ESTABLISHMENT OF GRAVIMETRIC NETWORK FOR THE AREA OF ZAGREB METRO

D. Markovinović¹, T. Bašić²

¹ University of Zagreb, Faculty of Geodesy, Kaciceva 26, 10 000 Zagreb, Croatia, e-mail: danko.markovinovic@geof.hr
² Croatian Geodetic Institute, Savska 41/XVI, 10 144 Zagreb, p.p. 19 Croatia, e-mail: tomiislav.basic@cgi.hr

Abstract

The paper offers the overview of the establishment of gravimetric network on the wider area of the City of Zagreb, the capital of the Republic of Croatia. The network consists of 34 points the stabilization of which has been taken over from the previously established fundamental GPS network of the City of Zagreb. There is also the implementation of precise GPS, levelling and relative gravimetric measurements presented. In gravimetric measurements the relative gravimeter Scintrex Autograv CG-5 has been used. For the purpose of defining gravimetric datum two absolute gravimetric points were used. The relative gravimeter was calibrated on the part of the Gravimetric calibration base of the Republic of Croatia before and after the measurement. In the process of adjusting the gravimetric network, the input values were absolute values of gravity and gravity differences between the points that had been previously corrected for the height of instrument, the change of Earth pole position and for the air pressure, as well as for the instrumental drift. Standard deviations of gravity differences and the distance between points were used for weights. The obtained results of the gravity value, as well as their accuracy estimation indicate that there was extremely reliable gravimetric network established using applied methodology.

Key words: gravity, network, gravimeter drift, linear, quadratic

Introduction

In June 2005 the gravity network of the City of Zagreb was established at the wider territory of the City of Zagreb, as a part of the II. Order Gravity Network of the Republic of Croatia. The purpose of establishing the gravity network was to define precisely the values of gravity at the locations of selected points belonging to the previously defined GPS network of the City of Zagreb. The points needed to be connected with relative gravimetric measurements to absolute gravimetric points, and also to the nearest point of the I. Order Gravimetric Network. GPS-RTK technology was used in determining the positions of gravimetric points. The height component was determined with precise levelling connection to the bench marks of the II. NVT.

Revision of the geodetic base at the territory of the city of Zagreb

The 0. Order Gravity network of the Republic of Croatia consists of 6 points (Fig. 1 – red colour). At the territory of the City of Zagreb there are two absolute points, i.e.: AGT02 Maksimir and AGT03 Puntijarka (Fig. 2). The I. Order Gravity Network of the Republic of Croatia consists of 36 points (Fig. 1 – blue colour). The nearest I. Order point, GT117 was used in relative gravimetric measurement [1,2,4,6].

The fundamental GPS network of the City of Zagreb consists of 38 points. The majority of points are stabilised with concrete column 1,3 m high at the bottom of each point a chromium-plated bolt is installed (Fig. 3.). All relative gravimetric measurements done refer to the bolt height. In GPS points, where it was not possible to do gravimetric measurements, adequate excenter was observed instead of them. Levelling survey required field reconnaissance to be carried out for the purpose of finding the most reliable II. NVT (highest order leveling network) bench marks. At the territory of interest there were altogether 5 bench marks found that levelling measurements were referred to.

Gravimetric, GPS and levelling measurements

Relative gravimetric measurement was done with relative gravimeter Scintrex AutoGrav CG-5. This is a quartz instrument controlled with microprocessor and highly automated. Operational range of the gravimeter is over 8000 mGal (1 mGal = 10⁻⁵ ms⁻²), and standard gravimeter resolution is 1 μGal (1 μGal = 10⁻⁸ ms⁻²). To be used in gravimetric measurement at the territory of the City of Zagreb, the gravimeter had been calibrated on the part of gravimetric calibration base of the Republic of Croatia [5]. For this purpose the points were used the values of which are more difficult to be known in absolute amount, i.e. in the points AGT03-Puntijarka and AGT02-Maksimir. On the basis of multiple measurements on the part of the calibration base between these two absolute gravimetric points and their adjustment, new scale changing factor was computed for the elative gravimeter CG-5 that was used in measurements. Table 1 provides the calibration factor that was used during the measurement.

¹ MSc (PhD Candidate) in Technical Sciences, assistant, Faculty of Geodesy
² PhD in Technical Sciences, full professor, Director of the Croatian geodetic institute
Relative gravimetric measurements in the gravimetric network of the City of Zagreb were done in the period between 13th and 29th of June 2005. Depending on the measurement plan and field conditions, the star methods were used primarily in the measurement, and when necessary, a profile method. In the process of planning the measurement the economic efficiency, distance between points and reliable calculations of gravimeter drift were considered.

### Table 1. Calibration factor of the gravimeter Scintrex CG-5

<table>
<thead>
<tr>
<th></th>
<th>GCAL1 ($10^{-5}$ms$^{-2}$)</th>
<th>$K_1$</th>
<th>GCAL1' ($10^{-5}$ms$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG-5</td>
<td>8428.017</td>
<td>1.000071310</td>
<td>8428.618</td>
</tr>
</tbody>
</table>

The calculation of linear and quadratic gravimeter drift was done always in the starting point of the gravimetric day. There are the measurement parameters given in the Table 2 that were used during the measurement in the Gravimetric network of the City of Zagreb.

### Table 2. Parameters of gravimetric measurement in the network of the City of Zagreb.

<table>
<thead>
<tr>
<th>Read time</th>
<th>60</th>
<th>#of Cycles</th>
<th>5</th>
<th>Seismic filter</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto reject</td>
<td>Yes</td>
<td>Save raw data</td>
<td>Yes</td>
<td>Terrain corr</td>
<td>No</td>
</tr>
<tr>
<td>Cycle time</td>
<td>77</td>
<td>Start delay</td>
<td>20</td>
<td>Cont. tilt corr</td>
<td>Yes</td>
</tr>
<tr>
<td>Tide correct</td>
<td>Yes</td>
<td>Auto repeat</td>
<td>No</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The instruments and additional equipment were transported with personal car. During the measurement 6000 km were made driving. There were altogether 54 gravimetric figures closed in the measurements of the Gravity Network of the City of Zagreb (Fig. 3). During the measurement in the gravimetric Network of the City of Zagreb the star method and the profile method were used. The mentioned methods were used for observing the closed figures, and the calculation of the daily gravimeter drift was taken into consideration. For that purpose the starting point was used that was observed in the middle of the day wherever it was possible, except as the final point.

Relative gravimeter Scintrex CG-5 has the possibility to yield measurement reductions immediately in the field. Gravimetric readings are in the instrument, in real time, reduced for the change of gravimeter reading scale referring to the calibration constant, height of gravimeter, measuring sensor temperature and the Earth’s tide waves [8,9].
Besides the mentioned reduction contained in the measurements in the field, the gravimetric readings should be additionally reduced for the influence of the instrument height, air pressure change and the change of Earth pole position, and finally for the gravimeter drift [3, 7, 11]. The time variation of gravimeter reading in the zero position of the instrument (drift) is given by the expression [10]:

\[ D(t) = z(t_0) + d_1 (t - t_0) + d_2 (t - t_0)^2 + \ldots \]

where \(z(t_0)\) is the gravimeter reading at the beginning of measurement \(t_0\), \(d_1\) and \(d_2\) are the drift parameters, and \(t\) is the moment of measurement. For the purpose of eliminating gravimeter drift and preparing the adjustment measurement, a daily linear and quadratic drift has been determined for CG-5 on the basis of the measuring method, i.e. of the repeated measurement in the point of origin. The example of determining the drift of CG-5 gravimeter is shown in Figure 5, for the 20th of June, 2005, where the drift is modelled on the basis of raw measurements as a linear – straight line with the corrected measurements obtained at the end (red), and quadratic – dash red line.

Figure 3. The part of II. Order Gravimetric network at the territory of the City of Zagreb

Figure 4. Typical GPS pillar of the Zagreb network

Figure 5. Linear and quadratic drift of the CG-5 gravimeter for the day 20.06.2005
For the purpose of determining the position of gravimetric points GPS-RTK method was used. Basic GPS points of the City of Zagreb were used as connection points. Official transformation parameters [1] were used for the transformation of coordinates from ETRS'89 system into Croatian Official Coordinate System. Thus obtained heights served for the comparison with orthometric heights obtained in levelling measurements.

Table 3. Statistics of levelling network adjustment

<table>
<thead>
<tr>
<th>N = 34</th>
<th>Height (m)</th>
<th>St.dev. (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>187.3971</td>
<td>0.0132</td>
</tr>
<tr>
<td>St.dev.</td>
<td>116.0126</td>
<td>0.0081</td>
</tr>
<tr>
<td>Min.</td>
<td>102.3099</td>
<td>0.0055</td>
</tr>
<tr>
<td>Max.</td>
<td>601.9932</td>
<td>0.0333</td>
</tr>
</tbody>
</table>

The height differences in the II. Order Gravity Network at the territory of the City of Zagreb were measured in one direction using digital levels Leica NA2000 and DNA03. Thus levelled height differences were controlled in closed figures of 3 or more points. There were altogether 20 figures levelled. The largest discrepancy of levelling with the length of levelling side being 1 km is 1.4 mm/1km. This discrepancy refers to the largest figure that is connected with the points on the southern and northern side, and its total length is 81.8 km. In the procedure of the levelling network adjustment the obtained mean reference error is \( m_0 = 1.44 \text{ cm} \).

Further on there is the comparison made of othometric heights obtained by means of levelling network adjustment and the heights in GPS network of the City of Zagreb. In order to carry out the comparison correctly, the difference of 17.2 cm related to the old system (Trieste) is added to the heights from the new height datum (HVRS71), and it was found out on the basis of the known heights of six bench marks in the old and new system and in this area. There were altogether 21 points compared with the existing heights and new height on the basis of the carried out leveling within the scope of this project. Table 4 gives statistical indicators of the comparison made.

**Adjustment of gravity network**

The measurements in the Gravimetric network for Zagreb area have included the total of 34 gravimetric points (2 absolute and 1 I. order points). The most probable values of gravity were obtained by means of adjustment using the least squares method. The connection between the measured and the requested values can be expressed with the correction equation [9]:

\[
\Delta z_{i,j} + v_{i,j} = g_{j} - g_{i},
\]

where \( \Delta z_{i,j} \) is the difference of gravimetric readings, \( v_{i,j} \) is the accompanying correction of gravimetric reading, \( g_{i,j} \) is the gravity in the points \( i \) i.e. \( j \) at the moments of measurements \( t_{i,j} \).

The adjustment of gravity network has been carried out using the expression for immediate measurement. In the adjustment the a priori datum has been defined using the absolute gravity values in the points AGT02 Maksimir and AGT03 Puntijarka.

Table 5. Statistics of obtained st. dev. in linear and quadratic drift (only AGT03 defines the datum)

<table>
<thead>
<tr>
<th>No. of meas. &amp; st.dev.</th>
<th>CG5 – lin. drift</th>
<th>CG-5 – quadr. drift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. (10^{-5}ms^{-2})</td>
<td>534.9096</td>
<td>31.3981</td>
</tr>
<tr>
<td>Max. (10^{-5}ms^{-2})</td>
<td>674.3315</td>
<td>18.0781</td>
</tr>
<tr>
<td>St.dev. (10^{-5}ms^{-2})</td>
<td>534.9108</td>
<td>31.3981</td>
</tr>
<tr>
<td>Mean (10^{-5}ms^{-2})</td>
<td>31.3981</td>
<td>31.3981</td>
</tr>
</tbody>
</table>

Apart from absolute values of gravity, the input values were the differences of gravity determined between two points. The gravity differences were obtained from arithmetic means of 60 second readings in five epochs in one point. On the basis of the reading there were also their belonging standard deviation determined, and the standard deviations of the gravity differences themselves were determined by applying the error growth law. Tables 5 and 6
give the overview of statistical indicators in the adjustment of free and fixed gravity network. In the free network the st. dev. is better as related to the network computed with square drift, and in fixed network the statistical indicators are adjusted due to the measurements eliminated in the network computed with quadratic drift.

Figure 6 presents graphic comparison (differences) of gravity values obtained by means of the adjustment with linear and quadratic model of gravimeter drift. There are some deviations noticed in four points (TM5, TM6, TM16 and TM17), that are grouped at the western edge of the gravity network and were observed during two gravimetric days when the gravimeter drift was somewhat bigger.

### Conclusion

On the basis of the procedures applied in measuring the gravity differences as well as the height differences between the points of the gravity network of the City of Zagreb as a part of the future II. Order Gravity Network of the Republic of Croatia, and of the processing and adjustment of gravimetric, GPS and levelling measurements it can be concluded that extremely reliable gravity network has been established with equally reliable heights. It can be seen from the mean standard deviation of the heights being 1.32 cm, and the minimal value of the standard deviation is 0.55 cm, and the maximal is 3.33 cm. Regarding the gravity, the mean standard deviation is 2.45x10^-8 ms^-2), which corresponds excellently with the adjustment obtained with the reference error of 2.30x10^-8 ms^-2, the minimal value of standard deviation is 1.15x10^-8 ms^-2, and the maximum value is 4.85x10^-8 ms^-2.

### References

11 URL1: International Earth Rotation Service (IERS), http://hpiers.obspm.fr/coppe/bul/bulb/ (30.05.2005.)

![Figure 6. Gravity differences obtained between the adjustment with linear and the adjustment with quadratic drift in the Zagreb Gravity Network](image-url)