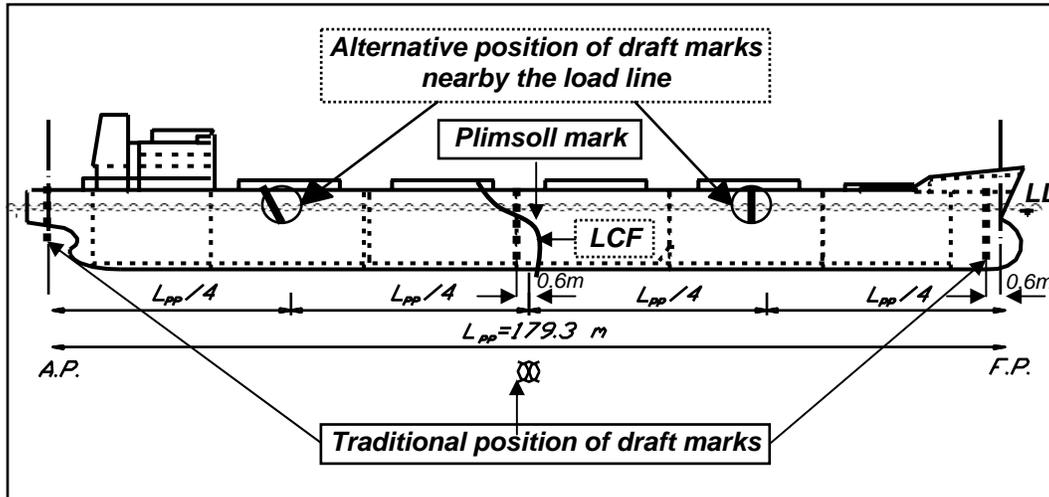


# Displacement of a Deflected Ship Hull

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*This note summarizes relevant approximations based on permanent waterplane characteristics for a more accurate assessment of the effects of longitudinal deflections of an elastic ship's hull on drafts and displacement during marine surveys of merchant ships. The possibility of a different practice for placement of draft marks alongside a ship, ensuring the same precision in displacement assessment of a deflected and trimmed hull by only two draft observations, is investigated by employing the waterplane properties. Correction factors for conventional ships, based on waterline coefficient for changes of hydrostatic load, arising due to hull deflections, are also summarized. The results are demonstrated on a bulk-carrier built in Croatia as an example.*

## Nomenclature

$A_{wl}$ =area of the waterplane;  
 $B_{wl}$ =beam of the waterplane;  
 $C_d$ =correction factor for draft of a deflected hull;  
 $C_M$ =correction factor for bending moments;  
 $C_Q$ =correction factors for the shear force;  
 $C_{WP}$ =waterline coefficient;  
 $d$ =draft in general;  
 $I_L$ =moment of inertia of the waterline about a transverse axis through the center of flotation;  
 $LCF$  = center of flotation of the waterplane;  
 $L_{wl}$ =length of the waterplane;

$MTI$ =moment to change trim one unit;  
 $M^d$ =change of the bending moment;  
 $Q^d$ =change of the shear force;  
 $TPI$ =tons per unit of immersion;  
 $x_d$ =positions of equal equivalent and observed drafts;  
 $w$ =hull deflection in general;

### Greek symbols

$\gamma_{sea}$ =specific gravity of sea water;  
 $\Delta$ =displacement in general;

### Subscripts

$a, m, f$ =related to aft, mean and forward;

### Superscripts

$e, s, o$ =related to equivalent, standard and observed;  
 $d$ =related to deflected hull

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## Introduction

A ship's displacement plays the most important role in the validation of ship's operational efficiency. Nevertheless, the determination of displacement is not always accurate. Numerous related conditions on-board are practically imperceptible and immeasurable. Some uncertainties come from different global, local, thermal, longitudinal or transverse, as well as permanent or temporary deformations of an immersed ship hull. Additionally, a hull is subjected to fouling, corrosion, damages, reparations and aging. Normally in the end, hull deformations cannot even be considered in practical determination of the displacement on-board during draft and deadeight surveys in the ship's service. Hence, it is nearly impossible to accurately determine the ship's displacement. Although ships operate under different loading conditions and with widely varying amounts of trim and hull deflections, the commonly used hydrostatic particulars are traditionally defined for the ship floating at successive plane waterlines parallel to each other and usually parallel to the base. In most cases when the trim is not too great and the deflection is small, however, it is entirely satisfactory, to make use of approximate practical calculations based on hydrostatic particulars and ordinary displacement curves, otherwise numerical lengthwise integration over inclined and deflected sections using Bonjean's curves is more appropriate (Comstock, 1967). This note tries to add impetus to a more accurate and practical assessment of a ship's displacement provided primarily for deadweight survey of loaded merchant ship (Durham, 1982). A comprehensive procedure applicable during marine surveys on-board for relatively small longitudinal deflection effects of an elastic ship's hull on drafts and displacement is suggested. The displacement assessment is based on standard hydrostatic particulars derived for a ship's hull as a rigid body, as well as direct observations during a draft survey. The idea presented in the note makes use of a parabolic approximation of the deflection line, as do most methods. The method suggested here takes the approach further by considering the actual ship's hull form via the waterplane properties (Ziha, 1997). The note also demonstrates how a different placement of draft marks alongside a ship hull, nearby the load line, may be appropriate during a draft survey. Suggested placement of draft marks at once provides an equivalent input draft accounting simultaneously for deflection and trim corrections for displacement assessments using only two draft readings, aft and forward at specified positions. The proposed placement of draft marks provides at least an additional accuracy checking during a deadweight survey. Finally, the methods are illustrated by a bulk-carrier example.

### 1. Assessment of the drafts and longitudinal deflections

In practice of a draft survey, the means of the drafts measured on draft marks placed on ship's port and starboard sides, not necessarily exactly on the aft and forward perpendiculars and in the midship section, are recalculated to drafts on positions in virtual coincidence with respective perpendiculars and midship section, Fig. 1., and are designated as observed drafts aft, amidships and forward, as shown:

$$d_a^o, d_m^o, d_f^o.$$

Reading and averaging both port and starboard draft marks is useful but rarely done in service due to the expense. The standard mean draft amidships used for conventional hydrostatic calculations during the deadweight survey of a hypothetically rigid ship hull on an assumed even keel position, Fig. 1., is obtained as the mean of the observed drafts aft and forward as follows:

$$d_m^s = \frac{d_a^o + d_f^o}{2} \quad (1)$$

The effect of a small trim on displacement can be assessed on the basis of the known position of the longitudinal center of flotation *LCF*, and superimposed during a deadweight survey to the displacement of a ship hypothetically on an even keel (Comstock, 1967) and floating on a standard mean draft (1).

The ship hull deflection is usually considered as hog or sag only amidships, defined as the deviation of the observed draft amidships from the standard mean draft amidships (1) and can be presented as:

$$w_m = d_m^s - d_m^o \quad (2)$$

It follows from (2), that the deflection amidships is positive for hogging condition and negative for sagging condition, Fig. 1.

A commonly adopted method of marine surveys for assessment of the effects of the hull longitudinal deflection on the ship displacement is to perform standard hydrostatic calculations with an equivalent value of draft amidships (Comstock, 1967), denoted as "quarter mean draft" or "mean of mean draft", as shown:

$$d_m^{w1/4} = \frac{d_a^o + 6 \cdot d_m^o + d_f^o}{8} \quad (3)$$

By substitution of (1) and (2) into (3), the following equivalent term explains the meaning of (3), as follows:

$$\begin{aligned} d_m^{w1/4} &= d_m^o + \frac{1}{4} \frac{d_a^o + d_f^o}{2} - \frac{d_m^o}{4} = \\ &= d_m^o + \frac{1}{4} \cdot (d_m^s - d_m^o) = d_m^o + \frac{1}{4} \cdot w_m \end{aligned} \quad (4)$$

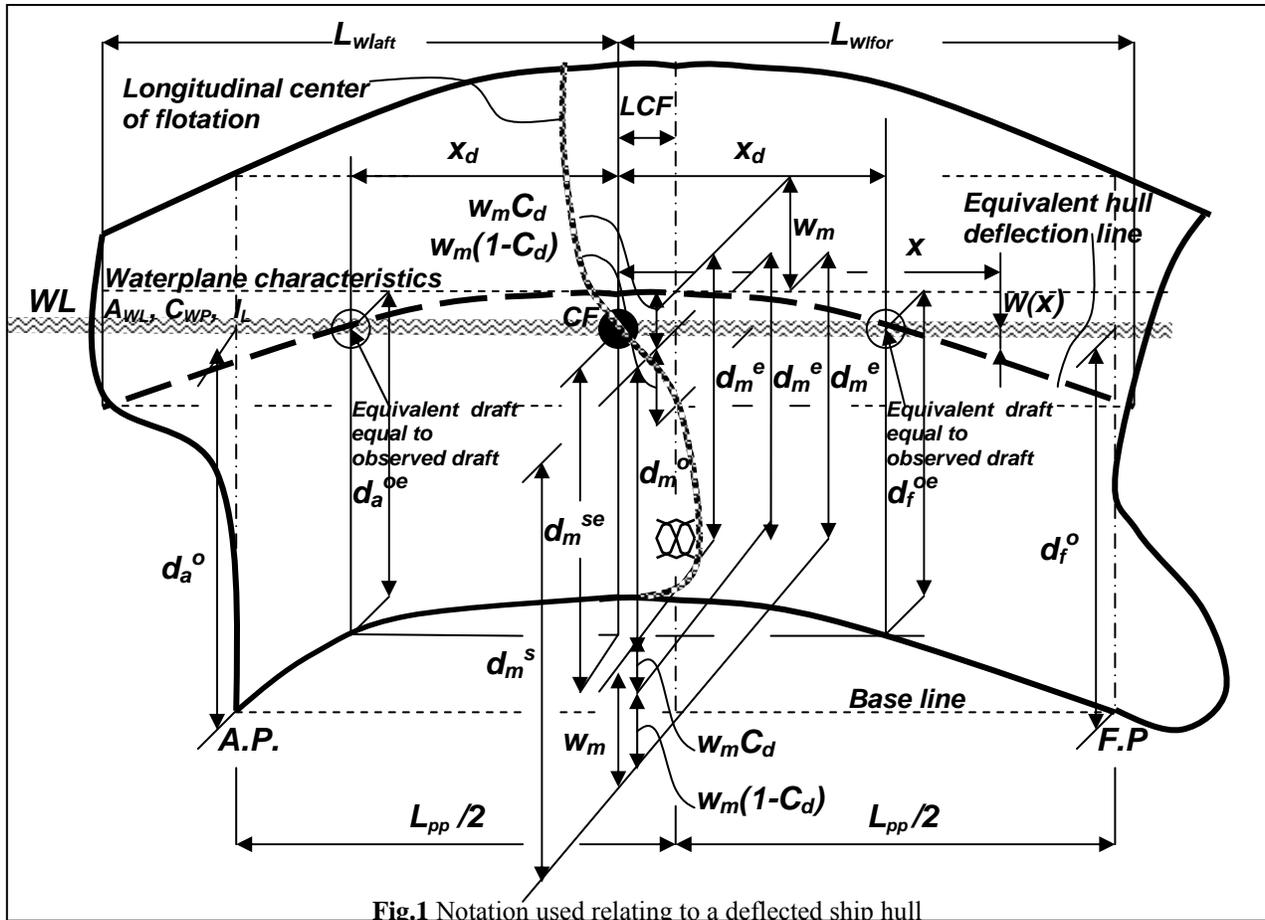


Fig.1 Notation used relating to a deflected ship hull

Put succinctly, the equivalent “quarter mean” draft (3) for assessment of a displacement of a deflected hull is obtained by modification of the mean of draft readings on port and starboard sides amidships for one quarter of the deflection observed amidships (2) during a draft survey.

The term (4) will be subjected to further critical investigation, especially with respect to the constant draft correction factor due to deflection amounting to 1/4. Let us assume a more general case of an equivalent draft (3) and (4), when instead of a constant 1/4, a variable draft correction factor  $C_d$  and its inverse denoted as draft correction coefficient  $c_d = 1/C_d$  are introduced:

$$d_m^e = \frac{d_a^o + 2d_m^o(c_d - 1) + d_f^o}{2c_d} = d_m^o + C_d \cdot w_m \quad (5)$$

Note that for  $C_d=1/4$  ( $c_d=4$ ) the equivalent draft (3) appears as a special case of the more general term for equivalent draft (5). In the sequel, a rational and more accurate method for assessments of the draft correction factor  $C_d$  based on an assumption of parallel ship's sides for small parabolic deflections for true waterplane geometric characteristics, will be investigated.

## 2. Assessment of the hull deflection effects

A second order symmetric parabola is often used to present the deflection line of the ship's hull:

$$w(x) = w_m \frac{x^2}{(L_{wl}/2)^2} \quad (6)$$

The parabolic deflection line after rectification represents the equivalent waterline defining the actual displacement of a ship as a rigid body, Fig. 1. It is obvious that the deflection line of a ship's hull is neither symmetrical nor parabolic, but the deviations of the parabolic form are usually of limited order of significance (Ziha, 1997). Experimentally and numerically determined hull deflection (Mackney and Ross, 1999) show that it can be satisfactorily fitted by parabola. On the other hand, it is not practical to determine the hull deflection shape on board more precisely; moreover, in most cases it is impossible.

A further assumption about the position of the maximal deflection close to the center of flotation  $LCF$ , since the exact position is practically indeterminable, may lead to simplification of draft survey procedure without significant effect on displacement calculation accuracy.

The hydrostatic characteristics considering small longitudinal deflections of the ship hull depend mostly on waterplane geometrical properties, Fig. 1.

The longitudinal change in buoyancy for hypothetically parallel ship sides due to assumed small longitudinal hull deflection  $w(x)$ , represents the change in the hydrostatic load relatively to the observed waterline and can be expressed at any abscissa 'x', using the waterplane breadth  $b(x)$  on the considered position, Fig. 1, as:

$$q_b(x) = \gamma_{sea} \cdot w(x)b(x) \quad (7)$$

The displacement of the deflected hull part in water of specific gravity  $\gamma_{sea}$ , supposing a parabolic deflection shape (6) with the extreme deflection assumed at position close to the center of flotation  $CF$ , either for hog or sag, is obtained by lengthwise integration of (7), as shown:

$$\begin{aligned} \Delta^d &= \int_{L_{wlf}}^{L_{wfor}} q_b(x) dx = \gamma_{sea} \cdot \frac{w_m}{(L_{wl}/2)^2} \int_{L_{wlf}}^{L_{wfor}} b(x)x^2 dx = \\ &= \gamma_{sea} \cdot w_m \cdot A_{wl} \frac{I_L/A_{wl}}{(L_{wl}/2)^2} = \gamma_{sea} \cdot w_m \cdot A_{wl} \cdot C_d \end{aligned} \quad (8)$$

The correction of draft amidships relative to the observed amidships draft, due to the displacement of the deflected hull part, can be determined as a parallel immersion or emersion employing area of a waterplane  $A_{wl}$  and moment of inertia  $I_L$  of the waterline with respect to the transverse axis through the center of flotation  $CF$ , as follows:

$$d_m^d = \frac{\Delta^d}{\gamma_{sea} \cdot A_{wl}} = w_m \cdot \frac{I_L/A_{wl}}{(L_{wl}/2)^2} = w_m \cdot C_d \quad (9)$$

The dimensionless draft correction factor  $C_d$  due to deflection amidships is derived by comparison of (5) and (9), only by basic waterplane geometric characteristics, and can be easily calculated from commonly available ship's permanent hydrostatic particulars, as shown:

$$C_d = \frac{I_L/A_{wl}}{(L_{wl}/2)^2} \quad (10)$$

Alternatively, in some practical cases, the correction of draft amidships may be approximated by substitution of

the moment to change trim one unit  $MTI$  and tons per unit of immersion  $TPI$  in (8) and can be expressed in consistent units (for inconsistent units, additional units conversion is required) as

$$d_m^d \approx w_m \cdot \frac{MTI}{TPI} \cdot \frac{L_{pp}}{(L_{wl}/2)^2}. \quad \text{Consequently, the}$$

alternatively defined draft correction factor is as shown:

$$C_d \approx \frac{MTI}{TPI} \cdot \frac{L_{pp}}{(L_{wl}/2)^2} \quad (10a)$$

For simplicity, the true waterplane shape of beam  $B_{wl}$  and waterplane area coefficient  $C_{WP} = A_{wl}/(L_{wl}B_{wl})$  can be approximated by a symmetric general parabola of order  $k$ , in the following form:

$$b(x) = B_{wl} \frac{(L_{wl}/2)^k - x^k}{(L_{wl}/2)^k} \quad (11)$$

The substitution of (11) in (7) and the repeated integration as in (8), may lead to satisfactory assessments of waterplane area characteristics as follows:

$$A_{wl} \approx B_{wl} L_{wl} \frac{k}{k+1} = B_{wl} L_{wl} C_{WP} \quad (12)$$

$$I_L \approx A_{wl} \cdot (L_{wl}/2)^2 \cdot C_d \quad (13)$$

The approximate draft correction factor for merchant ships with high waterplane area coefficient  $C_{WP}$ , based on assumption (11), is presented next:

$$C_d \approx \frac{1}{3(3-2C_{WP})} \quad (14)$$

Note also that  $k = C_{WP}/(1-C_{WP})$ .

It is easily recognizable from (14) and Fig. 2., that  $C_d$  significantly differs from 1/4 and that it is an increasing function of  $C_{WP}$ . Specifically,  $C_d=1/4$  only for a unique value of  $C_{WP}$  amounting to  $C_{WP}=0.834$ . The maximum value of  $C_d=1/3$  is attained for  $C_{WP}=1$ , i.e. for rectangular waterline shape. The minimum value of  $C_d=1/6$  is attained for  $C_{WP}=1/2$ , i.e. for triangular waterline shape. The draft correction factors for a bulk-carrier (10) and (10a) almost coincide and the approximation draft correction factor (14) based on  $C_{WP}$  differ insignificantly, Table 1. and Fig. 5.

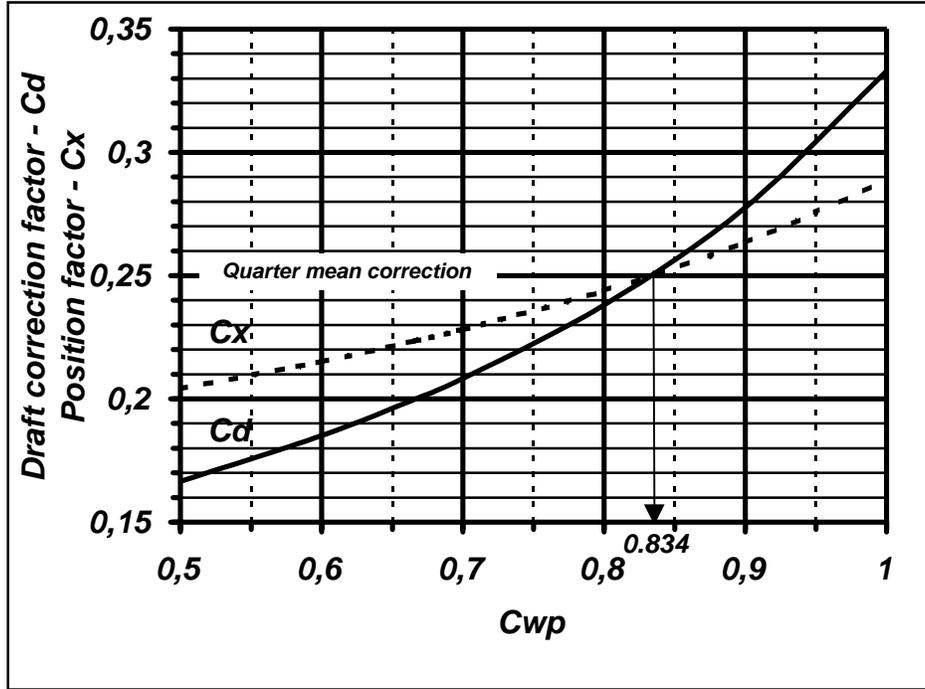


Fig. 2 Deflected ship hull draft correction factor and position factor for a generally parabolic waterplane shape

### 3. Assessment of the displacement

The displacement copied from the ship's permanent hydrostatic particulars (displacement curve), denoted as observed displacement  $\Delta^o$ , corresponds to the observed amidships draft during a draft survey, Fig. 3.

The standard displacement  $\Delta^s$  for a ship hull considered as a rigid girder on an even keel can be copied from hydrostatic data using the standard mean amidships draft  $d_m^s$  (1), Fig. 3.

The displacement  $\Delta^{w/4}$  for a deflected ship hull considered as an elastic body can be assessed from hydrostatic data for a rigid hull on an even keel, using the "quarter mean draft"  $d_m^{w/4}$  (4), Fig. 3.

Note that due to the assumption of parallel ship's sides for small deflections, the displacement curve is linear with first derivative equal to  $\partial\Delta/\partial d = \gamma_{sea} A_{wl}$ .

To obtain the actual displacement  $\Delta$  of the deflected hull from the permanent hydrostatic particulars for a rigid hull on an even keel, the equivalent mean draft can be used, as clarified on Fig. 3. The observed amidships draft is corrected to the equivalent amidships draft  $d_m^e$  using deflections amidships (2) and the draft correction factor (10) instead of constant 1/4 in (4), as shown:

$$d_m^e = d_m^o + d_m^d = d_m^o + C_d \cdot w_m = d_m^o + \frac{1}{c_d} \cdot w_m = \frac{d_a^o + 2d_m^o(c_d - 1) + d_f^o}{2 \cdot c_d} \quad (15)$$

The equivalent draft (15) is identical to the term (5) and represent a more rational basis for comprehensive and accurate displacement assessment of deflected ship hull of the commonly used equivalent "mean of mean" draft (3), since the draft correction factor  $C_d$  (10, 10a or 14) accounts appropriately for the hull form via the waterplane geometrical characteristics.

Alternatively, the standard mean draft amidships (1) can be corrected to the equivalent amidships draft using deflections amidships (2) and the draft correction factor (10, 10a or 14) as:

$$d_m^e = d_m^s - (w_m - d_m^d) = d_m^s - w_m(1 - C_d) \quad (16)$$

The actual displacement of a deflected ship hull  $\Delta$  can be obtained directly from standard and permanent hydrostatic particulars (displacement curve), using the equivalent amidships draft  $d_m^e$ , Fig. 3.

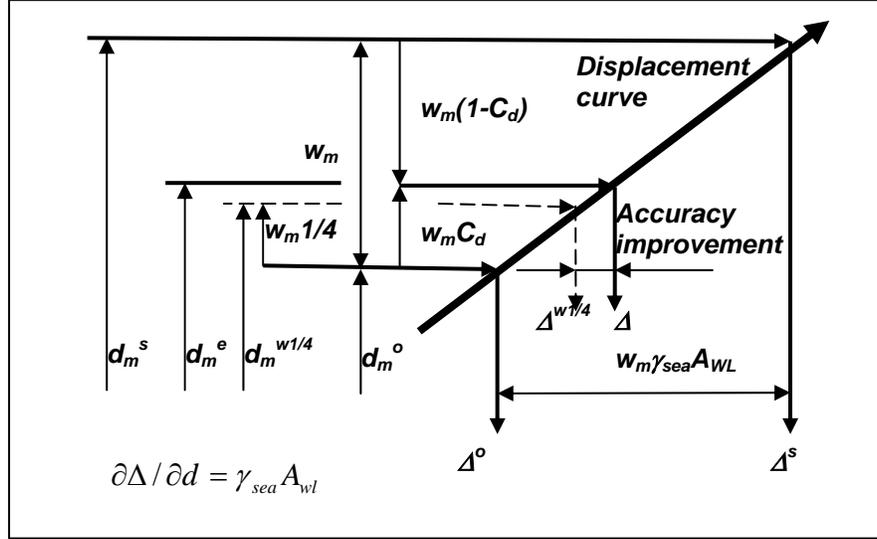


Fig. 3 Drafts and displacement of a deflected ship hull

Hence, the actual displacement of a deflected ship hull on a hypothetically even keel can be defined relative either to the standard displacement or to the observed displacement, Fig. 3., as follows:

$$\Delta = \Delta^o + \Delta_d^d = \Delta^o + w_m \gamma_{sea} A_{wl} C_d \quad (17)$$

$$\begin{aligned} \Delta &= \Delta^s - (w_m \gamma_{sea} A_{wl} - \Delta_d^d) = \\ &= \Delta^s - w_m \gamma_{sea} A_{wl} (1 - C_d) \end{aligned} \quad (18)$$

The expressions  $\gamma_{sea} A_{wl} C_d$  in (17) and  $\gamma_{sea} A_{wl} (1 - C_d)$  in (18) represent the displacement correction per unit of longitudinal deflection of the ship hull amidships, relatively to the observed displacement or standard displacement, respectively.

If there is a trim encountered on board, displacement of a hypothetically rigid hull on an even keel position (17-18) have to be additionally corrected for changes due to the trim. When the trim is not too great, the axis of rotation for change of trim without change of displacement may be assumed to pass through the center of flotation  $LCF$  of the even-keel waterline. In order to find the true displacement for a trimmed waterline the ship is imagined as rotate back to a waterline parallel to the base. Then the solution may be reached algebraically (Comstock, 1967). For very large trims and deflections, it is best to make use of Bonjean's curves for direct lengthwise integration of curves of areas of inclined and deflected sections. Water density correction is normally included during a deadweight survey.

#### 4. Alternative placement of draft marks

The draft marks on ship's port and starboard sides aft and forward, are traditionally placed as close as possible to the ship's aft and forward perpendiculars. The draft marks amidships are placed as close as possible to the middle of the length between the perpendiculars.

However, the traditional placement of draft marks was instituted when the ships were not as long as modern merchant vessels. The next consideration will demonstrate how an alternative displacement evaluation or at least a supplementary displacement checking procedure may be provided during a deadweight survey of loaded merchant ships by some other placement of draft marks nearby a load line alongside ship's hull, at positions specified in the sequel, see frontispiece.

When the deflection is not too great, a second order symmetric parabola (6), shifted vertically relatively to the observed waterline, for amount of parallel immersion defined as a fraction of the maximal deflection supposed at a position close to the center of flotation in amount of  $w_m C_d$  in (6), Fig. 1., can be used as an approximation denoted equivalent deflection line of the hull, as shown:

$$w(x) = w_m \frac{[x^2 + C_d (L_{wl} / 2)^2]}{(L_{wl} / 2)^2} \quad (19)$$

In order to find the true displacement of a deflected hull, it is imagined that the equivalent waterline is rectified back to a plane waterline by vertical translation of hull sections at some position "x", amounting to the value  $w(x)$  defined by the equivalent deflection line (19), Fig. 1. The positions along the ship's hull with respect to

longitudinal center of flotation  $LCF$ , where the observed drafts are equal to the equivalent draft can be obtained in intersections between the equivalent deflection line and the observed waterline by employing the draft correction factor  $C_d$  (10), (10a) or (14), Fig. 1., from the condition  $w(x)=0$  in (19):

$$x_d = \pm L_{wl} \frac{\sqrt{C_d}}{2} \quad (20)$$

The position factor denoted  $C_x$  in (20), is defined as shown:

$$C_x = \frac{\sqrt{C_d}}{2} \quad (21)$$

The position factor (21) employing the simplified draft correction factor (14) is presented on Fig. 2.

For a bulk carrier, the position factor (21) using improved draft correction factor (10) is calculated in Table 1., and presented on Fig. 5.

The drafts observed on ship sides aft and forward, on lengthways positions  $x_d$  from the center of flotation  $CF$ , are denoted as  $d_a^{oe}$ ,  $d_f^{oe}$ , Fig. 1.

The distance defined by (20) can be interpreted as the lengthways position of draft observations which provide directly immediate values for equivalent draft calculation. Following the assumption about the position of the maximal deflection close to the center of flotation  $LCF$ , the standard equivalent draft is defined by only two draft readings, compared to the conventional term (3) which requires three draft readings, as it is shown:

$$d_m^{se} = \frac{d_a^{oe} + d_f^{oe}}{2} \quad (22)$$

The standard equivalent draft defined by (22) virtually coincides with respective center of flotation and yields to displacement assessment of a deflected hull very close to the displacement defined by equivalent draft (15).

Moreover, in the same time, the standard equivalent draft defined by (22) accounts for the correction due to the ship's trim.

The trim and drafts, with respect to the perpendiculars, for displaced aft and forward draft observations, can be calculated as presented next:

$$trim_{P.P.} = (d_f^{oe} - d_a^{oe}) \frac{L_{pp}}{2x_d} \quad (23a)$$

$$d_a = d_a^{oe} - \frac{trim_{P.P.}}{L_{pp}} \cdot \left( \frac{L_{pp}}{2} - x_d + LCF \right) \quad (23b)$$

$$d_f = d_f^{oe} + \frac{trim_{P.P.}}{L_{pp}} \cdot \left( \frac{L_{pp}}{2} - x_d - LCF \right) \quad (23c)$$

Since the position of the maximal deflection is practically unobservable, it can be assessed more or less

accurately of (2), only by employing the observed amidships draft  $d_m^o$ , if available, as follows:

$$w_m = \frac{d_m^e - d_m^o}{C_d} \quad (24)$$

Hence, for the determination of the equivalent draft (22) and trim (23), the aft, amidships and forward drafts must not be observed at all, until the assessment of the amount of a hog or sag amidships (24) is required.

Note that for  $C_d=1/4$ , it follows from (21) that  $C_x=1/4$ , from (20), replacing  $L_{wl}$  with  $L_{pp}$ , it follows that  $x_d=\pm L_{pp}/4$  and that  $d_m^{se}$  (22) is identical to  $d_m^{wl/4}$  (4).

In other words, when the draft marks are placed on quarters of the ship's length between the perpendiculars, the calculated standard equivalent mean draft (22) is equal to the "quarter mean draft" or "mean of mean draft" (3).

## 5. Assessment of the changes in bending moments and shear forces due deflection

The standard still water hydrostatic calculation provide drafts and displacement and the longitudinal strength calculation provide shear forces and bending moments for the ship hull considered as a rigid girder. Additionally, deflection line calculation can be performed, using ship's hull vertical sectional moments of inertia. The deflection of the ship hull changes the hydrostatic loads and in reverse it effects the drafts, the shear forces and the bending moments. The corrections of displacement and loads due to deflection of the hull can be accomplished during a longitudinal strength calculation using an iterative procedure (Ziha, 1997).

In addition, approximate terms employing the general parabolic approximation of the waterplane shape (11) allow the assessment of changes in shear forces and bending moments due to observed deflection of the hull, relatively to the values determined for the ship hull considered as rigid girder (Ziha, 1997). It is easily recognizable that the bending moments calculated for the conventional ships considered as rigid girders, are in general reduced when the resulting deflection of the ship hull is taken into account. The direction of changes of shear forces due to hull deflection depends on load cases and can not be anticipated in general.

The maximum changes of the shear forces due to deflection relative to the rigid body occur at the positions  $x_d$ , and this change is obtained by lengthways integration of (7) as follows:

$$Q^d = \int_{L_{wlaft}}^{x_d} \gamma_{sea} q_b(x) dx = w_m \gamma_{sea} A_{wl} C_d C_Q = \Delta^d C_Q \quad (25)$$

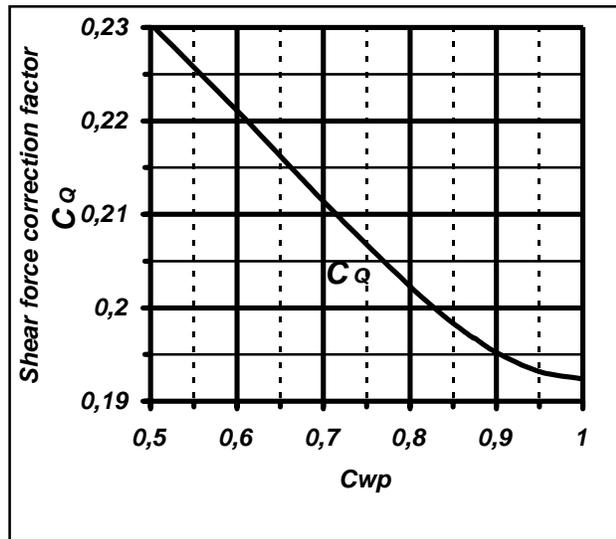
The shear force correction factor  $C_Q$  in (25) based on approximation (11), Fig 4a., is expressed as:

$$C_Q = 2 \frac{C_x}{C_{WP}} \left[ \frac{1}{3} - 3(1 - C_{WP})^2 C_d \frac{1 - C_{WP}}{2} \right] \quad (26)$$

The maximum change of the amidships bending moment due to deflection relative to the rigid body calculation is obtained by lengthways integration applied on (7), as:

$$M^d = \int_{L_{wlaft}}^0 \gamma_{sea} q_b(x) x dx = w_m \gamma_{sea} A_{wl} L_{wl} C_d C_M =$$

$$= w_m \gamma_{sea} 2 \frac{I_m}{L_{wl}} C_M = \Delta^d L_{wl} C_M \quad (27)$$



**Fig. 4a** Shear force correction factor for a generally parabolic waterplane shape

The amidships bending moment correction factor  $C_M$  in (27) based on general parabola waterplane shape assumption (11), Fig. 4b., is expressed as:

### 6. Example

A typical bulk-carrier built in Split Shipyard on the Adriatic coast of Croatia as Hull 399 is investigated according to the methods presented in the note.

The principal characteristics of the considered ship are as follows:

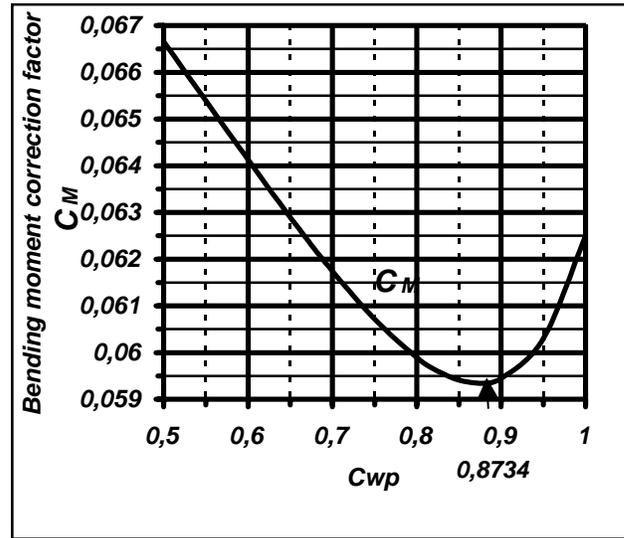
$L_{oa}=187.63$  m,  $L_{pp}=179.3$  m,  $B=30.8$  m,  $D=15.45$  m,  $d=10.8$  m,  $DWT=41600$  t,  $Lightshipweight=8400$  t,  $Speed=14.5$  knots. Load line (LL) is indicated on Fig. 5. and in frontspiece.

The basic ship's characteristics, the correction factors  $C_d$  (10, 10a), the position factor  $C_x$  (21) and position indicator  $x_d$  (20), based on true waterplane shape, the

$$C_M = \frac{1}{16} \frac{6C_{WP}^2 - 15C_{WP} + 10}{3C_{WP}^2 - 10C_{WP} + 8} \quad (28)$$

The minimal  $C_M = 0.05934$  is attained for  $C_{WP} = 0.8734$ .

It is demonstrated earlier (Ziha, 1997) as well as by the bulk-carrier example, Table 1., that the effects of longitudinal deflections of the hulls of conventional merchant ships are of limited order of significance even for relatively great deflections.



**Fig. 4b** Bending moment correction factor for a generally parabolic waterplane shape

correction factors  $C_d$ ,  $C_Q$ ,  $C_M$  (14), (26), (28), based on general parabola waterplane shape, Table 1.

The alternative position of draft marks are indicated on the figure in frontspiece. The humps on the correction factor curves are a consequence of the ship's bulbous bow. Note that the draft correction factor (10) and (10a), based on the true waterplane shape differs significantly from 1/4, but it differs insignificantly from the simplified correction factor based on parabolic waterplane (14), particularly for high values of  $C_{WP}$  nearby the load line, Fig. 5.

Considering the scantling draft  $d=10.8$  m, which corresponds to the summer load line and displacement of 51000 t, the changes of shear forces (25) and

bending moments (27) are assessed for any hull deflection as  $Q^d=w_m$  270 t and  $M^d=w_m$  15000 t m, respectively.

Maximal still water shear force and hogging bending moment of a loaded ship is copied from ship's loading manual for heavy cargo and 100% stores loading condition, amounting to about 4000 t and 90000 t m, respectively. Assuming a tolerable hog of maximally  $w_m < L_{wl}/1000$  m, the upper limits in reduction of bending moments of a loaded ship can be assessed up to 3% and changes of shear forces up to 1%. For homogeneous loaded ship the changes of shear forces and bending moments are in general smaller due to smaller bending moments and deflections. However,

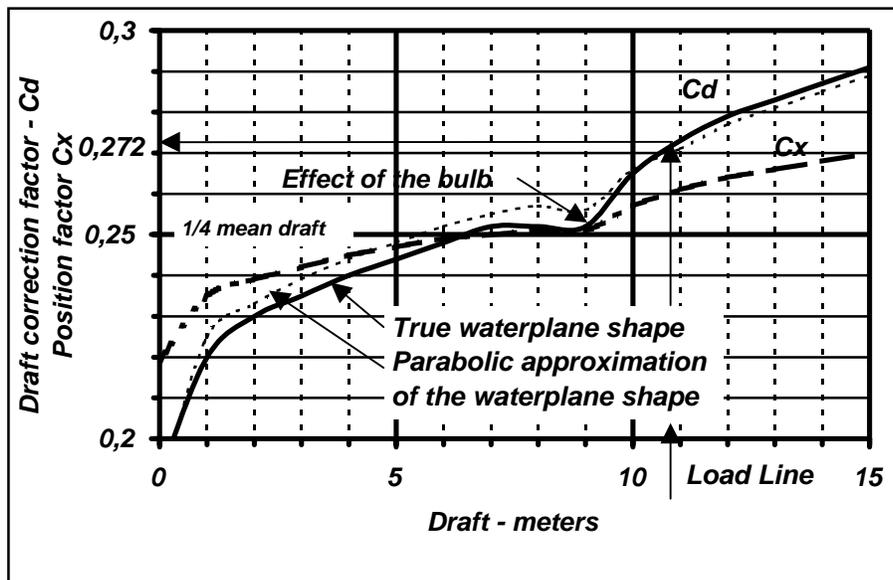
the reduction of bending moments due to deflection can be even greater for hogging conditions in light ballast or sagging conditions in heavy ballast.

The alternative placements of draft marks providing equivalent mean drafts for hydrostatic calculations of a deflected ship hull are presented on the profile of the bulk carrier Hull 399, Figure in frontispiece. Some may find useful to place draft marks at the position of the longitudinal center of flotation *LCF* on the ship's side instead of amidships, see frontispiece.

Note that for the considered bulk-carrier the forward and the amidships draft marks are displaced from the common positions for 0.6 m astern.

**Table 1** Hydrostatic data and correction factors for the bulk carrier Hull 399

<i>d</i>	<i>L<sub>wl</sub></i>	<i>C<sub>wp</sub></i>	<i>LCF</i>	<i>A<sub>wl</sub></i>	$\Delta$	<i>I<sub>L</sub></i>	<i>TP1</i>	<i>MT1</i>	<i>C<sub>d</sub></i>	$\frac{c_d}{1/C_d}$	<i>C<sub>d</sub></i>	<i>C<sub>x</sub></i>	<i>X<sub>d</sub></i>	<i>A<sub>wl</sub>C<sub>d</sub></i>	<i>C<sub>d</sub></i>	<i>C<sub>Q</sub></i>	<i>C<sub>M</sub></i>
<i>m</i>	<i>m</i>		<i>m</i>	<i>m<sup>2</sup></i>	<i>t</i>	<i>10<sup>3</sup>m<sup>4</sup></i>	<i>t/m</i>	<i>tm/m</i>	<i>Eq.10</i>		<i>Eq.10a</i>	<i>Eq. 21</i>	<i>m</i>	<i>m<sup>2</sup></i>	<i>Eq. 14</i>	<i>Eq. 26</i>	<i>Eq. 28</i>
Basic ship data, hydrostatic particulars									True WL shape				Parabolic WL shape				
0	167.38	<b>0.608</b>	4.000	3136	0	4202	3214	23921	<b>0.191</b>	5.227	0.191	0.219	<b>36.604</b>	600	<b>0.187</b>	0.220	0.064
1	174.95	<b>0.760</b>	4.520	4048	3816	6827	4149	38873	<b>0.220</b>	4.537	0.220	0.235	<b>41.066</b>	892	<b>0.225</b>	0.206	0.061
2	177.20	<b>0.785</b>	4.961	4283	8136	7696	4398	43998	<b>0.229</b>	4.369	0.229	0.239	<b>42.390</b>	983	<b>0.233</b>	0.204	0.060
3	178.42	<b>0.803</b>	5.031	4407	12605	8225	4527	47011	<b>0.234</b>	4.264	0.234	0.242	<b>43.200</b>	1038	<b>0.239</b>	0.202	0.060
4	178.27	<b>0.818</b>	5.007	4485	17175	8539	4636	49372	<b>0.240</b>	4.173	0.240	0.245	<b>43.633</b>	1082	<b>0.244</b>	0.201	0.060
5	177.99	<b>0.829</b>	4.734	4542	21811	8781	4664	50471	<b>0.244</b>	4.097	0.245	0.247	<b>43.969</b>	1114	<b>0.248</b>	0.200	0.060
6	178.10	<b>0.838</b>	4.174	4596	26503	9042	4719	51686	<b>0.248</b>	4.031	0.248	0.249	<b>44.354</b>	1145	<b>0.252</b>	0.199	0.059
7	178.46	<b>0.847</b>	3.353	4656	31252	9354	4781	53470	<b>0.252</b>	3.963	0.252	0.250	<b>44.822</b>	1181	<b>0.255</b>	0.199	0.059
8	180.92	<b>0.852</b>	2.223	4725	36068	9761	4853	55630	<b>0.252</b>	3.961	0.251	0.251	<b>45.451</b>	1209	<b>0.257</b>	0.198	0.059
9	184.40	<b>0.849</b>	0.754	4817	40963	10301	4944	58889	<b>0.252</b>	3.975	0.251	0.251	<b>46.244</b>	1212	<b>0.256</b>	0.109	0.059
10	183.20	<b>0.873</b>	-0.977	4918	45956	10933	5047	62496	<b>0.265</b>	3.774	0.265	0.257	<b>47.149</b>	1318	<b>0.266</b>	0.197	0.059
11	<b>183.60</b>	<b>0.886</b>	<b>-1.929</b>	<b>5010</b>	<b>51051</b>	<b>11518</b>	<b>5141</b>	<b>65897</b>	<b>0.273</b>	<b>3.666</b>	<b>0.273</b>	<b>0.261</b>	<b>47.947</b>	<b>1362</b>	<b>0.271</b>	<b>0.196</b>	<b>0.059</b>
12	184.00	<b>0.898</b>	-2.620	5088	56204	11983	5215	68450	<b>0.278</b>	3.594	0.278	0.264	<b>48.531</b>	1416	<b>0.277</b>	0.195	0.059
13	184.42	<b>0.908</b>	-2.840	5155	61408	12409	5284	70812	<b>0.283</b>	3.532	0.283	0.266	<b>49.064</b>	1459	<b>0.282</b>	0.195	0.060
14	184.85	<b>0.916</b>	-3.060	5213	66714	12782	5343	72945	<b>0.287</b>	3.484	0.287	0.268	<b>49.518</b>	1496	<b>0.285</b>	0.194	0.060



**Fig. 5** Draft correction factor  $C_d$  and position factor  $C_x$  for Hull 399

## 7. Conclusion

It becomes clear that the influence of longitudinal deflections of the hulls of merchant ships on bending moments and shear forces can be assessed by approximate terms presented in the note, but these effects are found conservative and of limited order of significance, and there is normally no need for more precise calculations of these quantities.

However, more accurate and practical hog or sag corrections of drafts and displacement due to hull deflection, may be useful to marine surveyors and of some benefit to shipowners and to the crew during a draft and deadweight survey in ship's service, particularly for loaded conditions of merchant ships.

It is demonstrated that the usually applied "quarter mean draft" or "mean of mean draft" correction for hog or sag is not appropriate for modern large merchant ships with relatively high waterplane coefficients. Therefore, this note offers quick, practical and more accurate rational methods based on the standard and permanent hydrostatic particulars for assessments of drafts and displacement of a sensibly deflected and trimmed ship hull during a draft and deadweight survey. It is argued in the note that the draft correction factor based on the true waterplane shape and parabolic deflection line differs significantly from the traditionally applied constant correction factor of 1/4 for most of the modern merchant ships. However, in the same time, the improved draft correction factor differs insignificantly from the simplified correction factor based only on a waterplane coefficient also derived in this note. Moreover, the presented correction factors can be easily calculated only on the basis of common hydrostatic particulars and attached permanently to the ship's trim, stability and loading instruction book for routine application during marine surveys.

The simplest application of the improved draft correction factors presented in the note on traditional draft readings aft, amidships and forward yields the more accurate displacement of a deflected ship during a deadweight survey.

Moreover, it appears feasible to assess a deflected ship's displacement with sufficient accuracy by placing draft marks nearby the load line only in two positions alongside a ship hull preferably port and starboard, which provide identical equivalent and observed drafts. Since the standard equivalent draft resulting from the alternative placement of draft marks virtually coincides with respective ship's center of flotation, the survey procedure yields to improved displacement assessment of a deflected hull, and moreover, in the same time, it accounts for the correction due to the ship's trim.

The draft survey based on alternative position of draft marks can be used as a stand-alone procedure also retaining the traditional Plimsoll mark, or at least as an additional checking useful in deadweight survey employing also the commonly placed draft marks. The improved correction factors allow definition of alternative placement of draft marks on about one quarter of the ship's length from amidships.

Consider that many merchant ships can not have draft marks on perpendiculars and amidships due to the hull form and additional recalculations due to draft marks position are needed anyhow. Since the traditional placement of draft marks was instituted when the ships were not as long as modern merchant vessels, the alternate draft marks placed closer to the amidships, may lead to more accurate and less troublesome draft readings on large ships due to less intense motions and due to easier near-simultaneous view by observers.

In the cases when the ship's trim or the deflection of the hull are too large, the additional draft readings improve the accuracy of the lengthwise integration over inclined and deflected sections using Bonjean's curves.

However, the alternative placement of draft marks would possibly reduce the precision of draft assessment at the stern, which affects propeller immersion and rudder effectiveness, and at the bow, which affects maneuverability and slamming, implying that the limiting drafts aft and forward should be indicated. But for a given observation tolerance, the gross error in definition of the standard equivalent mean draft using the alternate location of draft marks closer to the midship section will not exceed the one by traditional positioning of the draft marks. In many cases, the error can be less.

## Acknowledgments

The work on this note is based on lectures of Professor Josip Uršić, deceased in 1994. The loading manual for Hull 399 obtained from Shipbuilding Industry in Split was very helpful. The procedure presented in the note is submitted to the Croatian Intellectual Property Office.

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