MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

ON AMBIGUITIES AND UNCERTAINTIES IN PWA

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Establishing a well defined point of comparison between experimental results and theoretical predictions has for decades been one of the main issues in hadron spectroscopy, and the present status is still not satisfactory. Experiments, via partial wave (PWA) and amplitude analysis (AA), can give reliable information on scattering matrix singularities, while quark model calculations usually give information on resonant states spectrum in the first order impulse approximation (bare/quenched mass spectrum). And these two quantities are by no means the same. Up to now, in the absence of a better recipe, these quantities have usually been directly compared, but the awareness has ripen that the clear distinction between the two has to be made. One either has to dress quark-model resonant states spectrum and compare the outcome to the experimental scattering matrix poles, or to try to take into account all self-energy contributions which are implicitly included in the measured scattering matrix pole parameters, make a model independent undressing procedure and compare the outcome to the impulse approximation quark-model calculations. The first options seems to be feasible but complicated [1], but the latter one seems to be impossible [2] due to very general field-theory considerations [3, 4].

Hence, it seems fairly reasonable to focus our interest onto investigating detailed features of scattering matrix singularities.

The general structure of all coupled-channel models is identical: the same type of Dyson-Schwinger integral equation is always solved, but the channel-resonance vertex interaction is treated differently - the approach varies from phenomenological to microscopic [5]. Consequently, all coupled-channel models contain two types of scattering matrix singularities: full and bare. While the full scattering matrix singularities are unanimously identified with "measurable" scattering matrix poles, the interpretation of bare poles, related to the vertex interaction, have not yet been reached. Desirous ones attempt to relate them to quark-model resonant states: in [6–8] the possibility has been opened that $\gamma N \to \Delta$ helicity amplitudes and transition form factors of constituent quark models should be compared to the bare coupled-channel functions, in [9] a simple well defined model is devised for understanding

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Roper and Δ resonances in terms of Cloudy Bag Model form factors [10], and in [11, 12] this idea is used for understanding charmed/strange resonant states in meson-meson scattering sector. Second, more cautious ones give them certain measure of physical importance, but strongly refrain from giving them such a tempting physical meaning. The main reason for such a disagreement lies in believing that the poles of the interaction potential do arise only from the assumed model, and as such do not reveal much dynamics of the interaction [2, 13].

Here we discuss various modes how to numerically quantify both types of scattering matrix singularities, bare and dressed ones. First we focus onto dressed scattering matrix singularities.

Dressed scattering matrix singularities

Dressed scattering matrix poles are nowadays quantified in two dominant ways: either as Breit-Wigner parameters, i.e. parameters of a Breit-Wigner function which is used to locally represent the experimentally obtainable T-matrix, or as scattering matrix poles (either T or K). In spite of the fact that it is since Hoehler's analysis [14] quite commonly accepted that Breit-Wigner parameters are necessarily model dependent quantities, they are still widely used to quantify the scattering matrix poles. Only recently the scattering matrix poles are being shown in addition [15].

The possibility to define model independent "Breit-Wigner like" parameters parameters by parameterizing K-matrix poles with a Breit-Wigner function has been discussed by our group [16], but in this contribution we focus the presentation on extracting the dressed scattering matrix poles as complex numbers in the complex energy plane, quantities which are to be extracted by knowing only real and imaginary part of the scattering matrix on the real energy axes. We analyze the reliability of the speed-plot technique in particular.

As a self-consistency test to extracting the scattering matrix pole positions using the inherent multi-channel analytic continuation methods [17], we have applied the standard speed plot technique (single-channel) to the amplitudes which describe various different-channel reactions. Surprisingly, we obtained the values which differed from those obtained when using the original method. In addition, the obtained parameters were not identical for different channel processes. This anomalous behavior challenged common sense and the conclusion was drawn that either our partial-wave analysis or the applied pole extraction methods were incorrect. The single-channel extraction methods were carefully examined, and those methods were determined to be at fault. This effort resulted in a new model-independent

Table 1: The N^* resonance pole parameters obtained by the analytic continuation method and speed plot in various channels. The $N(????)$ stands for resonances unnamed in the RPP.

		Continuation method		Speed plot method					
				πN elastic		$\eta N \to \eta N$		$\pi N \to \eta N$	
N*	$L_{2I\,2J}$	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)
$N(1535)$	S_{11}	1517	190	1506	83	1531	388	-	-
$N(1650)$	S_{11}	1642	203	1657	183	1601	208	1632	179
$N(2090)$	S_{11}	1785	420	1764	133	-	-	1917	423
$N(1440)$	P_{11}	1359	162	1355	154	st^2	st^a	st^a	st^a
$N(1710)$	P_{11}	1728	138	1722	121	1733	154	1679	151
$N(????)$	P_{11}	1708	174	-	-	-	-	-	-
$N(2100)$	P_{11}	2113	345	2131	394	2122	357	2116	360
$N(1720)$	P_{13}	1686	235	1706	219	1617	289	1641	252
$N(1520)$	D_{13}	1505	123	1505	129	1527	129	-	-
$N(1700)$	D_{13}	1805	130	1953	290	1809	129	-	-
$N(2080)$	D_{13}	1942	476	1960	270	-	-	-	-
$N(1675)$	D_{15}	1657	134	1657	136	1651	149	1620	108
$N(2200)$	D_{15}	2133	439	2134	375	2141	422	2130	401

extraction method free from this anomaly.

Using the speed plot technique we have extracted the pole parameters from the coupled-channel amplitudes of ref. [17] for $\pi N \to \pi N$, $\eta N \to \eta N$ and $\pi N \to \eta N$ processes. We summarized the results in Table 1, and compared them to the pole parameters of analytic continuation method. To our surprise, in some partial waves the obtained pole positions turned out to be different for each process, and shifted with respect to analytic continuation method by a few tens of MeV. And that is in obvious contradiction with the input, because the pole positions in ref. [17] are manifestly identical for all T-matrix matrix elements by the very construction. Therefore, something was wrong.

In order to understand, explain and remedy this, we devise a new single-channel method: the T-matrix regularization procedure, the method in which the speed-plot technique is nothing but the first order approximation.

We start with a very general set of assumptions.

Let there be an analytic function $T(z)$ of complex variable z which has a first-order pole at some complex point μ. The function $T(z)$ can be any of the T-matrix matrix elements, and variable z can be either Mandelstam s or center-of-mass energy $\sqrt{s}$. In order to achieve a full correspondence with the speed plot technique, from now on we are going to use the latter choice. Since all physical processes occur for real energy values, we are allowed to directly determine only $T(x)$ for x being a real number. To be able to successfully continue $T(x)$ into complex energy plane (to search for its poles), we should regularize this function (i.e. remove the pole). In that case, any simple expansion of the regularized function would converge in the proximity of the removed pole.

The T-matrix matrix amplitudes are parameterized as:

$$T(z) = \underbrace{\frac{r}{\mu - z}}_{\text{resonant part}} + \underbrace{\left(T(z) - \frac{r}{\mu - z} \right)}_{\text{smooth background}}, \tag{1}$$

where μ and r are pole position and pole residue, and the varable z stands for center-of-mass energy ($\sqrt{s}$).

The function $T(z)$ with a simple pole at μ, is regularized by multiplying it with a simple zero at μ:

$$f(z) = (\mu - z)\, T(z). \tag{2}$$

From this definition and Eq. (1), it is evident that the value of $f(\mu)$ is equal to the residue r of $T(z)$ at point μ. As we have the access to the function values on real axis only, the Taylor expansion of f is done over some real x to give the value (residue) in the pole μ (where background is highly suppressed)

$$f(\mu) = \sum_{n=0}^{N} \frac{f^{(n)}(x)}{n!} (\mu - x)^n + R_N(x, \mu). \tag{3}$$

The expansion is explicitly written to the order N, and the remainder is designated by $R_N(x, \mu)$. Using the mathematical induction one can show that the nth derivative of $f(x)$, given by Eq. (2), is given as:

$$f^{(n)}(x) = (\mu - x)\, T^{(n)}(x) - n\, T^{(n-1)}(x). \tag{4}$$

Insertion of this derivative into Taylor expansion conveniently cancels all consecutive terms in the sum, except the last one

$$f(\mu) = \frac{T^{(N)}(x)}{N!} (\mu - x)^{(N+1)} + R_N(x, \mu), \tag{5}$$

where $T^{(N)}(x)$ is the Nth energy derivative of T-matrix element. To simplify the notation, the pole can be written as a general complex number $\mu = a + i\, b$.

Table 2: The comparison of N^* resonance pole parameters obtained by the analytic continuation method, and the Regularization method for πN , $\eta N \to \eta N$ and $\pi N \to \eta N$ processes. Numbers in subscript are the expansion order required to obtain convergent result.

		Analytic Contin.		Regularization Method					
				$\pi N \to \pi N$		$\pi N \to \eta N$		$\eta N \to \eta N$	
N^*	$L_{2I\,2J}$	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)
N(1535)	S_{11}	1517	190	$1522_{(7)}$	$146_{(7)}$	-	-	-	-
N(1650)	S_{11}	1517	203	$1647_{(7)}$	$203_{(7)}$	$1645_{(10)}$	$211_{(10)}$	-	-
N(2090)	S_{11}	1785	420	-	-	-	-	-	-
N(1440)	P_{11}	1359	162	$1354_{(8)}$	$162_{(8)}$	st^3	st^a	st^a	st^a
N(1710)	P_{11}	1728	138	$1729_{(8)}$	$150_{(8)}$	$1733_{(5)}$	$133_{(5)}$	$1728_{(7)}$	$142_{(7)}$
N(????)	P_{11}	1708	174	-	-	-	-	-	-
N(2100)	P_{11}	2113	345	$2120_{(6)}$	$347_{(6)}$	$2120_{(6)}$	$347_{(6)}$	$2120_{(6)}$	$347_{(6)}$
N(1720)	P_{13}	1686	235	$1691_{(5)}$	$235_{(5)}$	$1691_{(5)}$	$234_{(5)}$	$1691_{(5)}$	$235_{(5)}$
N(1520)	D_{13}	1505	123	$1506_{(4)}$	$124_{(4)}$	-	-	-	-
N(1700)	D_{13}	1805	130	$1806_{(5)}$	$132_{(5)}$	$1806_{(4)}$	$130_{(4)}$	-	-
N(2080)	D_{13}	1942	476	-	-	-	-	-	-
N(1675)	D_{15}	1657	134	$1658_{(5)}$	$138_{(5)}$	$1657_{(3)}$	$137_{(3)}$	$1658_{(5)}$	$138_{(5)}$
N(2200)	D_{15}	2133	439	$2145_{(6)}$	$439_{(6)}$	$2144_{(4)}$	$435_{(4)}$	$2144_{(6)}$	$438_{(6)}$

Once the Taylor series converges the remainder $R_N(x,\mu)$ can be disregarded, and the absolute value of both sides of Eq. (5) is given as:

$$|f(\mu)| = \frac{\left|T^{(N)}(x)\right|}{N!}\,|a + i\,b - x|^{(N+1)}\,. \tag{6}$$

To keep the form as simple as possible, Eq. (6) is raised to the power of $2/(N+1)$. After simple rearrangement of terms, in which we have collected the information on the T-matrix values on the right hand side, and the information on the pole position and residuum on the left hand side, the elemental second-order polynomial emerges:

$$\frac{(a-x)^2 + b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{\left|T^{(N)}(x)\right|^2}}, \tag{7}$$

This is the equation which enables us to directly extract the pole position ($a = \mathrm{Re}\ \mu$, $b = \mathrm{Im}\ \mu$) and the absolute value of the function residue $|f(\mu)|$

from the T-matrix values at the real axes, namely from the quantities directly attainable from the energy-dependent partial wave analysis and evaluated at factual energy points x.

What we actually do is the following: we first find the N-th derivative of the T-matrix, and then we calculate the right-hand side of Eq. (7). Observe that the ***exact*** knowledge of the right-hand side of Eq. (7) ***in only three points*** uniquely determines the pole parameters. The problem is that we can never know the right-hand side of Eq. (7) exactly. Therefore, we have two options: either i) to take various three-point sets, evaluate the right-hand side of Eq. (7), solve the equation for pole parameters, and make a statistical analysis of obtained results; or ii) to fit the right-hand side of Eq. (7) with the three parameter parabolic function. We have chosen the latter option, and the obtained fitting parameters are our final result.

The pole parameters attained in this way, with the subscript N denoting the number of required Taylor series terms, are for all three calculated processes given in Table 2. Discrepancies are eliminated.
Observe:

The standard speed plot method turns out to be the "regularization" method in the first order approximation! (To get the speed plot, one should reduce the expansion given by Eq. (3) to $N = 1$ term.) The developed regularization method represents an improvement of contemporary single-channel pole extraction methods. We demonstrate that it successfully finds resonance pole parameters from a T-matrix in a model-independent way, i.e. without having to assume a specific T-matrix functional form.

Bare scattering matrix singularities

Coupled-channel T-matrix formalisms (CC_T) [17, 19, 20] by construction distinguish between scattering-matrix poles and bare Green function (bare propagator) poles. The bare Green function poles, which are the subset of CC_T model fit parameters, can not be detected experimentally. To became observable they have to interact. Through the formalism described by resolvent Dyson-Schwinger equation the self-energy term is generated; the self-energy term shifts the initial real-value bare propagator poles into the complex energy plane; and eventually the measurable complex scattering-matrix poles are generated as dressed Green function poles.

Following the ideas formulated in a dynamical coupled-channel model of refs. [6, 21], but baring in mind controversies raised in ref. [2], we propose that in any CMB type model one should as well try to identify the position of a bare Green function pole with the mass of a quark-model resonant state (QMRS), and to correlate the imaginary part of the scattering-matrix pole

(SMP), which is created when the interaction effects shift QMRS into the complex energy plane, with its decay width.

Such an identification simultaneously solves two problems: establishes a missing link between QMRS and SMP offering a better control over the missing-resonance problem, and at the same time creates a mechanism how to distinguish between genuine scattering-matrix resonant states (SMRS) being states which are produced by a nearby bare propagator pole, and dynamically generated ones which are only an interference effect of distant ones. By accepting this assumptions, we are able to: a) identify which QMRS are needed to explain a chosen collection of experimental data; b) determine a nature of a given SMRS (genuine or dynamic).

Even before showing the results, we want to warn the reader that the simplicity of the model, i.e. the fact that we are effectively using only three out of at least seven accessible and contributing channels, will produce only qualitative results. Complexity of the coupled-channel model (simultaneous mixing of all channels) requires considerable number of parameters, and we expect that the absence of constraining data in more than two channels will necessarily produce instabilities in obtained fitting solutions.

The first four partial waves in I=1/2 channel (S_{11}, P_{11}, P_{13} and D_{13}) were analyzed. We use a model with three channels: two physical two-body channels πN and ηN, while the third, effective channel represents all remaining two- and three-body processes in a form of a two-body process.
For the πN elastic partial waves we used the VPI/GWU single-energy solutions [22, 23].
For the $\pi N \rightarrow \eta N$ partial-wave data we used the coupled-channel amplitudes from Batinić *et al.* [17], but instead of using smooth theoretical curves, we constructed the data points by normally distributing the model input (see ref. [16]).
Fitting strategy was taken over from ref. [16].
The obtained curves correctly reproduce all input partial wave data for πN elastic and $\pi N \rightarrow \eta N$ process, but are because of lack of space given elsewhere [24].

In Fig. 1 we show two groups of results: scattering matrix poles for two lowest negative and two lowest positive parity partial waves.

First group of results, the two lowest negative parity partial waves S_{11} and D_{13}, pretty well confirm our assumption. All three bare propagator poles for both partial waves can be naturally identified with lowest QMRS of refs. [25]. We do see some discrepancies in mass position, but each required bare propagator pole does qualitatively correspond to a particular QMRS, and all lowest QMRS have found their bare propagator counter partners.

The obtained CC_T scattering-matrix pole positions correspond reasonably well to the experimental values reported in ref. [15]. The only disagreement, the unexpected position of the third SMP of the S_{11} partial wave (too far in the complex energy plane), is again a consequence of the fit-results instability, and is expected to disappear with including more channels. All three experimentally detected SMPs for the D_{13} partial wave are reproduced, but the lowest two are somewhat shifted in mass.

We also symbolically visualize the influence of the interaction upon bare propagator poles, and their "journey" from the initial QMRS to the final SMP positions. In the world without interaction mixing matrices γ vanish, we have no "dressing", and bare propagator and scattering-matrix poles are identical. In the real world, in the world with interaction, the γ matrices are non-vanishing, and are obtained by fitting the partial wave data. Arrows represent the way how bare propagator poles travel from the world without interaction (γ=0) to the real world scattering-matrix singularities ($\gamma \neq 0$).

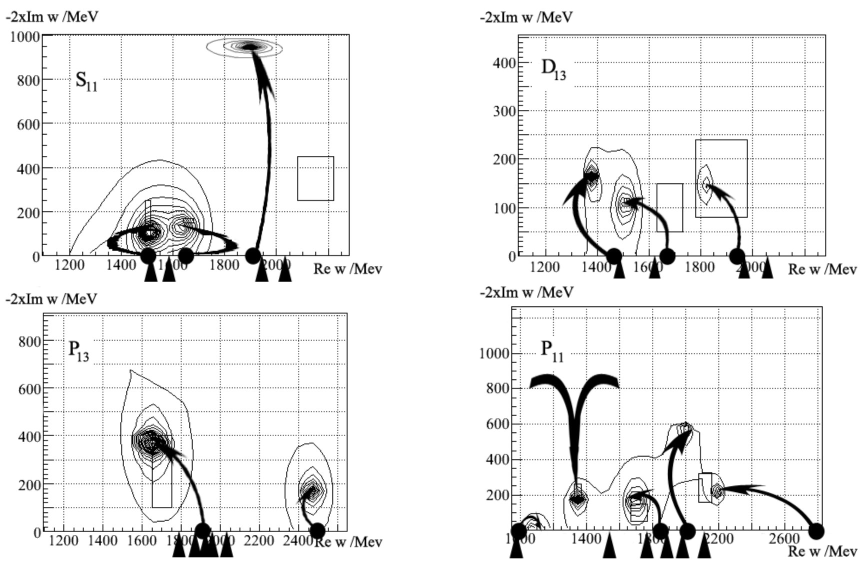

Figure 1: Scattering-matrix singularities and bare propagator pole positions. Full dots denote bare propagator pole positions, triangle arrows denote the few lowest quark-model resonant state masses of refs. [25].

Next group of results, the two lowest positive parity partial waves P_{13} and P_{11}, is still consistent with the hypothesis of the article, but some problems appear. Only one out of five QMRS of ref. [25] for the P_{13} partial wave is identified with the bare propagator pole, while other states remain yet to be identified. The second required bare propagator pole should either be identified with one of the higher lying QMRS, or will be shifted downwards when results of the fit stabilize.

The notoriously problematic P_{11} partial wave, however, remains a troublemaker as in majority of theoretical considerations. First, we needed four bare propagator poles in order to achieve acceptable fit to the input data. Having in mind that we should identify bare propagator poles with *all* quark-model states (resonant *and* bound), we have fixed the value of the first bare propagator pole to the mass of the sub-threshold nucleon pole, and left the remaining three poles unconstrained.

Problems start with the identification of QMRS with bare propagator pole position. In ref. [16] we have demonstrated that the presence of inelastic channels directly produces the *N(1710)* P_{11} SMP, and in Fig. 1 we show that it is generated by dressing the 1.854 GeV bare propagator pole. This pole can be directly associated with one of the quark-model states of ref. [25], either 1.770 or 1.880. The nucleon state is producing an insignificant, sub threshold and experimentally inaccessible pole at 1.1 GeV; remaining two poles at 2.018 and 2.759 produce SMP of 2.2 GeV which can be identified with poorly determined *N(2100)* P_{11}, and an experimentally not yet established state at 2 GeV.

However, our model with constraining data in only two channels shows two very interesting features for P_{11} partial wave: *i) no bare propagator pole which would correspond to the 1.540 quark-model state is needed; ii) one of experimentally confirmed SMPs, namely the N(1440)* P_{11} *state - Roper resonance, is not produced by any nearby bare propagator pole as it was the case for all other scattering-matrix poles; it is generated differently.*

The CMB model in conjunction with our interpretation of physical meaning of bare propagator poles offers us a natural way to characterize the nature of scattering-matrix resonant state. We propose a criteria: the *genuine* SMRS is a state which is produced by a nearby bare propagator pole; the *dynamic* SMRS is a state which is created out of distant bare propagator poles through the interaction mechanism itself.

According to this definition, we have no need for such an entity as the "Roper quark-model resonant state", in our model Roper resonance is a dynamic resonant state.

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