# Modeling of Control Loop in Production Scheduling for Overall Inventory Cost Reduction

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## Abstract

*Performance of manufacturing organization depends on production scheduling which is an essential part of the management of production systems. Effective scheduling can lead to performance which results in reducing work in process inventories [1]. According to the lean production principles, excessive production and high inventory level are the biggest waste in production process [2]. Inventory level higher then it is necessary for fulfilling the costumer demands, leads to additional costs in warehouse. Objectives of production control are graphically represented over time in diagrams: inventory, lead time, input orders and output of production process. In this paper, the fact that the input and output amount of work coming from released orders of one product are not equal in every period, results with a technique to balance input against output continuously, to establish a control loop.*

## 1. Introduction

The input and output amount of work coming from released orders are not equal in every period, so any shop floor control system has to look for a technique to balance input against output continuously, and to establish a control loop [3]. The scheduling is carried out by an employee, a computer program, or a combination of both and acts as a controller, fixing the planed status and thus leading in the control line to the actual production process. The process is monitored by feedback records of the inventory level to minimize influence of disruptive factor that is inconstant released orders. The aim of this paper is to define adequate mathematical model which is used to determine the influence of input parameters, release orders quantity and forecast on inventory level. The closed-loop scheduling and control cycle responsible for a continuous flow of manufacturing process and inventory level maintenance is one of internal parts of production scheduling and control process.

## 2. Production process simulation model

To schedule production of one product, wood casement with standard dimensions and shape, which consists of three production processes, the simulation model has been made. The material flow in production process, information flow, and all process data necessary for building a simulation model are shown on Figure 1.

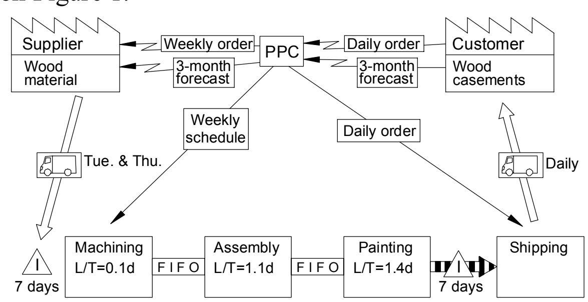


**Figure 1. Material and information flow in the production process**

The examined model, created using ProModel software tool, has all necessary properties of production process. The total lead time L/T for the product is 2.6 working days. The production process is organized in one shift and five working days per week. The inventory level in the warehouse of finished products is set to value which is suitable for fulfilling the costumer orders for at least 7 working days. The costumer orders quantity is not exceeding value of 60 products per day. To avoid every day change of minimum inventory level, two values are set for all possible situations. The lower value, 150 products, is set if the required inventory level is lower then 200 products, and for required inventory level higher then 200 products, value of 200 finished products is set in simulation. The required inventory level is calculated from three month forecast information from costumers.

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The simulation also has possibility to increase inventory level in 100 product increment, according to the increased costumer demand for a longer period of time.

The warm up period of 40 days has also been included in the simulation. This period provides stabilization of production process, and establishment of closed loop scheduling, before the statistical data have been gained. After end of the warm up period, when the predefined mistakes included into start period become negligible, the inventory level of the warehouse of finished product is being maintained only by contribution of mathematical model. The both start inventory level in warehouses and work in process material are predefined upon experience knowledge in the start of the simulation, and have to be accurate enough to simulate warm up period without reaching any of constraint situation.

The input and output curves of the work center within a period generally do not follow a straight line, and vary quite strongly at times, and lead to producing an unsteady inventory level. Because of that, simulation of the model was examined and optimized for the orders entered into simulation with external file generated by random number generator with addition of noises in generation like difference of order level for different periods of year. Orders used in simulation for optimizing the production plan are shown on Figure 2. for one year simulated period.

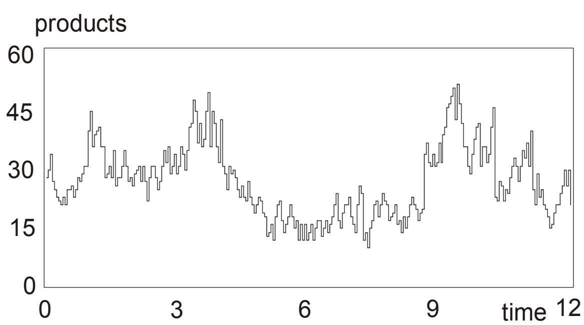


**Figure 2. Released orders within one year period**

Input parameters used to establish a closed loop production control have been evaluated at the end of the scheduled week, at Wednesday morning. The production demand according to the weekly plan quantity $Q$ can be expressed by a linear equation:

$$Q = x_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 \qquad (1)$$

The value $x_0$ is evaluated at the end of scheduled week, and gives a production plan according to the produced quantity in the last week of simulation. This value is the referent value for closed loop control, and other three members in equation increase or decrease that value. In case those other members in equation are equal to zero, the production plan is the same for next week like it was for previous simulated week.

The first parameter $x_1$ is difference between current inventory level in the final product warehouse and inventory level which has to be maintained. It shows the number of products per day which have to be added to the production plan, in order to achieve the required inventory level at the end of next simulated week. The evaluated number is then multiplied with the factor of signification $a_1$. The value of this product gives the partial influence on the production demand for next week of simulation in the way that it increases or decreases the production demand compared to last week of simulation.

The second parameter $x_2$ is the difference between quantity in orders from last simulated week and the week before, and shows how many products per day the average daily order is changed. This number presents possible gradient of orders quantities, and shows both new trend in order quantities in next period and change of inventory level during next week caused by difference in order quantity. The evaluated number is then multiplied with the factor of signification $a_2$ to apply partial influence of this parameter on the production demand.

The third parameter $x_3$ is the difference between the current production demand and forecast for next three months gathered by the costumer. This value represents the difference in number of product per day which has to be added to production demand on a shop floor to achieve the level of production set by the three month forecast. This parameter changes production demand to assure the fulfilling of costumer demands if those follow the three month forecast quantity. Demand forecasting is frequently different from the actual production plan [4]. Those three months forecasts is based on the expected orders in next three months period according to the external file order list, but modified with normal distribution factor which make those data individually different up to 50% then simulated future orders quantities.

## 3. Optimization process

Optimization of the model has been executed by changing input factors of signification and monitoring a result of an objective function. The objective function is maintanance of the accumulated inventory level in the final product warehouse at the lowest possible level. According to the lean production principles, excessive production and high inventory level are the biggest waste in production process [2]. Inventory level higher then it is necessary for fulfilling the costumer demands, leads to additional costs in warehouse. To decrease overall costs, the inventory has to be maintained at the lowest possible level, but high enough to fulfill all customer orders in next period.

Simulation has been executed with a goal to minimize number of product days in the final product warehouse, by changing factors of significations $a_1$, $a_2$ and $a_3$. These factors, together with examined parameters, make changes in production week plan quantity.

During simulation, ProModel software constantly monitors content of finished product warehouse. The content is daily evaluated from the simulation and added to accumulated value in previous period. The value of total inventory shows how many finished product days are stored in the warehouse.

Optimization process has been executed by multiple simulation of the same model with different combinations of factors of signification. The optimal factors of significations are given in Table 1.

**Table 1. Optimal factors of signification**

| $a_1$ | $a_2$ | $a_3$ | Total inventory [product day] |
|---|---|---|---|
| 0.05 | 1.00 | 0.29 | 39965 |

The optimal mathematical model for the next week plan for the shop floor can be expressed by equation:

$$Q = x_0 + 0.05x_1 + 1.00x_2 + 0.29x_3 \tag{2}$$

On Figure 3. is shown required inventory level in warehouse of finished product for fulfilling 7 days order quantities according to the three month forecast. The inventory level is unsteady, due to a very big variation in orders quantity. The weekly production plan generated by optimal mathematical model is shown on Figure 4. In case of examined production process, customer demands increases up to four times different value in period within two months. The optimal mathematical model handles this situation with rapidly increasing of production demand on shop floor. After decreasing costumer order quantities and normalizing them on the average level for next period, the model needs additional period for stabilization of inventory level in warehouse of finished products. Every period of unsteady order quantities, produces variation of the inventory level. On the Figure 5. is shown inventory level in the finished product warehouse over one year simulated period. The growth of total accumulated inventory in warehouse of finished products is shown on Figure 6. The value at the end of simulated year is the reference value for the optimization process.

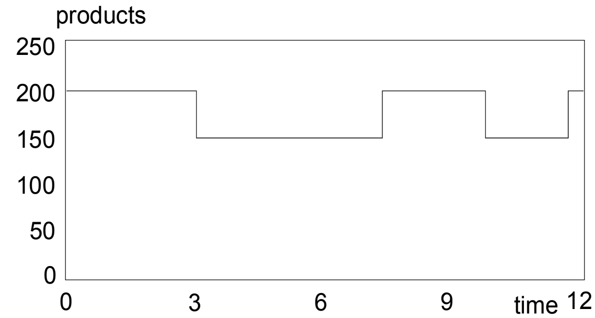


**Figure 3. Required inventory level for average 7 days order quantity**

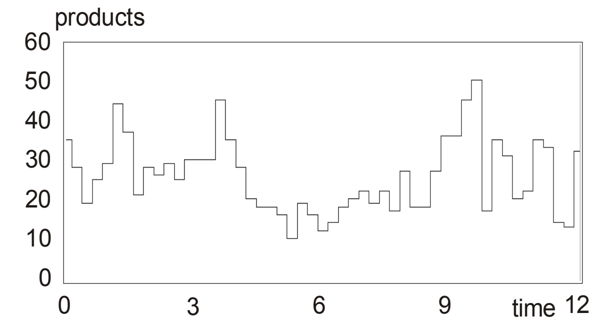


**Figure 4. Weekly production plan by the mathematical model**

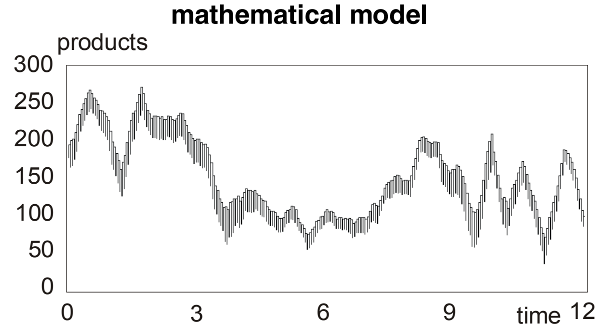


**Figure 5. Inventory level in finished product warehouse**


**Figure 6. Accumulated total inventory in product day in finished product warehouse**

During simulation, data about total inventory value is monitored for values of factors different from optimal ones. This list of collected results shows that any other combination of factors of significations gives the bigger total inventory value, and therefore worse result then optimal. For visualization of response surface, DesignExpert software is used. In a polynomial interpolation, all three input factors of significations were used at a same time. The total inventory value in warehouse of finished products is the response surface, which value is gained by 20 replications of the same simulation model with the different factors of signification for second order experiment plan. Number of replications, and factor values used in replications, have been determined by DesignExpert response surface program module. On Figure 7., 8. and 9. is shown the influence of two factors, at the optimal third factor level.

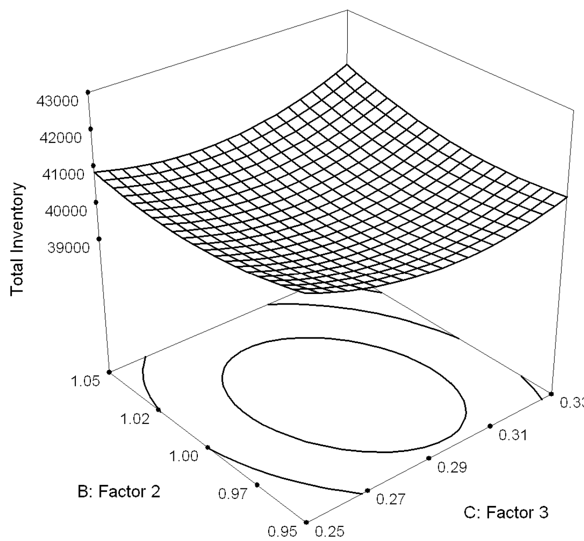


**Figure 7. Response surface for optimal Factor 1 value ($a_1 = 0.05$)**

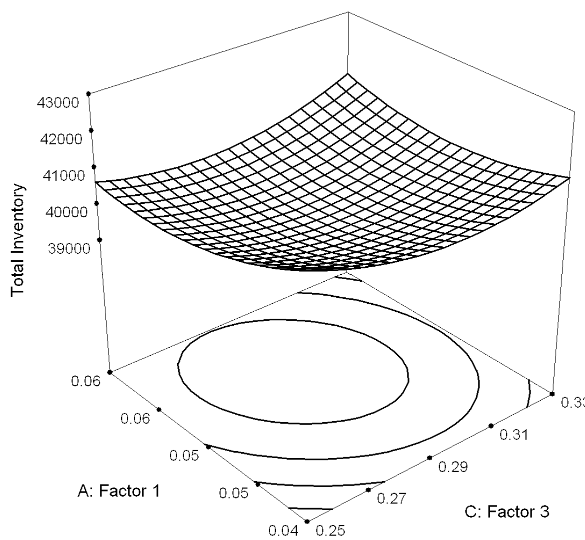


**Figure 8. Response surface for optimal Factor 2 value ($a_2 = 1.00$)**

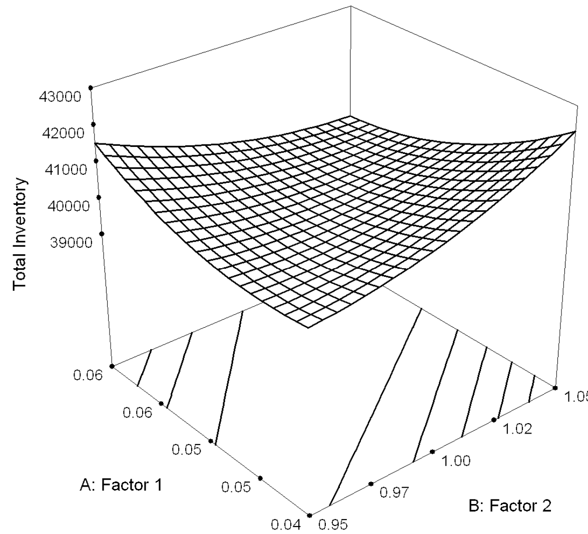


**Figure 9. Response surface for optimal Factor 3 value ($a_3 = 0.29$)**

With larger number of replication and with higher number of factors decimal places then two, the response surface will be even smother and more accurate. From the visualization of response surface, it is possible to recognize the interaction of influential factors and which factor change results with the biggest influence on total inventory value.

Besides the Factor 2, which has the biggest influence on the total inventory value, Factor 1 has very small influence. The optimal value $a_1$=0.05 gives the best response of mathematical model, and the biggest influence of this factor is represented in maintenance the inventory level close to the inventory level required by three month forecast gathered by the costumer. This factor is responsible for feedback information from warehouse of finished products. Simulation at the different values of Factor 1 then optimal one, responses with different total inventory values, which is shown on Figure 10.

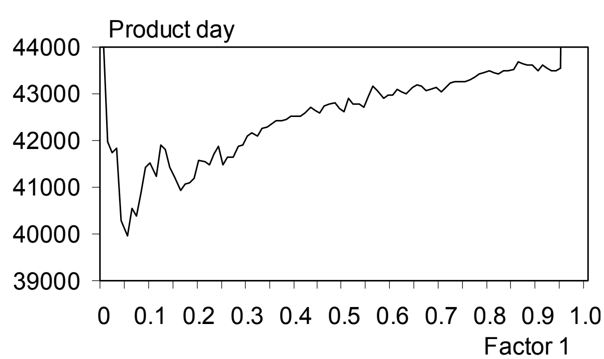


**Figure 10. Total inventory value at different Factor 1 values ($a_1$)**

During simulation at values of Factor 1 higher then 0.95, in one or more days within one year simulated period, the inventory level was not suitable for fulfilling the costumer demand. That is the constraint situation, which can not be permitted, and because of that, simulation stops without further examination of total inventory value at higher Factor 1 values then those shown on Figure 10.

The Factor 3 changes response with small reduction of the total inventory value. After achieving the optimal factors of signification, the examination of Factor 3 has been done to determine its influence on all possible values which could be entered into mathematical model. Other two factors, Factor 1 and Factor 2 were set at the optimal value. The range of possible values of Factor 3 and the resultant total inventory value according to the examined model are shown on Figure 11.

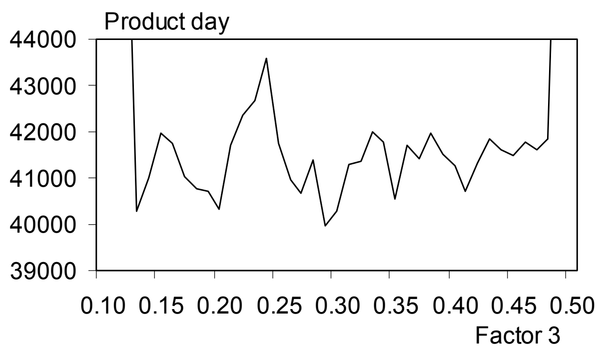


**Figure 11. Total inventory value at different Factor 3 values ($a_3$)**

It is obvious that the influence of Factor 3 is not significant, and it leads to unsteady total inventory value at the range from minimum value 0.13 to maximum value 0.48. For the value lower then 0.13, and higher then 0.48, the control process becomes not suitable for fulfilling the costumer demands.

Optimization process found the global minimum of objective function, and that was confirmed with this examination process. On the other hand, the result of total inventory for the value $a_3$=0.13 is 40284, which is the value very close to the optimal one. The fact that the result value is not significantly changed with the variation of Factor 3, gives a strong reason to investigate the changes in inventory level control to prove the importance of implementation the three month forecast data, provided by the costumer, in the control loop process. During the simulation process, the future order quantities are gained from the external file, changed individually by normal distribution factor to avoid correct three months forecast. On the Figure 12. is shown the difference in inventory level over one year period for two different Factor 3 values. The comparation of these two diagrams is used to made conclusions. The influence of three month forecast data on production scheduling is mostly used for decreasing the cost of inventory level and cost of work in process material during some periods of examined time. This influence is mostly presented in period of three months in the middle of simulated time. This period with decreased production demand values, and decreased required inventory level, has reduced total inventory cost. It is possible to use this periods of year for increased production of other products with bigger three month forecast, then average. But, for a whole year period, with the same produced amount of products, the saves in periods with decreased three month forecast are lost in the periods with increased forecast and total cost. The total inventory cost over one year period, with three month forecast included or not, presents the values very close to each other.

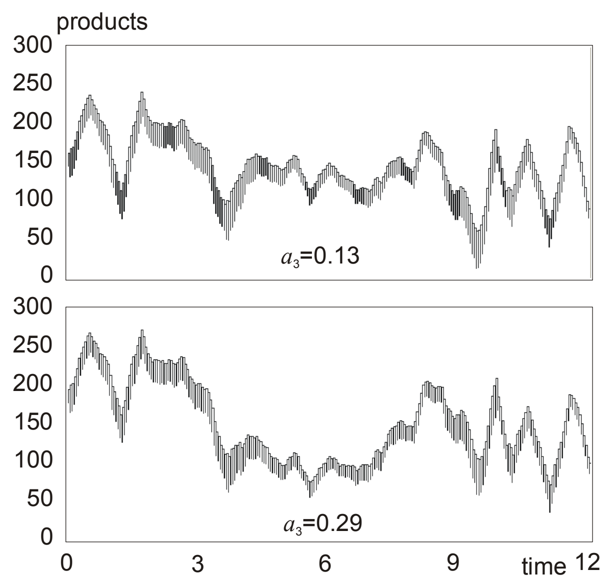


**Figure 12. Inventory level in finished product warehouse for the two different Factor 3 values ($a_3 = 0.13$ and $a_3 = 0.29$)**

Without implementation of Factor 3, which represents the influence of forecast, the required inventory level in the finished product warehouse is constant for the whole year. Planning the production with the constant inventory level disables the possibility to organize production on increased level for other products required by the costumer's demands.

The raw material is supplied two times in every simulated week, on Tuesday and Thursday. The three month forecast for raw material is provided by the planning and production control department and electronically delivered to the supplier. It is calculated from production demand for next week generated by the mathematical model and the three month forecast of finished products demand quantities.

The future research will be in the direction of implementation of this or similar simple mathematical models into simulation with two or more different products processed on the group of work centers on the same shop floor. In manufacturing systems with a wide variety of products, processes, and production levels, production schedules can enable better coordination to increase productivity and minimize operating costs [5].

## 4. Conclusion

The aim of this paper was to define adequate mathematical model which is used to determine the influence of input parameters, released order quantity and forecast on inventory level in the finished product warehouse.

The mathematical model, which is used in control loop, has to be optimized to give best response on actual released orders quantities. If mathematical model has the members of equation which gives a faster, or slower response then optimal, the production plan for next week will be or insufficient, or excessive for fulfilling the costumer demands. For given input parameters of simulation model in the mathematical model for production plan, any other factors of signification values will give the larger total inventory value which makes that mathematical model for control not good as optimal one. The optimal factor values give the relationship in influence between those three examined parameters on a production demand. Also, the interaction of all examined parameters has been made.

Conclusion can be made that value of orders quantity gradient parameter $x_2$ is the mayor parameter in calculation of production demand in the comparation to the other two parameters. The contribution of this parameter is the biggest, and it will result with expected rapid change of inventory level at the end of next simulated week. This parameter shows the possible trend in order quantities for next few weeks. It is used to produce the difference amount of products from last simulated week and the week before.

The parameter $x_1$ is responsible for achieving the required level in warehouse, but its contribution is small, because of interaction with Factor 2. For next simulated week, this factor is used to calculate the number of products which has to be added to the production plan for previous week. At the other side, every day warehouse inventory level is changed by the arrival of finished products. Production lead time of 2.6 working days results with a delay of arrival of products which were entered into production according to the previous planed week. This factor of signification has a stronger influence in cases of longer production lead times, when the delay is too high and therefore not sufficient to contribute in the regulation of the inventory level for one week planning.

The parameter $x_3$ is responsible for fulfilling the costumer demand according to the three month forecast. It has small contribution and depends mostly of forecast accuracy. It can be also used for defining the requested inventory level for fulfilling costumer orders for at least seven working days.

In the finished product warehouse, the minimum inventory level was about 30 finished products. The ideal situation would be the constant inventory level over whole year at the required inventory level. The constraint situation would appear if there was insufficient number of finished product for shipping. This constraint situation results in simulation break, and requires simulation restart with new factors of signification which will provide better maintenance of inventory level. Optimization process continues only with factors which provide the fulfilling of all costumer demand quantities. In this simulation and with daily order quantities by external file, the inventory level can be lowered by 30 products, without achieving a constraint situation. That would result in reduced total inventory value then shown in this paper.

For the production processes with different lead times the speed of response on changes in order and inventory level will be different, and the mathematical model can be optimized by the simulation for that particular process. This approach in production scheduling can be used for encouragement of production control personnel in defining the production plans for the shop floor.

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