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Redundancy and robustness of systems of events

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Abstract

The article aims to add a new impetus to rational and objective probabilistic evaluation of redundancy and robustness, based on uncertainties of systems and subsystems of events. An attempt is made to demonstrate the relevance of intuitive comprehension of redundancy and robustness of engineering systems of events. An event-oriented system analysis of a number of random observable operational and failure modes, with adverse probability distributions in a lifetime, may provide a deeper understanding of systems operational abundance and endurance. The system uncertainty analysis is based on the concept of entropy as defined in information theory and applied to probability theory. The article relates reliability, uncertainty, redundancy and robustness of systems of events and their application is illustrated in numerical examples. © 2000 Published by Elsevier Science Ltd. All rights reserved.

Keywords: Engineering; System analysis; System of events; Probability; Reliability; Uncertainty; Entropy; Information; Redundancy; Robustness; Event oriented system analysis

1. Introduction

Redundancy and robustness are inherent in the majority of complex engineering systems and have been viewed in different ways. Both the redundancy and the robustness are usually related to the system reliability and considered, on one hand, as a capacity of a system to operate even when some of the physical components have failed or on the other hand, as a system responsive to adverse conditions. Redundancy can be classified as either local or overall (e.g. for marine structural systems [1]). Whilst local redundancy refers to local reserve of operational capacity in system physical components, the overall redundancy can be expressed as a system reserve capacity or a residual system capacity (e.g. for structural systems [2]). System reserve capacity is the margin between the design demand and its limit state, or the ultimate capacity of the overall system to sustain the demands. The residual system capacity of operation is the remaining capacity after one or more components have failed or become non-operational. The redundancy of system's operation can be quantified in deterministic, probabilistic and semi-probabilistic terms (e.g. for structural systems [3]). The main disadvantage of deterministic measures of redundancy is that they do not take into consideration the statistical uncertainties of the system. Most widely used probabilistic measure of system redundancy is based on the conditional probability of system survival given if one component fails. The semi-probabilistic methods in redundancy assessment benefit from first-order reliability analysis. The optimisation of system redundancy in order to increase reliability of a system, with number of components under possible restrictions, also including cost functions, has been recognised as an effective method in reliability improvement [4]. The system reliability can also be linked with the robustness of the system. A system is considered reliable in terms of robustness if it is robust with respect to input and failure uncertainties, and consequently it has low reliability when even the small amounts of uncertainty entail the possibility of failure [5].

The scope of this article does not cover the physical or probabilistic measures for redundancy and robustness with respect to the components of the system, since these problems has been elaborated elsewhere (e.g. [4]). Moreover, the article investigates probabilistic measures for operational abundance and endurance, associated with engineering systems and subsystems of events.

The article demonstrates how system uncertainties, obtained as results of an event-oriented system analysis [6], can be interpreted in terms of system redundancy and system robustness associated with the subsystems of operational and failure modes [7]. Such an approach to system analysis may provide a more comprehensive assessment of system performance and hopefully a better system design. Intuition and common engineering reasoning are applied to a system surplus of probabilities and a number of operational and failure modes. The simplest system provides only one operational and one failure mode, and does not

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Nomenclature

A_i, E_i	random events in general
$H(\cdot)$	Shannon's entropy of a system of events
$H^1(\cdot)$	Renyi's/Shannon's entropy of order one
N, n	numbers of systems and of subsystems
n_o, n_f	number of operational and failure subsystems
N_o, N_f	number of operational and of failure events
$p(\cdot)$	probabilities of random events, (sub)systems
$P_f(\mathscr{S})$	probability of failure of the system
$R(\mathscr{S})$	system reliability
$\mathrm{RED}(\mathscr{S})$	system redundancy
$\mathrm{ROB}(\mathscr{S})$	system robustness
$\mathscr{S}$	system of events in general
$\mathscr{S}'$	system of subsystems of events
$\mathscr{O}$, $\mathscr{F}$	subsystems of operational and of failure events

render any superfluity in operational capacities or in responses to failures. If there are more operational and failure modes, with known probabilities, the system can be viewed as probabilistically ample in normal functions or in necessary working conditions. Such a system is denoted as probabilistically redundant and robust with respect to operational and failure modes. To the term redundancy, the article assigns the notion of a probabilistic abundance in a number and probabilities of operational modes. Analogously, the word robustness, in terms of failure events denotes an excessive capability to respond to all demands by more adequate failure modes. The article argues how system redundancy can be related to the uncertainty of operational modes, and how system robustness, contrary to system redundancy, can be related to the uncertainty of the failure modes. System redundancy and system robustness are defined as complementary basic characteristics of a system, apart from the system reliability and failure probability. The system uncertainties are expressed by entropy as defined in information theory [8–12] and as used in probability theory [13]. An example is given to illustrate the application of an event-oriented system analysis to a redundant and robust plane truss structure.

2. Engineering systems and subsystems of operational and failure modes

The probabilistic system analysis is based on statistical data about physical components and their interactions (e.g. [4,14–19]). In addition to the probabilistic system analysis, an event-oriented system analysis can be performed on the basis of known or observable lifetime events [6]. By operational modes and effects analysis, all, or at least all observable, events E_i of a system can be determined. Some of the events E_i are operational modes, denoted as E_i^{o} (status = O), and other events can be regarded as failure modes, denoted with E_i^{f} (status = F). The probabilities of possible modes can be hopefully calculated by using quantitative methods. They are denoted as $p(E_i)$, $i = 1, 2, \ldots, N$. N is the total number of all, or at least observable random events, constituting system of events $\mathscr{S}$. A distribution of system mode probabilities is presented in Fig. 1. Since the sequence of system modes is irrelevant in system performance analysis, the operational modes are collected and presented on the

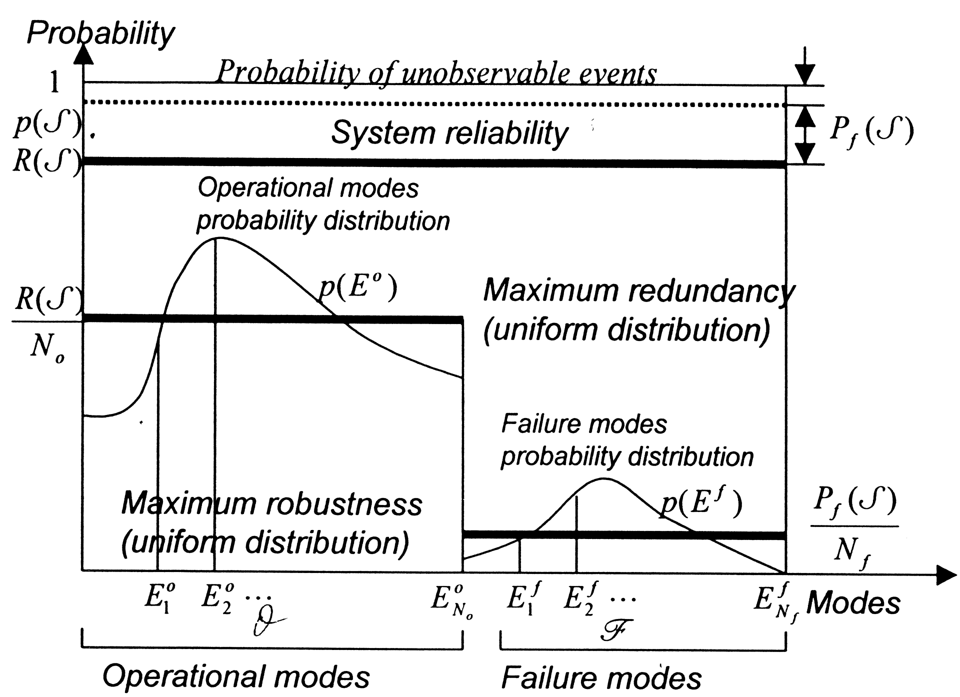

Fig. 1. Distribution of system mode probabilities.

left-hand side of Fig. 1, and the failure modes are presented in continuation, on the right-hand side of Fig. 1. The system of events $\mathscr{S}$ is a finite scheme [13], and can also be presented as summation of two subsystems $\mathscr{O}$ and $\mathscr{F}$, as shown:

$$\mathscr{S} = \begin{pmatrix} E_1 & E_2 & \cdots & E_N \\ p(E_1) & p(E_2) & \cdots & p(E_N) \end{pmatrix} = (\mathscr{O} + \mathscr{F})$$

The probability associated with the system of events $\mathscr{S}$ is as follows:

$$p(\mathscr{S}) = \sum_{i=1}^{N} p(E_i) \tag{1}$$

$\mathscr{S}$ is a complete system when $P(\mathscr{S}) = 1$, and an incomplete system when $p(\mathscr{S}) < 1$.

The subsystem $\mathscr{O}$ comprises all of the random events $E_i^{\mathrm{o}},\ i = 1, 2, \ldots, N_{\mathrm{o}}$ when the system is operating:

$$\mathscr{O} = \begin{pmatrix} E_1^{\mathrm{o}} & E_2^{\mathrm{o}} & \cdots & E_{N_{\mathrm{o}}}^{\mathrm{o}} \\ p(E_1^{\mathrm{o}}) & p(E_2^{\mathrm{o}}) & \cdots & p(E_{N_{\mathrm{o}}}^{\mathrm{o}}) \end{pmatrix}$$

The subsystem $\mathscr{F}$ consists of all the random events E_i^{f}, $i = 1, \ldots, N_{\mathrm{f}}$ when the system fails:

$$\mathscr{F} = \begin{pmatrix} E_1^{\mathrm{f}} & E_2^{\mathrm{f}} & \cdots & E_{N_{\mathrm{f}}}^{\mathrm{f}} \\ p(E_1^{\mathrm{f}}) & p(E_2^{\mathrm{f}}) & \cdots & p(E_{N_{\mathrm{f}}}^{\mathrm{f}}) \end{pmatrix}$$

The total number of events is equal to $N_{\mathrm{o}} + N_{\mathrm{f}} = N$. It may also be noted that the sequence of the events of a system or of subsystems is irrelevant with respect to reliability and uncertainty considerations.

The reliability of the system, denoted as $R(\mathscr{S})$, corresponds to all of the outcomes when the system is operating and can be calculated as the probability of the subsystem of operational modes $p(\mathscr{O})$ as:

$$R(\mathscr{S}) = p(\mathscr{O}) = \sum_{i=1}^{N_{\mathrm{o}}} p(E_i^{\mathrm{o}}), \tag{2}$$

The failure probability of the system denoted as $P_{\mathrm{f}}(\mathscr{S})$ corresponds to all of the outcomes when the system fails and can be calculated as the probability of the subsystem of failure modes $p(\mathscr{F})$ as:

$$P_{\mathrm{f}}(\mathscr{S}) = p(\mathscr{F}) = \sum_{i=1}^{N_{\mathrm{f}}} p(E_i^{\mathrm{f}}) \tag{3}$$

A system $\mathscr{S}'$ of two subsystems of events, each considered as a compound event denoted $\mathscr{O}$ for the system operating and denoted $\mathscr{F}$ for the failed system, can be defined as follows:

$$\mathscr{S}' = (\mathscr{O}, \mathscr{F}) = \begin{pmatrix} \mathscr{O} & \mathscr{F} \\ p(\mathscr{O}) & p(\mathscr{F}) \end{pmatrix}$$

Either for complete systems, when $p(\mathscr{S}) = 1$, or for incomplete systems, when $p(\mathscr{S}) < 1$, it holds:

$$p(\mathscr{O}) + p(\mathscr{F}) = p(\mathscr{S}) = p(\mathscr{S}') \tag{4}$$

2.1. Engineering system uncertainties

The entropy of complete or of incomplete systems of events $\mathscr{S}$ and $\mathscr{S}'$ expresses the system uncertainties and can be determined as Renyi's/Shannon's entropy of order one, defined as follows:

$$H_N^1(\mathscr{S}) = H^1(\mathscr{O} + \mathscr{F}) = H^1(\mathscr{S})$$
$$= \left[-\sum_{i=1}^{N} p(E_i) \log p(E_i) \right] \Big/ \sum_{i=1}^{N} p(E_i) = \frac{H(\mathscr{S})}{p(\mathscr{S})} \tag{5}$$

$$H_2^1(\mathscr{S}') = H^1(\mathscr{O}, \mathscr{F})$$
$$= H^1(\mathscr{S}') = [-p(\mathscr{O})\log p(\mathscr{O}) - p(\mathscr{F})\log p(\mathscr{F})]$$
$$/[p(\mathscr{O}) + p(\mathscr{F})] = \frac{H(\mathscr{S}')}{p(\mathscr{S}')} \tag{6}$$

The maximal entropy of systems $\mathscr{S}$ and $\mathscr{S}'$ are obtained for N and for 2 equally probable events, as:

$$H_N^1(\mathscr{S})_{\max} = -N \frac{1}{p(\mathscr{S})} \frac{p(\mathscr{S})}{N} \log \frac{p(\mathscr{S})}{N} = \log \frac{N}{p(\mathscr{S})} \tag{7}$$

$$H_2^1(\mathscr{S}')_{\max} = -2 \frac{1}{p(\mathscr{S}')} \frac{p(\mathscr{S}')}{2} \log \frac{p(\mathscr{S}')}{2} = \log \frac{2}{p(\mathscr{S}')} \tag{8}$$

Note that through in the mathematical expressions the base of logarithm is usually two.

3. Definition of the redundancy of the system of events

The redundancy in event-oriented system analysis is viewed as a capability of a system to continue operations by performing different random operational modes of given probabilities in case of random failures of components. Performing another operational mode, the system may be brought on equal or on the reduced operational capacity. The operational capacities can be viewed either as the probabilities of operational modes or as the physical or technical working capacities of a damaged system. However, it is intuited that a highly redundant system has more operational modes of similar level of reliability (Fig. 1). If so, there is an uncertainty about which of the operational modes is to be performing. The operational capacities of alternative operational modes are in general different from the principal operational mode and should also be taken into account in the assessment of system redundancy. Hence, the redundancy of a system is only related to the operational modes of the system.

The intuitive expectations based on common engineering reasoning on the measure of the redundancy of a system of events are listed next:

1. REDUNDANCY (System/Operational) = 0: If there is no operational modes or if there is only one operational mode or, if among a number of operational modes, one is

dominantly probable, i.e. sure, there is no redundancy at all. In all other cases, the redundancy should be positive.
2. REDUNDANCY (System/Operational) = maximum: Redundancy is greatest if all the alternative operational modes are of the same probability of occurrence and possibly, of the same operational capacity. The maximum redundancy can be associated with the set of equally probable alternative operational modes of the same level of operational capacity. If equally probable alternative operational modes of the same level of operational capacity are considered, the larger a set of operational modes, the more redundancy is gained.
3. REDUNDANCY (System/Operational) is independent of the sequence of operational modes. The sequence of the events within the system or within subsystems is irrelevant with respect to the redundancy.
4. REDUNDANCY (System/Operational) = 1 (by the definition of a unit): The definition of the unit of redundancy is not more or less arbitrary than the choice of the unit of some physical quantity. E.g., it can be maintained that the redundancy is equal to one if there are only two equally probable operational modes.

The intuitive description of the system redundancy based on the set of operational modes is very reminiscent of the definition of the uncertainty of a system of events, which is denoted in the information theory as entropy. The principal problem of the application of entropy on the definition of the redundancy is that the entropy can be applied to complete or incomplete systems of events, and that the subsystem of operational modes is obviously a partial distribution of probabilities of a subsystem of operational modes. In order to overcome this problem, let us consider the subsystem of operational modes under the condition that the system itself is operational, denoting the conditional operational events as $E_i^o/\mathcal{O}$.

The probability distribution of operational modes $p(E_i^o)$, $i = 1, 2, \ldots, N_o$ is considered as a partial distribution of probabilities. The subsystem of operational modes $\mathcal{O}$ is part of a distribution of probabilities of the entire system of events $\mathcal{S}$. However, every partial distribution $p(E_i^o)$ can be assigned an ordinary or complete distribution by substituting $p(E_i^o/\mathcal{O})$ instead of $p(E_i^o)$. It may be interpreted as a complete conditional distribution of probabilities with respect to the conditions that the system $\mathcal{S}$ of known i.e. observable modes is operational with probability $p(\mathcal{O})$.

It is clear that $p(E_i^o/\mathcal{O})p(\mathcal{O}) = p(E_i^o)$.

The subsystem of only operating modes $\mathcal{O}$ can be viewed under the condition that the system of all known modes $\mathcal{S}$ is wholly operational. Let it be denoted as $\mathcal{S}/\mathcal{O}$. It can be proven that the conditional probability of subsystem of operational modes $\mathcal{S}/\mathcal{O}$, with respect to the conditions that the system is wholly operational, depends only on the probability of a subsystem of operational modes $\mathcal{O}$ itself. Obviously, $p(\mathcal{S}/\mathcal{O}) = 1$.

The system $\mathcal{S}$ under the condition that it is operational, denoted as $\mathcal{O}$, can be presented as follows:

$$\mathcal{S}/\mathcal{O} = \begin{pmatrix} E_1^o/\mathcal{O} & E_2^o/\mathcal{O} & \cdots & E_{N_o}^o/\mathcal{O} \\ \dfrac{p(E_1^o)}{p(\mathcal{O})} & \dfrac{p(E_2^o)}{p(\mathcal{O})} & \cdots & \dfrac{p(E_{N_o}^o)}{p(\mathcal{O})} \end{pmatrix}$$

The uncertainty of the subsystem of operational modes $\mathcal{O}$ can be expressed as Shannon's entropy applied only to the system $\mathcal{S}$ under the condition that it is operational $\mathcal{S}/\mathcal{O}$. The condition that the system $\mathcal{S}$ is wholly operational with probability $p(\mathcal{O})$ is considered as a complete conditional distribution.

According to the definition of the conditional entropy of operational modes, it follows:

$$H_{N_o}(\mathcal{S}/\mathcal{O}) = -\sum_{i=1}^{N_o} \frac{p(E_i^o)}{p(\mathcal{O})} \log \frac{p(E_i^o)}{p(\mathcal{O})} \tag{9}$$

The following relation expresses the differences between the conditional entropy of a subsystem $\mathcal{O}$ represented by partial distribution of $\mathcal{S}$ denoted as $\mathcal{S}/\mathcal{O}$ and the entropy of order one of a system $\mathcal{O}$ viewed as an incomplete one:

$$H_{N_o}(\mathcal{S}/\mathcal{O}) = H_{N_o}^1(\mathcal{O}) + \log p(\mathcal{O}) \tag{10}$$

In Eq. (10), the following term is the Renyi's/Shannon's entropy of order one, corresponding to the entropy of incomplete system of events, which is defined as follows:

$$H_{N_o}^1(\mathcal{O}) = H_{N_o}(\mathcal{O})/p(\mathcal{O}) \tag{11}$$

In Eq. (11), partial summa within the system $\mathcal{S}$, corresponding to only operational modes is as shown:

$$H_{N_o}(\mathcal{O}) = -\sum_{i=1}^{N_o} p(E_i^o) \log p(E_i^o) \tag{12}$$

The maximum entropy of operational modes is attained for N_o equally probable events with probabilities $R(\mathcal{S})/N_o$, Fig. 1, and amounts to:

$$H_{N_o}(\mathcal{S}/\mathcal{O})_{\max} = -N_o \frac{R(\mathcal{S})}{N_o} \log \frac{R(\mathcal{S})}{N_o} = \log N_o \tag{13}$$

Common characteristics of the entropy (e.g. [11–13]) applied only to the subsystem of operational modes considered as a complete conditional distribution with respect to the condition that the system in a whole is operational are resumed next:

1. $H(\mathcal{S}/\mathcal{O}) = 0$ if none or only one of operational modes can occur;
2. $H(\mathcal{S}/\mathcal{O}) = H_{\max}$ if all the operational modes are equally probable;
3. $H(\mathcal{S}/\mathcal{O})$ does not depend on the sequence of the events;
4. $H(\mathcal{S}/\mathcal{O}) = 1$ definition of a unit is arbitrary; in general the base of the applied logarithm is used. For example, for logarithm base of two, the unit entropy 1 bit, corresponds to uncertainty associated with a system of two equally probable events.

The intuitive expectations of redundancy measure comply with the characteristics of the conditional entropy of the subsystem of operational modes considered as a complete conditional distribution with respect to the condition that the system in a whole is operational with known probability. The above considerations show that the redundancy of the system of events can be rationally and objectively measured by the entropy of operational modes under the condition that the system of all known or observable modes is operational and can be written succinctly as follows:

$$\text{REDUNDANCY (System/Operational)} = \text{RED}(\mathcal{S}/\mathcal{O})$$

$$= \text{RED}(\mathcal{S}) = H_{N_o}(\mathcal{S}/\mathcal{O}) \tag{14}$$

Note that the redundancy optimisation has been recognised earlier as the maximisation of the reliability of a series system composed of N_o independent redundant groups, denoted as subsystems, by introducing additional redundant components [4]. A redundant group is not necessarily a separate part of a system. It may be a group of units of the same type, which uses the same redundant units. For instance, in spare-parts problems a redundant group might be a set of identical units allocated throughout the entire system in different places. In this context, the system reliability index can be expressed as the probability of successful operation of a series system consisting of independent groups of redundant units, as presented:

$$p(R^o) = p(R_1^o, R_2^o, \ldots, R_{N_o}^o) = \prod_{i=1}^{N_o} p(R_i^o) = 1 - \sum_{i=1}^{N_o} [1 - p(R_i^o)] \tag{15}$$

In Eq. (15) $R^o = R_1^o, R_2^o, \ldots, R_{N_o}^o$ is a set of the system's redundant groups of units R_i^os and $p(R_i^o)$ is the reliability index of the ith redundant group, $i = 1, 2, \ldots, N_o$. The reliability index in Eq. (15) can be also presented in additive form as:

$$L(R^o) = L(R_1^o, R_2^o, \ldots, R_{N_o}^o) = -\log p(R^o) = \sum_{i=1}^{N_o} -\log p(R_i^o) \tag{16}$$

The term (16) is used as a reliability measure in terms of redundancy. However, such a measure does not account for the probability distribution of operational events and has an entirely different meaning from the redundancy defined here by the term (14).

4. Definition of the robustness of the system of events

The principal difference between the robustness and the redundancy of a system of events as viewed in the event-oriented system analysis is that robustness is regarded as the system's capability to respond to all possible random failures uniformly. A robust behaviour is intuited when the system can provide more adequate failure modes to adverse demands with more or less equal probabilities, Fig. 1. A robust system can become non-operational at different levels of failure. The failure levels can be viewed either as the probabilities of failure of alternative failure modes or as the levels of physical or technical failures of the system. When the system responds to all demands uniformly, there is a high uncertainty about which of the failure modes could occur. Hence, the system robustness is related only to the failure modes of the system in the same manner, as the redundancy is related to the operational modes.

The intuitive expectations and common engineering reasoning about the measure of the robustness of a system of events are listed next:

1. ROBUSTNESS (system/fails) = 0: If there is no failure or non-operational modes or if there is only one failure or non-operational mode or, if among a number of failure or non-operational modes, one is dominantly probable, i.e. sure, there is no robustness at all. In all other cases, the robustness should be positive.
2. ROBUSTNESS (system/fails) = maximum: Robustness is greatest if all the alternative failure or non-operational modes are of the same probability of occurrence and possibly of the same level of failure. The maximum robustness can be associated with the set of equally probable alternative failure or non-operational modes with the same level of failure. If equally probable alternative failure or non-operational modes of the same level of failure are considered, the larger the set of failure modes, the more robustness is gained.
3. ROBUSTNESS (system/fails) is independent of the sequence of the failure modes. The sequence of the events within the system or within the subsystems is irrelevant with respect to robustness considerations.
4. ROBUSTNESS (system/fails) = 1 (the definition of a unit): The definition of the unit of robustness is arbitrary just as it was in the definition of redundancy. Let us, for example, agree that the robustness equals one, if there are only two equally probable failure modes.

The intuitive description of the robustness of a system of events based on the set of failure modes strongly reminds of the definition of redundancy defined on the set of operational modes. The same approach to the definition of robustness can be applied as in the definition of redundancy. The subsystem of only failure modes $\mathcal{F}$ can be viewed under the condition that the system of all known modes $\mathcal{S}$ is failed, let it denote $\mathcal{S}/\mathcal{F}$. It can be proven that the conditional probability of the subsystem of failure modes $\mathcal{S}/\mathcal{F}$, with respect to the conditions that the system of known or observable modes is failed in a whole, depends only on the probability of the subsystem of failure modes $\mathcal{F}$ itself. Obviously, $p(\mathcal{S}/\mathcal{F}) = 1$.

The system $\mathcal{S}$ under the condition that it is failed, denoted as $\mathscr{F}$, can be presented as follows:

$$\mathscr{S}/\mathscr{F} = \begin{pmatrix} E_1^f/\mathscr{F} & E_2^f/\mathscr{F} & \dots & E_{N_f}^f/\mathscr{F} \\ \frac{p(E_1^f)}{p(\mathscr{F})} & \frac{p(E_2^f)}{p(\mathscr{F})} & \dots & \frac{p(E_{N_f}^f)}{p(\mathscr{F})} \end{pmatrix}$$

The uncertainty of the subsystem of failure modes $\mathscr{F}$ can be expressed as Shannon's entropy applied only to the systems $\mathscr{S}$ under the condition that it is non-operational $\mathscr{S}/\mathscr{F}$. It is considered as a complete conditional distribution with respect to the condition that the system fails with probability $p(\mathscr{F})$ and, according to the definition of the entropy of failure modes, it follows:

$$H_{N_f}(\mathscr{S}/\mathscr{F}) = -\sum_{i=1}^{N_f} \frac{p(E_i^f)}{p(\mathscr{F})} \log \frac{p(E_i^f)}{p(\mathscr{F})} \quad (17)$$

The following relation expresses the differences between the conditional entropy of a subsystem $\mathscr{F}$ represented by a partial distribution of $\mathscr{S}$ denoted as $\mathscr{S}/\mathscr{F}$ and the entropy of order one of a system $\mathscr{F}$ viewed as an incomplete one, where:

$$H_{N_f}(\mathscr{S}/\mathscr{F}) = H_{N_f}^1(\mathscr{F}) + \log p(\mathscr{F}) \quad (18)$$

In Eq. (18), Renyi's/Shannon's entropy of order one, corresponding to the entropy of incomplete system of event, is defined as shown:

$$H_{N_f}^1(\mathscr{F}) = H_N(\mathscr{F})/p(\mathscr{F}) \quad (19)$$

In Eq. (19), partial summa within the system $\mathscr{S}$, corresponding to only failure modes, is as follows:

$$H_{N_f}(\mathscr{F}) = -\sum_{i=1}^{N_f} p(E_i^f)\log p(E_i^f) \quad (20)$$

The maximum entropy of the failure modes is attained for N_f equally probable events with probabilities of $P_f(\mathscr{S})/N_f$ (Fig. 1), and amounts to:

$$H_{N_f}(\mathscr{S}/\mathscr{F})_{\max} = -N_f \frac{P_f(\mathscr{S})}{N_f} \log \frac{P_f(\mathscr{S})}{N_f} = \log N_f \quad (21)$$

The common characteristics of entropy, now applied only to the subsystem of failure modes considered as a complete conditional distribution with respect to the condition that the system in whole is non-operational are resumed next:

1. $H(\mathscr{S}/\mathscr{F}) = 0$ if only one of the failure modes can occur;
2. $H(\mathscr{S}/\mathscr{F}) = H_{\max}$ if all the failure modes are equally probable;
3. $H(\mathscr{S}/\mathscr{F})$ does not depend on the sequence of the events;
4. $H(\mathscr{S}/\mathscr{F}) = 1$ definition of a unit is arbitrary; in general, the base of applied logarithm is used. For example, for logarithm base of two, the unit entropy 1 bit, corresponds to uncertainty associated with a system of two equally probable events.

The intuitive expectations of the robustness measure of systems of events comply with the characteristics of the conditional entropy of the subsystem of failure modes, as it is presented earlier by consideration about the redundancy. Presented considerations allow that the robustness of the system of events can be measured by the entropy of failure modes under the condition that the system in whole is non-operational. It can be written in condensed form as follows:

$$\text{ROBUSTNESS (System/Fails)} = \text{ROB}(\mathscr{S}/\mathscr{F})$$
$$= \text{ROB}(S) = H_{N_o}(\mathscr{S}/\mathscr{F}) \quad (22)$$

5. The relation between the redundancy and the robustness and the system reliability

It can be proven that the following expressions bring into the relation the system redundancy, the system robustness and the overall uncertainty of the system and of the system of operational and non-operational subsystem [6]. The weighted summa of the entropy of the subsystems $\mathscr{O}$ and $\mathscr{F}$ of the system $\mathscr{S}$ can be represented, using mathematical notation, as shown:

$$p(\mathscr{O})H(\mathscr{S}/\mathscr{O}) + p(\mathscr{F})H(\mathscr{S}/\mathscr{F})$$
$$= p(\mathscr{S})[H_N^1(\mathscr{S}) - H_2^1(\mathscr{S}')]$$
$$= H(\mathscr{O} + \mathscr{F}) - H(\mathscr{O}, \mathscr{F}) \quad (23)$$

The same relation can also be expressed in terms of reliability, redundancy and robustness measures:

$$R(\mathscr{S})\text{RED}(\mathscr{S}) + P_f(\mathscr{S})\text{ROB}(\mathscr{S})$$
$$= p(\mathscr{S})[H_N^1(\mathscr{S}) - H_2^1(\mathscr{S}')]$$
$$= H(\mathscr{O} + \mathscr{F}) - H(\mathscr{O}, \mathscr{F}) \quad (24)$$

5.1. The subsystems of operational and failure modes

More generally, there can be n_o groups of modes with same operational capacities or n_f groups of modes with the same failure seriousness. Then the system of events can be decomposed into more than only two subsystems, with subscript A denoting different system of subsystems S_A', where $n = n_o + n_f$, as follows:

$$\mathscr{S} = (\mathscr{O}_1 + \mathscr{O}_2 + \dots + \mathscr{O}_{n_o} + \mathscr{F}_1 + \mathscr{F}_2 + \dots + \mathscr{F}_{n_f})$$

and

$$\mathscr{S}'_A = (\mathscr{O}_1, \mathscr{O}_2, \dots, \mathscr{O}_{n_o}, \mathscr{F}_1, \mathscr{F}_2, \dots, \mathscr{F}_{n_f}).$$

The subsystems of operational and failure modes are defined as compounds of subsystems as

$$\mathscr{O} = (\mathscr{O}_1 + \mathscr{O}_2 + \dots + \mathscr{O}_{n_o}) \qquad \mathscr{F} = (\mathscr{F}_1 + \mathscr{F}_2 + \dots + \mathscr{F}_{n_f})$$

and

$$\mathscr{O}'_A = (\mathscr{O}_1, \mathscr{O}_2, \ldots, \mathscr{O}_{n_o}), \qquad \mathscr{F}'_A = (\mathscr{F}_1, \mathscr{F}_2, \ldots, \mathscr{F}_{n_f}).$$

The relation of the reliability, failure probability, redundancy and robustness of subsystems to the system uncertainties can be expressed as follows:

$$\sum_{i=1}^{n_o} R(\mathscr{O}_i)\mathrm{RED}(\mathscr{S}/\mathscr{O}_i) + \sum_{i=1}^{n_f} P_f(\mathscr{F}_i)\mathrm{ROB}(\mathscr{S}/\mathscr{F}_i) = H_N(\mathscr{S}) - H_n(\mathscr{S}'_A) \tag{25}$$

The reliabilities and the failure probabilities in Eq. (25) associated with each of the groups are calculated as $R(\mathscr{O}_i) = \sum_{j=1}^{n_{o,i}} p(E^o_{ij})$ and $P_f(\mathscr{F}_i) = \sum_{j=1}^{n_{o,i}} p(E^f_{ij})$.

The redundancy and the robustness of each of the groups of modes of equal operational capacity and of equal failure seriousness in Eq. (25), are defined as follows:

$$\mathrm{RED}(\mathscr{S}/\mathscr{O}_i) = H_{n_{o,i}}(\mathscr{S}/\mathscr{O}_i) = -\sum_{j=1}^{n_{o,i}} \frac{p(E^o_{ij})}{p(\mathscr{O}_i)} \log \frac{p(E^o_{ij})}{p(\mathscr{O}_i)} \tag{26}$$

$$\mathrm{ROB}(\mathscr{S}/\mathscr{F}_i) = H_{n_{f,i}}(\mathscr{S}/\mathscr{F}_i) = -\sum_{j=1}^{n_{f,i}} \frac{p(E^f_{ij})}{p(\mathscr{F}_i)} \log \frac{p(E^f_{ij})}{p(\mathscr{F}_i)} \tag{27}$$

The relation among the system redundancy (14) and redundancies of the subsystem (26) is as follows:

$$R(\mathscr{S})[\mathrm{RED}(\mathscr{S}/\mathscr{O}) + \mathrm{RED}(\mathscr{S}'_A/\mathscr{O})] = \sum_{i=1}^{n_o} \mathrm{REL}(\mathscr{O}_i)\mathrm{RED}(\mathscr{S}/\mathscr{O}_i) \tag{28}$$

The relation among the system robustness (22) and robustness of the subsystem (27) is as follows:

$$P_f(\mathscr{S})[\mathrm{ROB}(\mathscr{S}/\mathscr{F}) + \mathrm{ROB}(\mathscr{S}'_A/\mathscr{F})] = \sum_{i=1}^{n_f} P_f(\mathscr{F}_i)\mathrm{ROB}(\mathscr{S}/\mathscr{F}_i) \tag{29}$$

In Eqs. (28) and (29), following terms with the meaning of the redundancy and robustness of the system of subsystems, are defined as follows:

$$\mathrm{RED}(\mathscr{S}'_A/\mathscr{O}) = \mathrm{RED}(\mathscr{S}'_A) = H_{n_o}(\mathscr{S}'_A/\mathscr{O}) = -\sum_{i=1}^{n_o} \frac{p(\mathscr{O}_i)}{p(\mathscr{O})} \log \frac{p(\mathscr{O}_i)}{p(\mathscr{O})} \tag{30}$$

$$\mathrm{ROB}(\mathscr{S}'_A/\mathscr{F}) = \mathrm{ROB}(\mathscr{S}'_A) = H_{n_f}(\mathscr{S}'_A/\mathscr{F}) = -\sum_{i=1}^{n_f} \frac{p(\mathscr{F}_i)}{p(\mathscr{F})} \log \frac{p(\mathscr{F}_i)}{p(\mathscr{F})} \tag{31}$$

The relation among the redundancy and robustness of the whole system to the redundancies and robustness of the groups of modes can be derived by the summation of Eqs. (30) and (31) as shown:

$$R(\mathscr{S})[\mathrm{RED}(\mathscr{S}/\mathscr{O}) + \mathrm{RED}(\mathscr{S}'_A/\mathscr{O})] + P_f(\mathscr{S})[\mathrm{ROB}(\mathscr{S}/\mathscr{F}) + \mathrm{RED}(\mathscr{S}'_A/\mathscr{F})] = \sum_{i=1}^{n_o} R(\mathscr{O}_i)\mathrm{RED}(\mathscr{S}/\mathscr{O}_i) + \sum_{i=1}^{n_f} P_f(\mathscr{F}_i)\mathrm{ROB}(\mathscr{S}/\mathscr{F}_i) \tag{32}$$

6. Example: a redundant plane truss structure

An event-oriented system analysis procedure will be demonstrated on a plane statically indeterminate truss structure [17] (Fig. 2) considering variations in component probability distributions. The aim of the example is to assess the redundancy and the robustness of the system of events, as well as their relations to the system reliability and system uncertainty.

6.1. System service modes and effects analysis

A system service modes and effects analysis is performed first in order to identify all the modes and appropriate probabilities of occurrence.

The structure fails if one of the edge elements 1, 2, 3, or 4 fails or if at least two of the remaining central elements 5, 6, 7, 8, 9 and 10 fail (Fig. 2). The minimal cut set is presented in Fig. 3.

There are $N = 2^{10} = 1024$ outcomes. Enumeration on a digital computer determines all possible operational and failure modes. There are $N_o = 7$ operational modes and $N_f = 1017$ failure modes.

Both the subsystems of operational modes $\mathscr{O}$ and the subsystem of failure modes $\mathscr{F}$ are collected in the system of events $\mathscr{S}$.

The system of operational modes $\mathscr{O}$, under the condition that the system is operational, consists of the following events with appropriate probabilities:

$$\mathscr{S}/\mathscr{O} = \begin{pmatrix} E^o_1/\mathscr{O} & E^o_2/\mathscr{O} & \ldots & E^o_7/\mathscr{O} \\ \dfrac{p(E^o_1)}{p(\mathscr{O})} & \dfrac{p(E^o_2)}{p(\mathscr{O})} & \ldots & \dfrac{p(E^o_7)}{p(\mathscr{O})} \end{pmatrix}$$

Note that $A_i,\ i = 1, 2, \ldots, 10$ are the events of operation of ith component. The operational modes are defined on the basis of the events of operations and failures of the components, with following probabilities:

$$p(E^o_1) = p(A_1)p(A_2)p(A_3)p(A_4)p(A_5)p(A_6)p(A_7)p(A_8)p(A_9)p(A_{10});$$

$$p(E^o_2) = p(A_1)p(A_2)p(A_3)p(A_4)p(\bar{A}_5)p(A_6)p(A_7)p(A_8)p(A_9)p(A_{10});$$

$$p(E^o_3) = p(A_1)p(A_2)p(A_3)p(A_4)p(A_5)p(\bar{A}_6)p(A_7)p(A_8)p(A_9)p(A_{10});$$

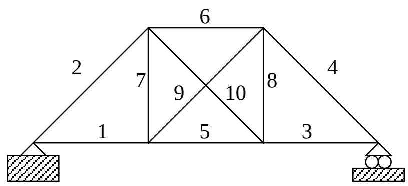

Fig. 2. Plane truss structure.

$$p(E_4^{\mathrm{o}}) = p(A_1)p(A_2)p(A_3)p(A_4)p(A_5)p(A_6)p(\bar{A}_7)p(A_8)p(A_9)p(A_{10});$$

$$p(E_5^{\mathrm{o}}) = p(A_1)p(A_2)p(A_3)p(A_4)p(A_5)p(A_6)p(A_7)p(\bar{A}_8)p(A_9)p(A_{10});$$

$$p(E_6^{\mathrm{o}}) = p(A_1)p(A_2)p(A_3)p(A_4)p(A_5)p(A_6)p(A_7)p(A_8)p(\bar{A}_9)p(A_{10});$$

$$p(E_7^{\mathrm{o}}) = p(A_1)p(A_2)p(A_3)p(A_4)p(A_5)p(A_6)p(A_7)p(A_8)p(A_9)p(\bar{A}_{10});$$

The dominant operational mode with the highest reliability is the first mode E_1^{o} with all its elements operational, i.e. with operational capacity $O = 10/10$. All other operational modes $E_i^{\mathrm{o}},\ i = 2, 3, \ldots, 7$ have reduced operational capacity $O = 9/10$, due to the failure of one of the central elements. Due to a large number of failure modes, they are not presented in the paper.

The system $\mathscr{S}$ is the complete system of events, since

$$p(\mathscr{S}) = p(\mathscr{O}) + p(\mathscr{F}) = \sum_{i=1}^{1024} p(E_i) = 1.0,$$

where

$$p(\mathscr{O}) = \sum_{i=1}^{7} p(E_i^{\mathrm{o}})$$

and

$$p(\mathscr{F}) = \sum_{i=8}^{1024} p(E_i^{\mathrm{f}}).$$

Furthermore, there is a range of potential views on the system of events representing the plane truss structure in Fig. 2. Let us consider the fully operational mode E_1^{o} with all elements operating, as an outstanding mode, denoted as subsystem $\mathscr{O}_1$ with only one mode. The remaining operational modes $E_i^{\mathrm{o}},\ i = 2, 3, \ldots, 7$ represent the redundant subsystem of six modes denoted as $\mathscr{O}_2$. The two subsystems of operational modes are:

$$\mathscr{O}_1 = \begin{pmatrix} E_1^{\mathrm{o}} \\ p(E_1^{\mathrm{o}}) \end{pmatrix}$$

and

$$\mathscr{O}_2 = \begin{pmatrix} E_2^{\mathrm{o}} & E_3^{\mathrm{o}} & E_4^{\mathrm{o}} & E_5^{\mathrm{o}} & E_6^{\mathrm{o}} & E_7^{\mathrm{o}} \\ p(E_2^{\mathrm{o}}) & p(E_3^{\mathrm{o}}) & p(E_4^{\mathrm{o}}) & p(E_5^{\mathrm{o}}) & p(E_6^{\mathrm{o}}) & p(E_7^{\mathrm{o}}) \end{pmatrix}.$$

The system of events $\mathscr{S}$ can be now viewed as a compound of three subsystems $\mathscr{S} = (\mathscr{O}_1 + \mathscr{O}_2 + \mathscr{F})$ and the system built of subsystems as $S'_A = (\mathscr{O}_1, \mathscr{O}_2, \mathscr{F})$. The subsystem of operational modes itself can be viewed as a set of two subsystems $\mathscr{O} = (\mathscr{O}_1 + \mathscr{O}_2)$ and as a subsystem built of sub-subsystems as $\mathscr{O}'_A = (\mathscr{O}_1, \mathscr{O}_2)$.

6.2. Redundancy and robustness analysis of a statically indeterminate plane truss structure

The parametric study of a plane truss structure by the variation of element's reliability is to be performed next. The edge elements A_1, A_2, A_3, and A_4 are assumed to be of the same quality, also having identical reliabilities. The central elements A_5, A_6, A_7, A_8, A_9 and A_{10} are of another quality, with the reliability different from the edge elements.

6.3. Discussion of the results of the event-oriented system analysis

The investigation of the effects of component reliabilities upon the system, for an assumed low target system reliability of 0.99, is illustrated in Fig. 4, Table 1, and discussed below:

1. The increase of the reliability of edge elements over 0.9999 does not have significant effects on system characteristics (Table 1).
2. For the reliability of central elements under 0.9731, the edge element reliability is unattainable.
3. For the reliability of edge elements under 0.99749, the central element reliability is unattainable.
4. The target system reliability of $p(\mathscr{O}) = 0.99$ is accomplished when all elements are of identical reliability of 0.9975142.
5. The redundancy $\mathrm{RED}(\mathscr{S})$ is an increasing function of the edge element's reliability. The maximum attainable redundancy amounts to $\mathrm{RED}(\mathscr{S})_{\max} = \log_2 7 = 2.8073$ bits. It can be of interest to express the maximum achieved redundancy relative to the maximum attainable redundancy as it follows: $\mathrm{RED}(\mathscr{S})/\mathrm{RED}(\mathscr{S})_{\max} = 0.9546/2.8073 = 0.34$.
6. The maximal robustness $\mathrm{ROB}(S)$ is encountered for the edge elements reliability equal to 0.9993 and the central

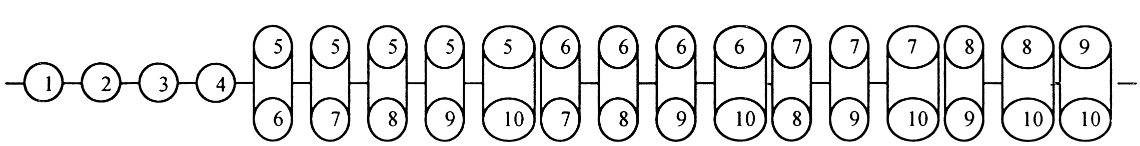

Fig. 3. Minimal cut set of the redundant plane truss structure on Fig. 1.

Table 1
Results of event-oriented system analysis for a given target system reliability $p(\mathcal{O}) = 0.99$ and various reliabilities of elements for a redundant plane truss structure

Reliabilities of elements		Operational modes		$H(\mathcal{O}+\mathcal{F})$ bits	$H(\mathcal{O},\mathcal{F})$ bits	$\mathrm{RED}(\mathcal{S})$ bits	$\mathrm{ROB}(\mathcal{S})$ bits	$H(\mathcal{S}) - H(\mathcal{S}')$ bits	$\mathrm{RED}(S'_A, O)$ bits
Edge	Central	$p(\mathcal{O}_1)$	$p(\mathcal{O}_2)$	Eq. (5)	Eq. (6)	Eq. (14)	Eq. (22)	Eq. (24)	Eq. (30)
0.997489	0.999999	0.9897	0.0003	0.1061	0.0808	0.0049	2.0430	0.0153	0.0038
0.997491	0.999752	0.9885	0.0015	0.1211	0.0808	0.0199	2.0594	0.0403	0.0164
0.9975	0.9984	0.9806	0.0094	0.2035	0.0808	0.1018	2.1828	0.1127	0.0774
0.997514	*0.997514*	0.9754	0.0146	0.2509	0.0808	0.1488	2.2791	0.1701	0.1108
0.9978	0.9967	0.9377	0.0523	0.5432	0.0808	0.4346	3.2114	0.4624	0.2983
0.998	0.9882	0.9293	0.0607	0.6382	0.0808	0.5269	3.6010	0.5574	0.3326
0.9985	0.9832	0.8980	0.0920	0.8026	0.0808	0.6861	4.2602	0.7218	0.4462
0.999	0.9794	0.8790	0.1110	0.9147	0.0808	0.7959	4.5884	0.8339	0.5063
0.9993	0.9774	0.8693	0.1207	0.9688	0.0808	0.8500	*4.6470*	0.8880	0.5348
0.9995	0.9761	0.8633	0.1267	1.0072	0.0808	0.8825	4.6223	0.9164	0.5519
0.9999	0.9738	0.8525	0.1375	1.0556	0.0808	0.9408	4.3452	0.9658	0.5813
0.99999	0.9733	0.8502	0.1398	1.0672	0.0808	0.9530	4.1837	0.9864	0.5874
0.999999	0.9732	0.8501	0.1399	1.0673	0.0808	0.9543	4.1555	0.9865	0.5876
0.9999999	0.9731	0.8497	0.1403	1.0674	0.0808	0.9546	4.1515	0.9868	0.5887

elements reliability of 0.9774. Then the entropy of failure modes enriches the maximum value of 4.6470, indicating that the probabilities of failure modes are maximally uniformly distributed. The maximum attainable robustness amounts to $\mathrm{ROB}(\mathcal{S})_{\max} = \log_2 1017 = 9.9901$ bits. It can be of interest also to express the maximum achieved robustness relative to the maximum attainable robustness as $\mathrm{ROB}(\mathcal{S})/\mathrm{ROB}(\mathcal{S})_{\max} = 4.6470/9.9901 = 0.4651$.

7. The system entropy $H(\mathcal{O}+\mathcal{F})$ is an increasing function of the edge elements reliability. The maximum attainable system entropy equals to $H(\mathcal{O}+\mathcal{F}) = \log 1024 = 10$ bits. The achieved system entropy is relatively low with respect to the maximum entropy of the system and amounts to $H(\mathcal{O}+\mathcal{F})/H(\mathcal{O}+\mathcal{F})_{\max} = 1.0674/\log 1024 = 0.10674$, indicating a low uncertainty.
8. The entropy of the system of operational and failure subsystems is constant $H(\mathcal{O},\mathcal{F}) = 0.0808$ due to the imposed system target reliability of $p(\mathcal{O}) = 0.99$. The maximum attainable entropy of the system of subsystems equals to $H(\mathcal{O},\mathcal{F}) = \log 2 = 1$ bit. It is apparent that there is a small uncertainty due to high reliability, and only two subsystems of events. The achieved entropy represents only a small fraction of the maximum entropy, as shown: $H(\mathcal{O},\mathcal{F})/H(\mathcal{O},\mathcal{F})_{\max} = 0.0808/\log 2 = 0.0808$.
9. The maximal system redundancy indicates the optimal system operational abundance.
10. The maximal system robustness indicates the optimal system endurance under distress.

6.4. Event oriented system analysis applied on the subsystems of different operational capacities

The reliability of the fully operational mode $\mathcal{O}_1$ is changing significantly with the variation of element's reliabilities even when the target system reliability is constant. The system reliability increases by the increase of the central element reliabilities (see Table 1). The system redundancy changes too. The redundancy of the fully operational mode $\mathcal{O}_1$ is $\mathrm{RED}(\mathcal{S}/\mathcal{O}_1) = H_1(\mathcal{S}/\mathcal{O}_1) = 0$, since there are no other modes with all elements operating. It is apparent that the subgroup $\mathcal{O}_2$ is perfectly redundant in case when all central elements are of the same reliability. The redundancy of the subsystem $\mathcal{O}_2$ is equal to $\mathrm{RED}(\mathcal{S}/\mathcal{O}_2) = H_6(\mathcal{S}/\mathcal{O}_2) = \log 6 = 2.5849$ and attains the maximal value due to the fact that all the probabilities $p(E_i^{\mathrm{o}}),\ i = 2, 3, \ldots, 7$ of alternative failure modes are identical. Since $H(\mathcal{S})$ is not changed, the difference in system redundancy appears only in $H(\mathcal{S}'_A/\mathcal{O}) = \mathrm{RED}(\mathcal{S}'_A)$ of Eq. (30), indicating that there is an increase in the redundancy of the system, see last column in Table 1. This increase in redundancy is the consequence of the decrease in reliability of a fully operational mode and the uniformity of the probabilities of operational modes. The increase of the system redundancy leads to the unification of the operational modes probabilities, i.e. $p(E_1^{\mathrm{o}})$ reduces and the

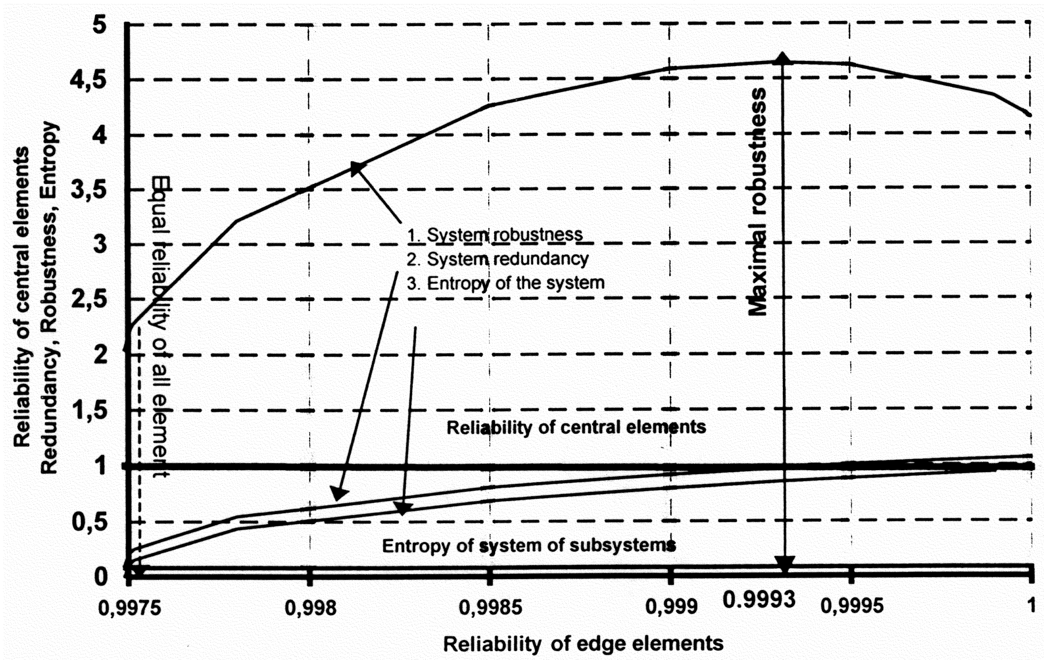

Fig. 4. Results of event-oriented system analysis for a given target system reliability $p(\mathcal{O}) = 0.99$ and various reliabilities of elements for a redundant plane truss structure.

$p(E_i^o)$, $i = 2, 3, \dots 7$ increase. Note that $H(\mathcal{S}'_A/\mathcal{O})_{\max} = RED(\mathcal{S}'_A)_{\max} = \log 2 = 1$ bit.

7. Conclusion

The idea underlined in the article is to relate the redundancy and the robustness to the operational abundance and failure endurance of systems of events. Such characterisations, based on common engineering reasoning, are expected to be relevant from the engineering point of view. These probabilistic system properties are either intuited or conceived by common engineering reasoning. If these intuited properties are properly chosen, then such arguments can show the real relevance of definitions of redundancy and robustness for engineering purposes. Moreover, it has been demonstrated how these properties are related to the uncertainties of a system of events. The redundancy and the robustness were interpreted in terms of uncertainties and expressed by entropy. In the axiomatic treatment of entropy it is shown that only entropy has the properties in full agreement with intuition about uncertainties. Consequently, redundancy and robustness, defined by entropy of operational and failure modes, are the only rational measures for system operational abundance and endurance to failures. The entropy as a measure of the system uncertainty does not depend on anything else than the possible events and in this sense is entirely objective. Hence, the redundancy and robustness as defined in the article by entropy are also objective measures of system operational abundance and endurance to failures.

Engineering systems employing more operational and failure modes, with known probabilities, are denoted in the article as probabilistically redundant and robust with respect to the operational and failure modes. The term redundancy in the article is assigned the notion of probabilistic abundance in a number and in probabilities of operational modes. Analogously, the word robustness in terms of failure events denotes an excessive capability to respond to all the demands by a number of failure modes with adequate probability distribution.

Some of the consequences of the definitions of the redundancy and robustness of the systems of events for the engineering design can be summarised as follows:

- The system reliability and the system redundancy are two independent system characteristics referring only to the operational states of the system.
- The system failure probability and system robustness are two independent characteristics but referring only to the failure modes of the system.
- The request for adequate system reliability leads to maximal attainable probabilities of operational modes, as well as to minimal attainable probabilities of failure modes.
- The requests for high system redundancy and high system robustness lead to maximally attainable uniformity of the probability distributions of operational modes and of failure modes, respectively, regardless of the system reliability or failure probability.

The same system of events can be viewed in different ways. Some of the operational modes can be considered more important from the designer's or from the user's point of view, for example when all elements operate, than all the other operational modes with reduced operational capabilities. On the other hand, some of the failure modes can endanger systems much more than any other of the failure modes. The presented approach allows separate and joined analysis of different groups of operational and failure modes of special interest, as well as their relations.

Computational problems involved in redundancy and robustness assessment of large systems arise from the complexity of the probability calculations of all the operational and failure modes. For highly redundant and robust systems there is usually a great number of operational and failure modes. The presented approach allows to concentrate the analysis only on the observable and important modes, being numerically feasible. It is not impossible that in the next future, thanks to an enormous increase in numerical capacities of new generations of computers, the redundancy and robustness of systems of events will be taken into consideration in the assessment of overall system effectiveness.

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