HYDROSTATICS OF A DEFLECTED SHIP HULL

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Approximate terms based on waterplane area characteristics are provided for easy and fast assessment of the effects of deflections of an elastic ship hull on draughts, displacement and still water hull girder loads. A numerically efficient iterative method based on repeated standard rigid girder calculation is proposed for the determination of draughts, displacement and hull girder loads of an elastic ship hull on an optional floating line.
The relevance of both of the methods is empirically justified.

1. Introduction

The paper presents first a comprehensive method for assessment of the deflection effects of an elastic ship hull, on draughts and displacement, as well as on the still water hull girder loads. The method is based on the data observed on-board, and uses a parabolic approximation of the deflection line as most of the methods do, e.g. Ref. [1], but taking into account the ship hull form, what the existing methods do not. Next, an iterative method for hydrostatics and hull girder loads determination of the deflected ship hull on an optional floating line, based on a repeated application of standard rigid body calculation of shear forces and bending moments, e.g. Ref. [2], is presented. Finally, the methods are tested on two ships: "A" - a large OBO-carrier and "B" - a river barge.

2. Approximate assessment of the hull deflection effects

The following considerations are based on the data available on-board. The draughts

observed on ship side(s) aft, amidships and forward, see Figure 1, are denoted as follows:

$$d_a^o, d_m^o, d_f^o$$

The observed amidships draught d_m^o corresponds to the "observed" displacement Δ_o, as determined from standard hydrostatic data, see Figure 2.
The standard amidships draught d_m^s used for conventional hydrostatic calculations is obtained from the observed draughts as follows:

$$d_m^s = \frac{d_a^o + d_f^o}{2} \tag{1}$$

The "standard" displacement Δ_s for a ship hull considered as a rigid girder can be obtained from standard hydrostatic data using the standard amidships draught d_m^s from eqn. (1).

2.1. Assessment of the hull deflection

The ship hull deflection w_m is usually considered only amidships as the deviation of the observed amidships draught of the standard draught amidships in eqn. (1), as shown:

$$w_m = d_m^o - d_m^s = d_m^o - \frac{d_a^o + d_f^o}{2} \tag{2}$$

It follows from eqn. (2), that the deflection amidships is positive for a sagging condition.
The second order symmetric parabola, can be used as an approximation for the deflection line of the hull, given in the following form:

$$w(x) = w_m \frac{x^2}{(L_{wl}/2)^2} \tag{3}$$

Table 1 presents a comparison between the parabolic approximation and the theoretically calculated deflection line for the sagging condition of test ship "A", normalized to unity.

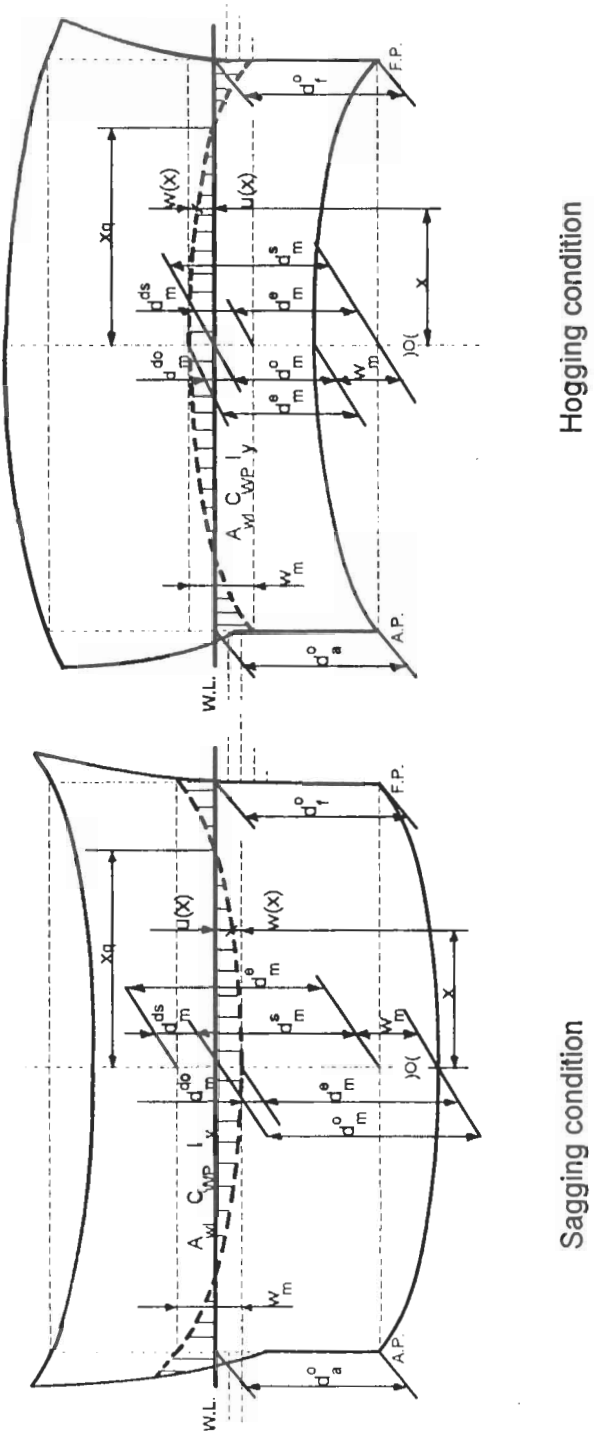

Figure 1. Notations for description of a deflected ship hull.

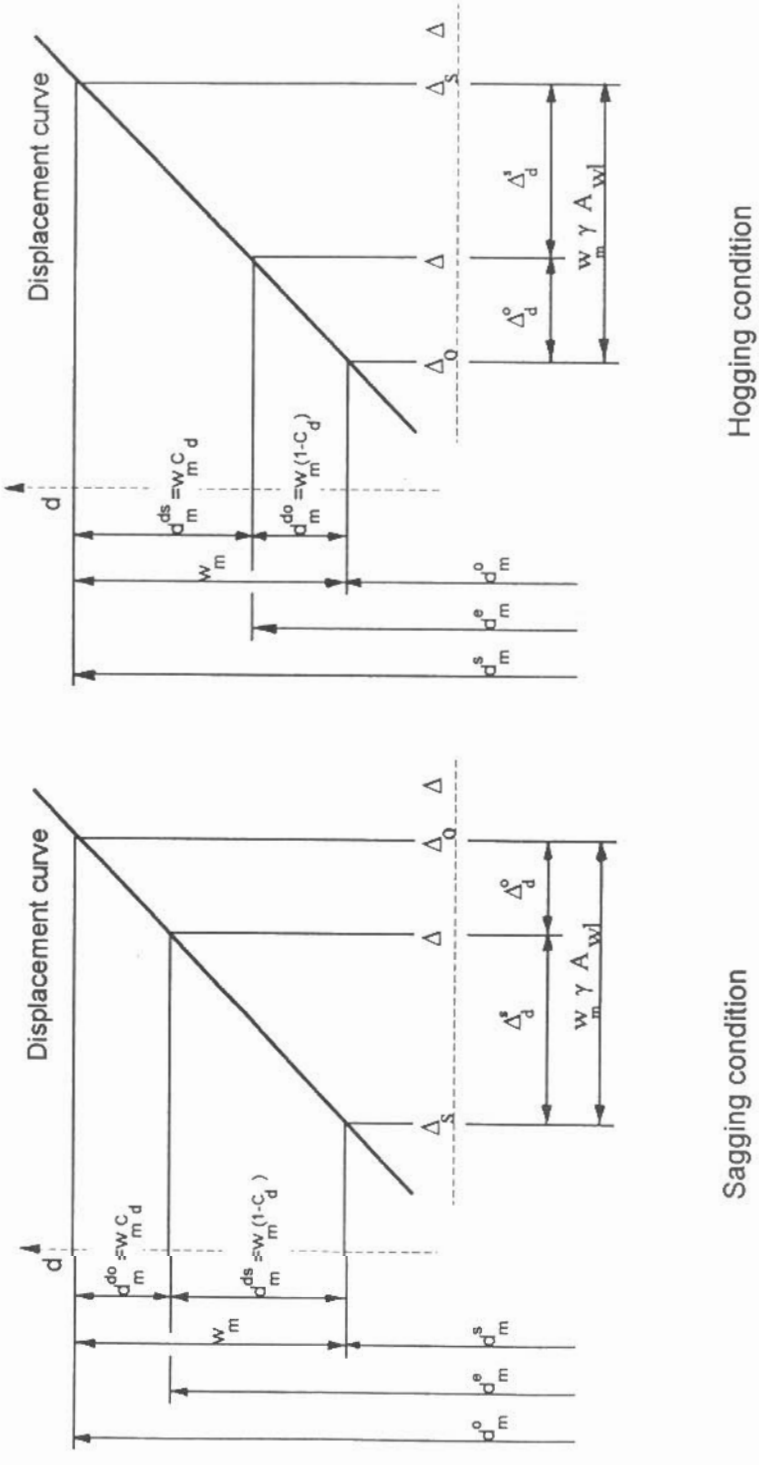

Figure 2. Relations of draughts and corresponding displacements of a deflected ship hull.

Table 1. Calculated and parabolic approximation of a deflection line normalized to unity, for a sagging condition of the test ship "A".

w↓	/	x→	.00L_{wl}	.05L_{wl}	.10L_{wl}	.15L_{wl}	.20L_{wl}	.25L_{wl}	.30L_{wl}	.35L_{wl}	.40L_{wl}	.45L_{wl}	.50L_{wl}
(calc)	w_{aft}		.000	.155	.307	.452	.587	.708	.809	.892	.952	.988	1.00
(calc)	w_{forw}		.000	.152	.302	.446	.579	.699	.803	.886	.948	.987	1.00
(parab)	$w_{aft\&forw}$		.000	.190	.360	.510	.640	.750	.840	.910	.960	.990	1.00

It is obvious from Table 1 that the calculated deflection line is neither symmetrical nor parabolic, but the deviations of the parabolic form are of limited order. On the other hand, it is not practical to determine the deflection shape on board more precisely.

2.2. Assessment of the draughts and displacement of a deflected ship hull

The hydrostatic characteristics of the ship hull for small deflections, depend mostly on waterplane properties, as can be seen from Figure 1. The longitudinal change in buoyancy due to hull deflection $w(x)$ for hypothetically parallel ship sides, can be defined at any abscissa x, using the waterplane breadth $b(x)$, as the change of the sectional area:

$$a_r(x) = w(x)b(x) \tag{4}$$

The decrease and increase of the displacement relative to the "observed" displacement Δ_o, of the deflected hull in sagging and hogging conditions respectively, in water of specific gravity γ, supposing a symmetric parabolic deflection shape as in eqn. (3), is obtained by lengthwise integration of eqn. (4) as follows:

$$\Delta_d^o = \gamma \int_{-\frac{L_{wl}}{2}}^{+\frac{L_{wl}}{2}} a_r(x)\mathrm{d}x = \frac{w_m\gamma}{(L_{wl}/2)^2} \int_{-\frac{L_{wl}}{2}}^{+\frac{L_{wl}}{2}} b(x)x^2\mathrm{d}x = w_m\gamma A_{wl}\frac{(I_m/A_{wl})}{(L_{wl}/2)^2} = w_m\gamma A_{wl}C_d \tag{5a}$$

The change of the displacement relative to the "standard" displacement can be expressed as

$$\Delta_d^s = w_m\gamma A_{wl} - \Delta_d^o = w_m\gamma A_{wl}(1 - C_d) \tag{5b}$$

The following symbols are used:

I_m longitudinal moment of inertia of the waterline plane about a transverse axis through the mid of its length; it can be obtained from standard hydrostatic data as follows:

$$I_m = A_{wl}(L_{wl}/2)^2 C_d = I_y + x_{wl}^2 A_{wl} \approx M_1 L_{wl} \tag{6}$$

I_y longitudinal moment of inertia of the waterline plane about a transverse axis through the center of flotation;
A_{wl} area of the waterplane of length L_{wl} and breadth B_{wl}, ($A_{wl} = L_{wl}\, B_{wl}\, C_{WP}$);
x_{wl} distance from the waterline center of gravity to the mid of length L_{wl};
M_1 the moment to change trim 1 meter;
C_{WP} waterplane area coefficient;
C_d the draught and displacement correction factor in eqn. (5) due to deflection is defined as shown:

$$C_d = \frac{(I_m/A_{wl})}{(L_{wl}/2)^2} \approx 4\frac{M_1}{A_{wl}L_{wl}} \tag{7}$$

The correction of the observed amidships draught due to deflection can be determined as a parallel emersion or immersion for sagging or hogging conditions respectively, as follows:

$$d_m^{do} = \frac{\Delta_m^o}{\gamma A_{wl}} = w_m C_d \tag{8a}$$

The correction of the standard amidships draught can be easily obtained as shown:

$$d_m^{ds} = w_m - d_m^{do} = w_m(1 - C_d) \tag{8b}$$

To obtain the actual displacement Δ of the ship from the standard hydrostatic data, the equivalent amidships draught can be used, as clarified on Figure 2. Either the observed amidships draught d_m^o or the standard amidships draught d_m^s can be corrected to the equivalent amidships draught d_m^e using deflections amidships w_m from eqn. (2) and the draught and displacement correction factor C_d from eqn. (7) as shown:

$$d_m^e = d_m^o - d_m^{do} = d_m^o - w_m C_d = d_m^s + d_m^{ds} = d_m^s - w_m(1 - C_d) \tag{9}$$

The actual displacement can be obtained directly from the standard hydrostatic data (displacement curve), using the equivalent amidships draught d_m^e (see Figure 2).

Hence, the actual displacement can be determined relative either to the "observed" displacement Δ_o or to the "standard" displacement Δ_s according to eqn. (5) (see Figure 2) as shown:

$$\Delta = \Delta_o - \Delta_d^o = \Delta_o - w_m \gamma A_{wl} C_d = \Delta_s + \Delta_d^s = \Delta_s + w_m \gamma A_{wl}(1 - C_d) \tag{10}$$

The expressions $A_{wl}C_d$ and $A_{wl}(1 - C_d)$ in eqn. (10) represents the volume of displacement correction per unit of deflection of the ship hull, relatively to the "observed" displacement and "standard" displacement, respectively.
For the test ship "A", hydrostatic data, correction factors C_d determined according to eqn. (7), as well as the values for $A_{wl}C_d$ and $A_{wl}(1 - C_d)$ are given in Table 2.

2.3. Approximation of the waterplane shape

For simplicity, the true waterplane shape will be approximated by a symmetric general parabola of order k:

$$b(x) = B_{wl}\left[1 - \frac{x^k}{(L_{wl}/2)^k}\right] \tag{11}$$

In that case, the following relations based on eqn. (11) can easily be proved:

$$k = \frac{C_{wp}}{1 - C_{wp}} \tag{12}$$

$$A_{wl} = \int_{-\frac{L_{wl}}{2}}^{+\frac{L_{wl}}{2}} b(x)\mathrm{d}x = B_{wl}L_{wl}\frac{k}{k+1} = B_{wl}L_{wl}C_{WP} \tag{13}$$

$$I_m = \int_{-\frac{L_{wl}}{2}}^{+\frac{L_{wl}}{2}} ba(x)x^2\mathrm{d}x = A_{wl}(L_{wl}/2)^2 C_d \tag{14}$$

$$C_d = \frac{1}{6(\frac{3}{2} - C_{WP})} \tag{15}$$

For some other types of vessels, e.g. barges or pontoons, a more appropriate approximation of the waterplane shape can be of the linear form given as follows:

$$b(x) = B_{wl}\left[\ 1 - 2\,\frac{1 - C_{WP}}{(L_{wl}/2)}x\ \right] \tag{16}$$

The draught and displacement correction factor for these linear waterplane forms according to eqn. (16), can be obtained as:

$$C_d = \frac{C_{WP} - \frac{1}{3}}{2C_{WP}} \tag{17}$$

2.4. Assessment of the bending moments and the shear forces

The standard still water calculations provide draughts, displacement, shear forces and bending moments for the ship hull considered as a rigid girder. The deflection line calculation can be performed using the ship hull girder vertical sectional moments of inertia. The deflection of the ship hull changes the hydrostatic loads and in reverse it affects the draughts, the shear forces and the bending moments. A second order symmetric parabola in eqn. (3), shifted vertically (in order to assure hydrostatic balance) over the parallel immersion $w_m C_d$, can be used as an approximation for lengthwise distribution of the changes of hydrostatic loads relatively to the ship hull considered as rigid girder:

$$u(x) = w(x) - w_m C_d = w_m\left[\ \frac{x^2}{(L_{wl}/2)^2} - C_d\ \right] \tag{18}$$

The longitudinal change in buoyancy (positive direction upward ↑) for hypothetically parallel ship sides due to hull deflection $u(x)$, relatively to the floating line of a hypothetically rigid ship hull, can be obtained on the basis of eqn. (18), as follows:

$$q_b = -\,u(x)b(x) \tag{19}$$

The maximum changes of the shear forces due to the deflection relative to the rigid body calculation occur at the positions $\pm x_q$, where $q_b(x_q) = 0$ in eqn. (19), as follows:

$$x_q = \pm\, L_{wl}\frac{\sqrt{C_d}}{2} = \pm\, L_{wl}C_{xq} \tag{20}$$

and the amount of this maximum in shear force change is as shown:

$$Q_d = \pm\,\gamma \int_{-\frac{L_{wl}}{2}}^{x_q} q_b(x)\mathrm{d}x \;= \pm w_m \gamma B_{wl} L_{wl} C_Q \tag{21}$$

The change of the amidships bending moment due to deflection relative to the rigid body calculation is obtained by lengthwise integration applied on eqn. (19), as shown:

$$M_d = \gamma \int_{-\frac{L_{wl}}{2}}^{0} q_b(x) x \mathrm{d}x \;= w_m \gamma B_{wl} L_{wl}^2 C_M \tag{22}$$

The shear force correction factor C_Q in eqn. (21) based on eqn. (11) is expressed as:

$$C_Q = C_d^{3/2}\left[\tfrac{1}{3} - 3(1 - C_{WP})^2 C_d^{\frac{1-C_{WP}/2}{1-C_{WP}}}\right] \tag{23}$$

It is easily recognizable that the bending moments calculated for the conventional ships considered as rigid girders, are generally reduced when the resulting deflection of the ship hull is taken into account. The amidships bending moment correction factor C_M in eqn. (22) based on general parabola assumption in eqn. (11), is expressed as:

$$C_M = \frac{C_{WP}}{48}\,\frac{6C_{WP}^2 - 15C_{WP} + 10}{-6C_{WP}^3 + 29C_{WP}^2 - 46C_{WP} + 24} \tag{24}$$

The shear force and amidships bending moment correction factors based on linear approximation of the waterplane shape in eqn (16), can be expressed as:

$$C_Q = C_d^{3/2}\left[\tfrac{1}{3} - \tfrac{1}{4}(1 - C_{WP})C_d^{1/2}\right] \tag{25}$$

$$C_M = \frac{12C_{WP}^2 + -8C_{WP} - 5}{-720C_{WP}} \tag{26}$$

For faster assessments of deflection effects, the linearized draught, amidships bending moment and shear force correction factors are given as follows (see also Figure 3.):

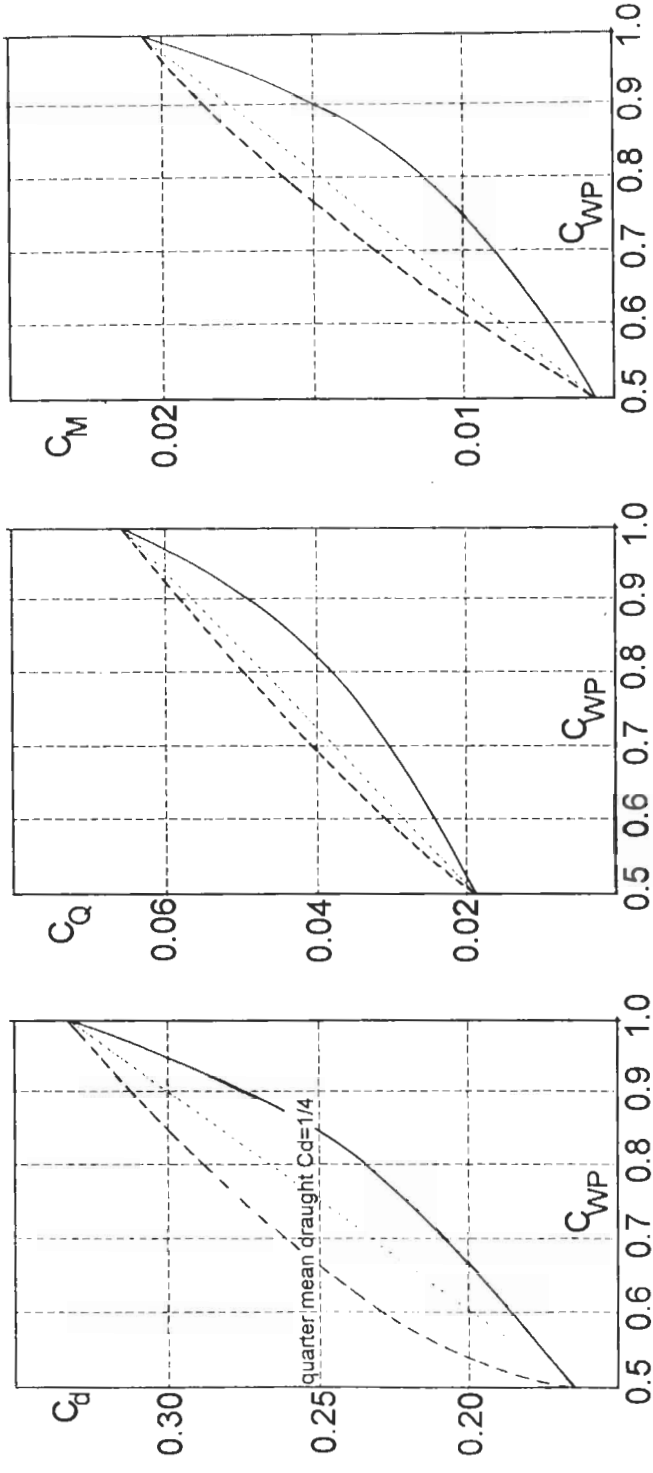

Figure 3. Correction factors based on C_{WP} for draughts C_d, for shear forces C_Q and for bending moments C_M.

- the lower curves correspond to general parabolic approximation of the waterline (hairline);
- the upper curves correspond to linear approximation of the waterline (dash line);
- the lines between correspond to linearized correction factors (dash-and-dot line);

$$C_d^l = \tfrac{1}{3} C_{WP} \tag{27}$$

$$C_Q^l = 0.08988 C_{WP} - 0.02573 \tag{28}$$

$$C_M^l = \tfrac{1}{720}(22 C_{WP} - 7) \tag{29}$$

The correction factors C_d, C_Q, C_M and C_{xq}, for the test ship "A" based on eqns. (15), (23), (24) and (20) respectively, are given in last four columns in Table 2.

Table 2. Hydrostatic data and correction factors for the test ship "A".

d [m]	C_{wp} –	L_{wl} [m]	A_{wl} [m^2]	I_y [m^4]	M_1 [tm]	C_d eq7	$A_{wl}C_d$ [m^2]	$Awl(1-C_d)$	C_d eq15	C_Q eq23	C_M eq24	C_{xq} eq20
Actual ship data, OBO-carrier						Parabolic deflection			Waterline approximation			
3	.80	227	6457	22095296	96061	.262	1691	4762	.238	.0385	.0114	.244
6	.83	232	6642	24166730	104166	.270	1793	4848	.249	.0413	.0123	.249
9	.85	236	6828	26065544	111390	.276	1884	4943	.256	.0432	.0129	.253
12	.88	238	7060	28654838	117436	.279	1969	5090	.269	.0464	.0140	.259
15	.91	240	7292	31589366	126863	.289	2107	5185	.282	.0500	.0153	.265
18	.93	248	7432	33315562	133276	.289	2147	5284	.292	.0527	.0163	.270
21	.94	250	7525	34351280	137491	.292	2197	5328	.298	.0541	.0168	.272

The factors C_d in eqns. (15, 17, 27), C_Q in eqns. (23, 25, 28) and C_M in eqns. (24, 26, 29) depend only on the waterplane area coefficient C_{WP}, and are represented on Figure 3.

It is obvious from Figure 3., that the usually applied 'quarter mean draught' correction, e.g. Ref. [1], corresponding to C_d =1/4, is valid only for a specific waterplane coefficient of about C_{WP}=0.834 for general parabolic approximation of the waterline. For other values of C_{WP}, the correction differs significantly from one quarter, that confirms the need for more precise approximation of the hull form, i.e. of the waterplane shape.

3. Iterative calculation of the hull deflection effects

The geometrical characteristics of a deflected ship hull on the optional floating line $d(x)$ at position x_r, (see Figure 4) can be defined by correction of the standard draughts $d^s(x_r)$ of the hypothetically undeflected hull, in successive iterations, for an amount of deflection $w(x_r)$ determined by a standard calculation method in previous iteration, as follows:

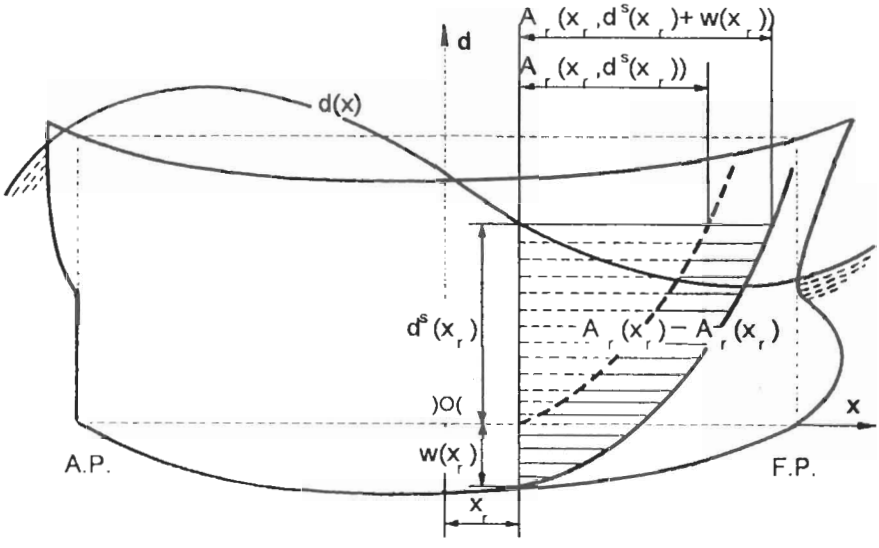

Figure 4. Geometry definition of a deflected ship hull.

$$d(x_r) = d^s(x_r) + w(x_r) \tag{30}$$

The sectional area A_r to draught $d(x_r)$ of a deflected hull is then as shown:

$$A_r[x_r,d(x_r)] = A_r[x_r,d^s(x_r) + w(x_r) \tag{31}$$

The correction in eqns. (30) and (31) is not applied when the bottom is out, or when the deck is immersed.

The geometry of a deflected hull can be described by eqns. (30) and (31), using standard data for a non-deflected hull (Bonjean's curves), and the standard calculation methods can be applied by iterations, as presented by a flowchart in Figure 5. Convergence criteria can be based on the relative changes in the bending moments or in the deflection line. Sufficient accuracy will normally be achieved in 3-5 iterations. The first iteration corresponds to the standard rigid body calculation, taking the initial hull deflection into account, if any.

The draughts obtained for the rigid ship hull can be regarded as the equivalent draughts with respect to the draughts of the deflected hull, as it is considered earlier.

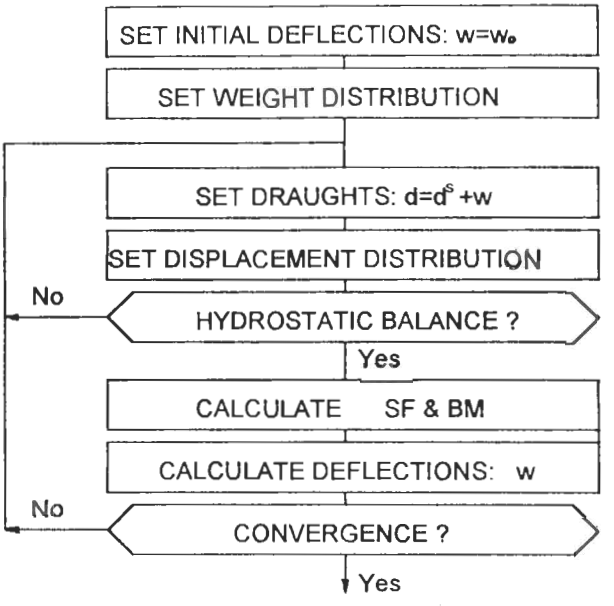

Figure 5. Flowchart of the iterative method for calculating draught and hull girder loads of an elastic ship hull.

4. Examples

Two test vessels: "A" and "B" are investigated, with principal data and loading conditions according to Tables 3. and 4.

Table 3. Principle data of test ships "A" and "B".

Ship	L_{oa} [m]	L_{bp} [m]	B [m]	D [m]	d [m]	Δ [t]	LS [t]	I_{sec} [m^4]
"A"	251.10	241.00	32.25	20.60	15.00	101948	18217	307.28
"B"	77.00	77.00	11.00	4.50	3.20	2650	650	1.64

Table 4. Load cases for test ships "A" and "B".

Ship	Load case 1	Load case 2	Load case 3
"A"	lightship (LS)	tanker service, fully loaded	light ballast, 100 % stores
"B"	ballast condition	2000 t bulk cargo	600 t heavy cargo

First, a standard calculation of hydrostatics, shear forces and bending moments for the

ship hull as a rigid body is performed. Next, the calculation for the ship hull considered as an elastic body is performed by iterations, taking the deflection line into account until the convergence is achieved, using the procedure described in section two.

Finally, approximate terms as presented in section one, are applied. The results are presented in Table 5.

Table 5. Displacements, deflections, draughts, bending moments and shear forces:
- (1st row) calculated for the ship hull as a rigid girder;
- (2nd row)calculated for elastic ship hull using iterative procedure;
- (3rd row) assessed by using the approximate term.

CASE	Δ	Δ_d^s	Δ_d^o	w_m	d_a^e	d_m^e	d_f^e	d_m^s	BM	M_d %	SF	Q_d%
C_d	Δ_s	Δ_o		w_m	d_a^o	d_m^o	d_f^o	d_m^s	BM	M_d	SF	Q_d
C_{WP}	Δ_s	Δ_o		w_m	d_a^o	d_m^o	d_f^o	d_m^s	BM	M_d	SF	Q_d
	[t]			[cm]		[m]			[kNm]		[kN]	
A-1	18217	548	195	-12.2	5.11	3.02	0.93	3.02	1486197	1.5%	28953	1.2%
.262	18765	18022		-12.0	5.22	2.99	1.00	3.11	1463311	22886	28673	280
.80	18802	18008		-12.0	5.19	2.99	1.01	3.10	1463338	22859	28614	339
A-2	100513	367	146	7.6	14.69	14.77	14.85	14.77	-996963	1.9%	24223	1.1%
.285	100143	100559		7.4	14.65	14.79	14.79	14.72	-977278	19685	23979	244
.90	100123	100668		7.4	14.64	14.79	14.80	14.72	-977088	19875	23951	272
A-3	45121	785	300	-15.4	8.39	7.00	5.62	7.00	1741587	2.1%	30497	1.5%
.277	45906	44821		-15.1	8.52	6.96	5.71	7.11	1705656	35931	30023	474
.86	45862	44837		-15.1	8.50	6.96	5.72	7.11	1705622	35965	29988	509
B-1	736	8	4	-1.3	0.88	0.90	0.92	0.90	8191	1.9%	-637	0.8%
.330	744	732		-1.3	0.89	0.89	0.93	0.91	8034	157	-637	0
.98	744	732		-1.3	0.89	0.89	0.93	0.91	8010	181	-632	5
B-2	2650	23	11	4.0	3.13	3.16	3.19	3.16	-23347	2.3%	-2364	0.6%
.330	2627	2661		3.9	3.10	3.17	3.16	3.13	-22827	-520	-2344	-20
.98	2627	2661		3.9	3.10	3.17	3.16	3.13	-22803	-544	-2349	-15
B-3	1250	20	10	3.4	1.51	1.53	1.55	1.53	-25780	1.8%	-1609	1.2%
.330	1230	1260		3.3	1.48	1.54	1.52	1.50	-25349	-431	-1589	-20
.98	1230	1260		3.3	1.49	1.54	1.53	1.50	-25314	-466	-1589	-20

5. Conclusions

Instead of the usually applied "quarter mean draught" correction, the presented terms based on the observed draughts, the parabolic approximation of the deflection line of

the ship hull and the true shape of the waterline, provide a fast practical method for determination of draughts, displacement, shear forces and bending moments of a deflected ship hull.

It is demonstrated that the deviations due to the parabolic form of the theoretical deflection line are in general of limited order for conventional ships. The presented expressions based on the application of the standard hydrostatic data as the moment to change trim one meter, the waterplane longitudinal moment of inertia, the waterplane area, can easily be used on-board.

The iterative application of a standard rigid-body calculation procedure for an optional floating line, as presented in the paper, allows theoretical determination of the ship hull deflection effect on draughts, displacement, shear forces and bending moments, in a limited number of iterations (3 to 5).

The approximate terms based only on the waterplane area coefficient, are provided for faster assessments and are in a good agreement with the results of the iterative procedure.

The maximum bending moments of a deflected ship are in general slightly less (approximately 2%) than the bending moments of a ship hull considered as a rigid body.

The shear forces of the deflected ship are also slightly changed compared to the shear forces determined for the ship as a rigid girder (approximately 1%).

More accurate and practical corrections of draughts and displacement due to deflection as demonstrated in the paper, can be beneficial in ship service.

References

[1] Durham, C. F.: Marine Surveys, Fairplay Publications, London, 1982.

[2] Comstock, J. P.(ed): Principles of Naval Architecture, SNAME, New York, 1957.

Table of notations

A_{wl}	area of the water plane;
A_r	sectional area;
BM	bending moments;
B_{wl}	waterline beam;
C_d	correction factor for draught and displacement of a deflected hull relative to the observed amidships draught;
C_M	correction factor for the bending moment amidships of a deflected hull relative to the value calculated for a hull as a rigid body;

C_Q	correction factor for the shear force at the position x_{wl} of a deflected hull relative to the value calculated for a hull as a rigid body;
C_{xq}	correction factor for the lengthwise position of the maximum change in shear force;
C_{WP}	waterline coefficient;
d_a^o, d_m^o, d_f^o	observed draughts aft, amidships and forward;
d_m^{do}	change of the observed amidships draught due to the hull deflection;
d_m^{ds}	change of the standard amidships draught due to the hull deflection;
d_m^e	equivalent amidships draught corresponding to the actual displacement;
d_m^s	standard amidships draught;
I_m	moment of inertia of the waterline about a transverse axis through the mid of length;
$I_{\sec}$	sectional moment of inertia,
I_y	moment of inertia of the waterline about a transverse axis through the center of flotation;
L_{wl}	waterline length;
M_d	change of the bending moment amidships for a deflected hull relative to the value calculated for a hull as a rigid body;
M_1	the moment to change trim 1 meter;
Q_d	change of the shear force at position x_{qm} for a deflected hull relative to the value calculated for a hull as a rigid body;
SF	shear forces;
x_{qm}	positions of maximal changes of shear forces;
x_{wl}	distance from the waterline center of gravity to the mid of length L_{wl};
w_m	hull deflection amidships (maximal deflection), considered as positive for hogging;
γ	specific gravity of water;
Δ	actual displacement;
Δ_o	"observed" displacement corresponding to the observed amidships draught;
Δ_s	"standard" displacement corresponding to the standard amidships draught;
Δ_d^o	change of the displacement for deflected hull relative to the "observed" displacement;
Δ_d^s	change of the displacement for deflected hull relative to the "standard" displacement;