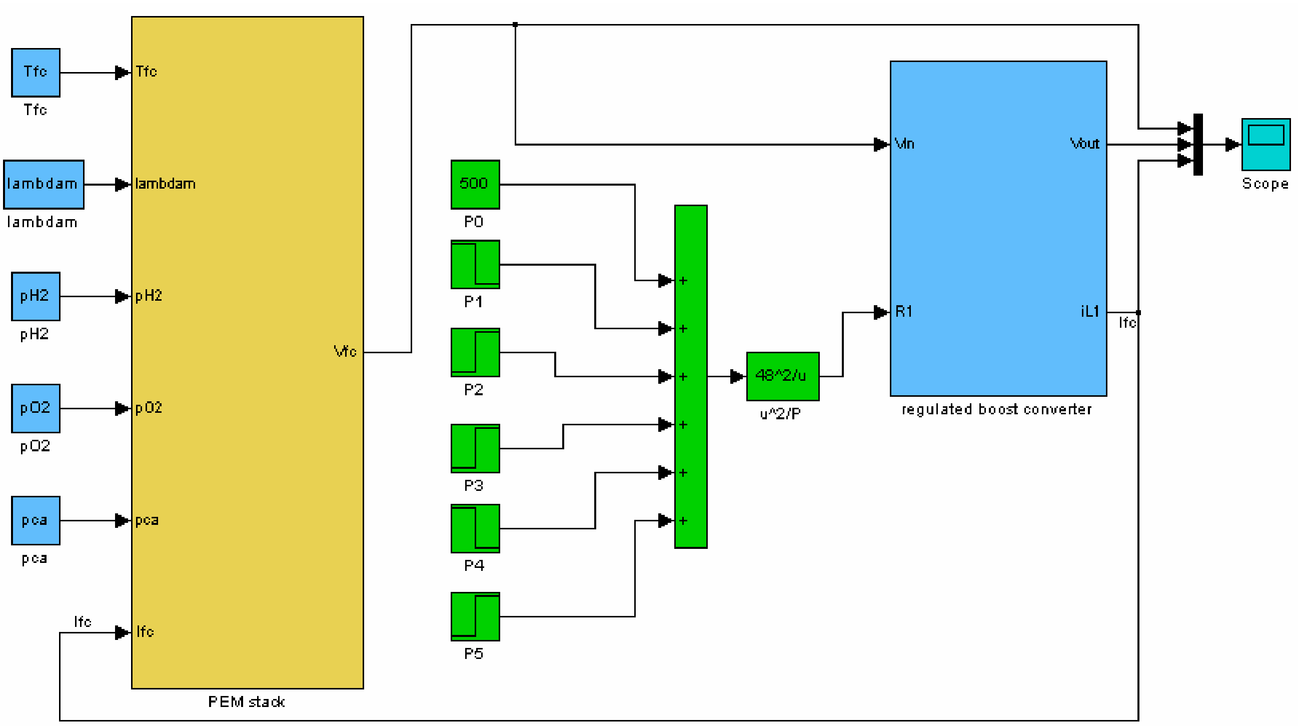

Fig. 8. MATLAB/Simulink model of system PEM + regulated boost converter

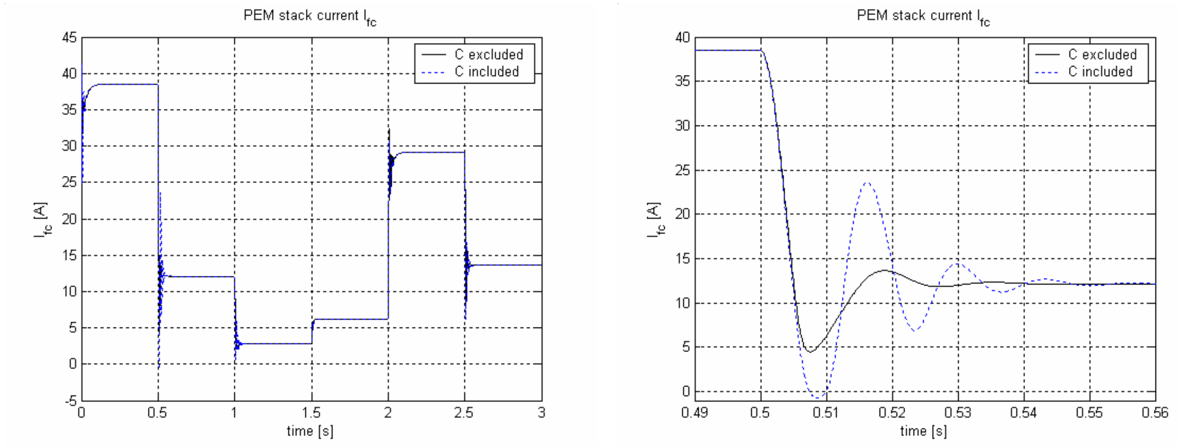

Fig. 9. Fuel cell stack current response I_{fc}

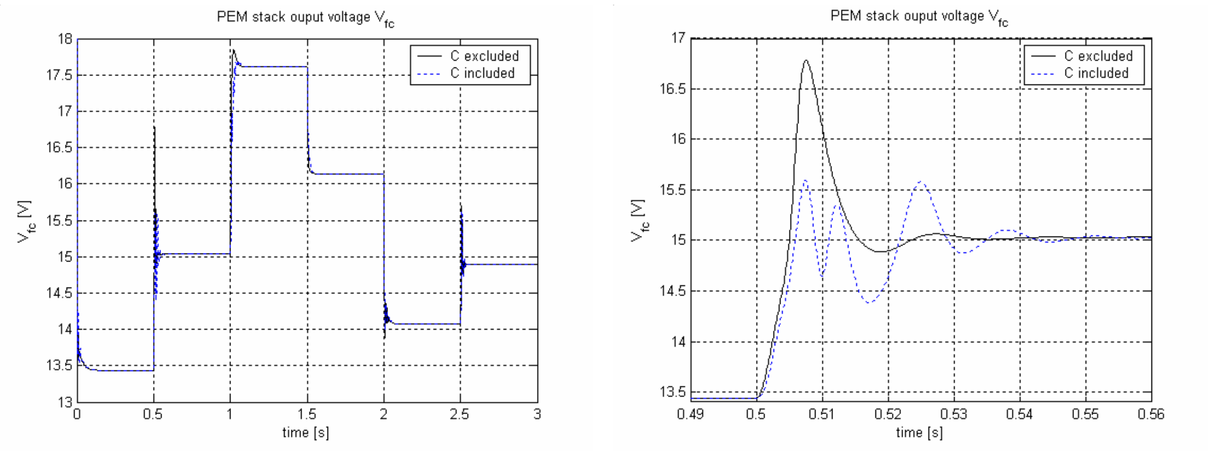

Fig. 10. Fuel cell stack output voltage response V_{fc}

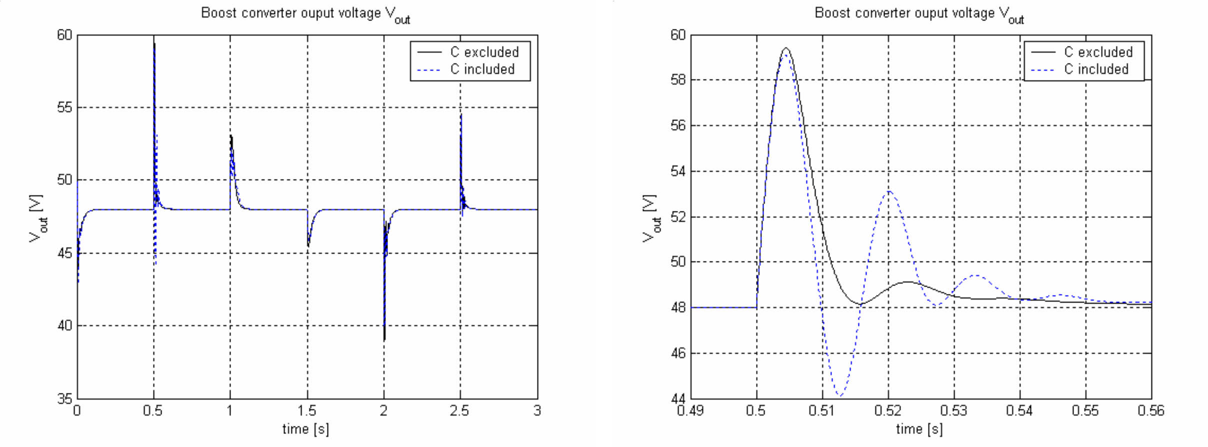

Fig. 11. Boost converter output voltage response V_{out}

V. CONCLUSION

Simulation results are showing that applying a model structure of fuel cell and boost converter, taking parasitic capacitances into consideration, results with better understanding of behavior of controlled system based on PEM fuel cell as power source. This allows new possibilities in research and can result with better system component dimensioning and controller design for such power systems.

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