UNCERTAINTY OF MULTI-LEVEL SYSTEMS OF EVENTS

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Abstract: The note presents probabilistic uncertainty modeling of multi-level systems of events, mutually linked by transitive events. A method for multi-level systems with potentials to emerge new functional states is suggested and the application is illustrated by a two-level example of systems pertinent to a game with a die and a coin. The presented procedure provides analogy to more complex transitive systems operating in uncertain circumstances with a redistribution of capabilities and demands in case of cascades of failures.

Keywords: probability, uncertainty, entropy, event oriented system analysis, engineering.

1. Introduction

The article tackles systems with transitive properties by event oriented systems analysis (EOSA). The systems involved acquire new functional states after configuration changes due to failures. A general procedure is anticipated for versatile engineering problems with multi-level transitive behavior. The feasibility is demonstrated on a game with a die and coins. The note reveals that the entropy of objects in case of transitive behavior always increase.

2. Multi-level systems

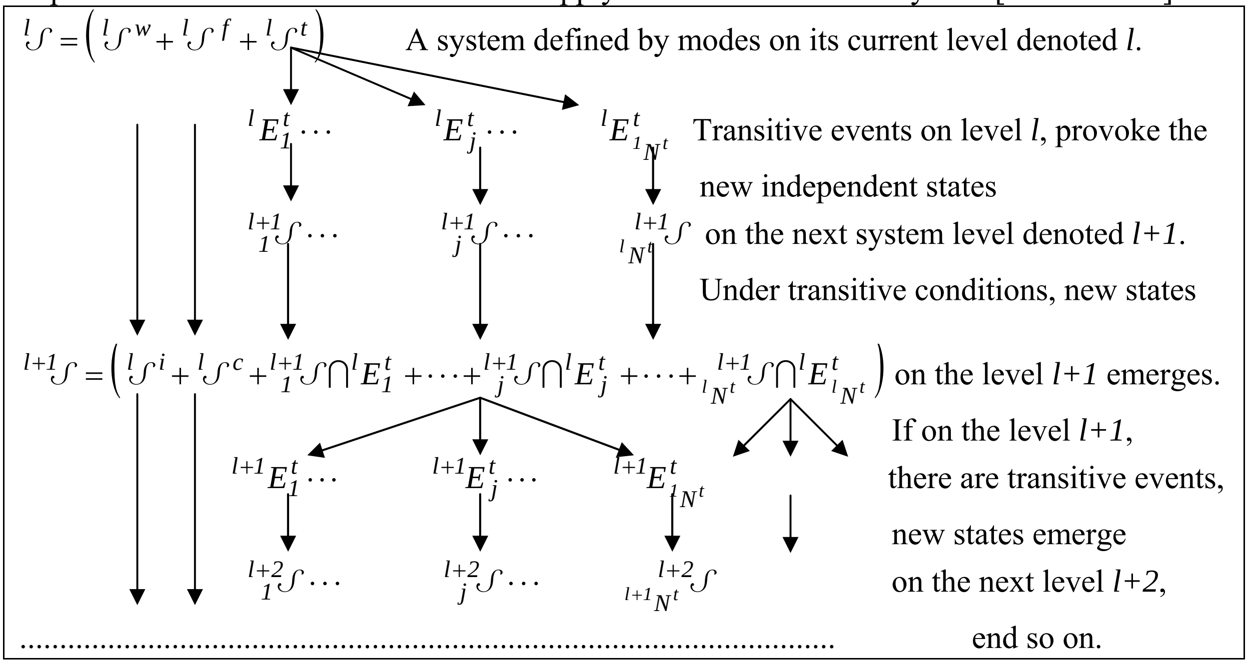
System levels, states, modes, events and system profiles represent a multi-level system with transitive events, denoted as follows:

- ${}^{l}\mathcal{S}$ A system level is a system of events comprising all states and modes of a system, where $l = 1,2,...,n$, and n is the number of system levels.
- ${}^{l}_{j}\mathcal{S}$ System states are systems composed of modes, which represent distinguished actions during the system's lifetime.
- ${}^{l}_{j}\mathcal{S}^{s}$ System modes are subsystems of events with a common outcome "s", presented as finite scheme [Khinchin 1957]: $${}^{l}_{j}\mathcal{S}^{s} = \begin{pmatrix} {}^{l}_{j}E^{s}_{1} & \cdots & {}^{l}_{j}E^{s}_{i} & \cdots & {}^{l}_{j}E^{s}_{{}^{l}_{j}N^{s}} \\ p({}^{l}_{j}E^{s}_{1}) & \cdots & p({}^{l}_{j}E^{s}_{i}) & \cdots & p({}^{l}_{j}E^{s}_{{}^{l}_{j}N^{s}}) \end{pmatrix}.$$

The outcomes "s" have meanings within the system context, for example: *w*-winning, *f*-failing, *t*-transitive, *n*-non-transitive, and combinations. On the considered system level, there are $j = 1,2,...,{}^{l}n$, system states, presented by system modes as ${}^{l}_{j}\mathcal{S} = \left({}^{l}_{j}\mathcal{S}^{w} + {}^{l}_{j}\mathcal{S}^{f} + {}^{l}_{j}\mathcal{S}^{t}\right)$.

- ${}^{l}_{j}E^{s}_{i}$ An event of a specific outcome "s", where $i = 1,2,...,{}^{l}_{j}N^{s}$, and ${}^{l}_{j}N^{s}$ is a number of events within a system level, state or mode. For only one mode index *j* may be omitted.
- ${}^{l}\mathcal{S}^{s}{}'$ A system profile is a compound system of modes, which briefs the actions, presented for example as ${}^{l+1}\mathcal{S}' = \left({}^{l}_{1}\mathcal{S}^{i}, {}^{l}_{1}\mathcal{S}^{c}, {}^{l+1}_{1}\mathcal{S}\cap{}^{l}E^{t}_{1}, \cdots, {}^{l+1}_{j}\mathcal{S}\cap{}^{l}E^{t}_{j}, \cdots, {}^{l+1}_{{}^{l}N^{t}}\mathcal{S}\cap{}^{l}E^{t}_{{}^{l}N^{t}}\right)$.
- ${}^{l+1}_{j}\mathcal{S}\cap{}^{l}E^{t}_{i}$ A compound system mode is a subsystem of those events, which represents a state in the case that a specific transitive event occurred on the previous system level.

A procedure below demonstrates how to apply EOSA to multi-level system [Žiha 2000-b]:



Apparent descriptive characteristics of the system uncertainties are the number of states, a number of transition levels and the numbers of events on subsequent levels.

Unconditional probabilities of system levels, states and modes are calculated as follows:

$$p({}^{l}\mathcal{S}) = \sum_{all\,E\in{}^{l}\mathcal{S}} p({}^{l}E) = \sum_{j=1}^{{}^{l}n} p({}_{j}^{l}\mathcal{S}) \tag{1}$$

$$p({}_{j}^{l}\mathcal{S}) = \sum_{all\,E\in{}_{j}^{l}\mathcal{S}} p({}_{j}^{l}E) = \sum_{all\ {}_{j}^{l}\mathcal{S}^{s}\in{}_{j}^{l}\mathcal{S}} p({}_{j}^{l}\mathcal{S}^{s}) \tag{2}$$

$$p({}_{j}^{l}\mathcal{S}^{s}) = \sum_{all\,E\in{}_{j}^{l}\mathcal{S}^{s}} p({}_{j}^{l}E^{s}) \tag{3}$$

The conditional probability of transition from one level to the next is defined as follows:

$$p({}_{j}^{l}\mathcal{S}\cap{}^{l-1}E_i^t) = p({}_{j}^{l}\mathcal{S}/{}^{l-1}E_i^t)\,p({}^{l-1}E_i^t) \tag{4}$$

When the states in no way affects the probability of transition, the independence is expressed by the relation $p({}_{j}^{l}\mathcal{S}/{}^{l-1}E_i^t) = p({}_{j}^{l}\mathcal{S})$.

Unconditional Renyi/Shannon's entropy of order one [Renyi 1970] for a system level is:

$$H^{1}({}^{l}\mathcal{S}) = -\frac{1}{p({}^{l}\mathcal{S})}\sum_{all\,E\in{}^{l}\mathcal{S}} p({}^{l}E)\log p({}^{l}E) \tag{5}$$

For complete systems of events, Renyi/Shannon's entropy of order one is equal to Shannon's entropy [Shannon and Weaver 1949]. All logarithms applied are of base two. The uncertainties are expressed in *bits*.

Conditional entropy of a state with respect to a selected mode is defined as follows:

$$H({}_{j}^{l}\mathcal{S}/{}_{j}^{l}\mathcal{S}^{s}) = -\sum_{i=1}^{{}_{j}^{l}N^{s}} \frac{p({}^{l}E_i^s)}{p({}_{j}^{l}\mathcal{S}^{s})}\log\frac{p({}^{l}E_i^s)}{p({}_{j}^{l}\mathcal{S}^{s})} \tag{6}$$

The maximal attainable values of (5) and (6) are $\log\sum_{j=1}^{{}^{l}n}{}_{j}^{l}N^{s}$ and $\log{}_{j}^{l}N^{s}$, respectively.

Unconditional Renyi/Shannon's entropy of order one for a system profile is calculated as:

$$H^{1}({}^{l}\mathcal{S}^{s\prime}) = -\frac{1}{p({}^{l}\mathcal{S}^{s\prime})}\sum_{j=1}^{{}^{l}_{j}n^{s}} p({}_{j}^{l}\mathcal{S}^{s})\log\ p({}_{j}^{l}\mathcal{S}^{s}) \tag{7}$$

The maximal attainable value of (7) is $\log {}^{l}_{j}n^{s}$ where ${}^{l}_{j}n^{s}$ is the number of system modes.

The unconditional entropy (5) overestimates the uncertainty, since it does not account for the knowlwdge of system actions. Therefore, the conditional entropy of a game level with respect to a system profile [Žiha 1998] expresses the reduced uncertainty more appropriately:

$$H({}^{l}\mathcal{S}/{}^{l}\mathcal{S}^{s\prime}) = p({}^{l}\mathcal{S})\left[H({}^{l}\mathcal{S}) - H({}^{l}\mathcal{S}^{s\prime})\right] = \sum_{j=1}^{{}^{l}N^{t}} p({}_{j}^{l}\mathcal{S}^{s})H({}_{j}^{l}\mathcal{S}^{s}) \tag{8}$$

Conditional entropy of the next system level can be expressed by the increment of unconditional entropy with respect to the previous level, equal to the weighted sum of the next level system states entropy, as follows:

$$H({}^{l+1}\mathcal{S}\ /\ {}^{l}\mathcal{S}) = H^{1}({}^{l+1}\mathcal{S}) - H^{1}({}^{l}\mathcal{S}) = \frac{1}{p({}^{l}\mathcal{S})}\sum_{j=1}^{{}^{l}N^{t}} p({}^{l}E_{j}^{t})H({}_{j}^{l+1}\mathcal{S}^{s}) \tag{9}$$

Conditional entropy of the next level system profile can be expressed by the increment of unconditional entropy with respect to the previous level profile and by the conditional entropy of the next level profile conditioned on transitive mode, as follows:

$$H({}^{l+1}\mathcal{S}^{s\prime}/\ {}^{l}\mathcal{S}^{s\prime}) = H^{1}({}^{l+1}\mathcal{S}^{s\prime}) - H^{1}({}^{l}\mathcal{S}^{s\prime}) = \frac{p({}^{l}\mathcal{S}^{t})}{p({}^{l}\mathcal{S})}H({}^{l+1}\mathcal{S}^{s\prime}/\ {}^{l}\mathcal{S}^{t}) \tag{10}$$

Relations (9, 10) indicate that the entropy in case of a transition to a higher level must increase. The change in conditional entropy can be expressed by changes of unconditional entropy of the system level and of the system profile, as follows:

$$H({}^{l+1}\mathcal{S}/{}^{l+1}\mathcal{S}^{s\prime}) - H({}^{l}\mathcal{S}/{}^{l}\mathcal{S}^{s\prime}) = p({}^{l}\mathcal{S})\left\{[H({}^{l+1}\mathcal{S}) - H({}^{l}\mathcal{S})] - [H({}^{l+1}\mathcal{S}^{s\prime}) - H({}^{l}\mathcal{S}^{s\prime})]\right\} \tag{11}$$

The relative measure of uncertainty can be expressed for either complete or incomplete systems of events, in dimensionless form with respect to a reference system [Žiha 1999]:

$$h_{n,N}^{1}(\bullet) = \frac{H_{n}^{1}(\bullet)}{H_{N}^{1}(\bullet)_{\max}} \tag{12}$$

The average number of events *F* and the average probability of occurrence *G* are in general defined for either complete or incomplete systems of events [Žiha 2000-a] as follows:

$$F_{N}(\bullet) = \frac{1}{G_{N}(\bullet)} = 2^{H_{N}^{1}(\bullet)} \tag{13}$$

The conditional entropy of operational and failure modes can be interpreted in terms of redundancy and robustness of systems of events [Žiha 2000-c].

3. AN EXAMPLE OF A HAZARDOUS GAME

The proposed game starts with tossing a die. Whenever a die shows, let us say, 2 or 5, or whatsoever is agreed among the participants, the game is going on with a coin. The winning strategy is not of primary interest in this example and it can be defined arbitrarily in agreement with the players. However, the purpose of the example is the representation of multi-level games by systems of events, the determination of the game probabilities and moreover, the assessment of game uncertainties. The following system of events can be assigned to the first level of the game with only one requisite, corresponding to a system state appropriate to an unbalanced die, as shown:

$${}^{1}\mathcal{S} = \left({}_{1}^{1}\mathcal{S}\right) = \left({}_{1}^{1}\mathcal{S}^{w} + {}_{1}^{1}\mathcal{S}^{f} + {}_{1}^{1}\mathcal{S}^{t}\right) = \begin{pmatrix} {}^{1}E_{1}^{w} & {}^{1}E_{2}^{w} & {}^{1}E_{1}^{f} & {}^{1}E_{3}^{f} & {}^{1}E_{1}^{t} & {}^{1}E_{2}^{t} \\ 2/16 & 1/16 & 1/16 & 1/16 & 1/16 & 2/16 \end{pmatrix}.$$

The primary probabilities of winning, failing, transition and a non-transition (3) amount to:

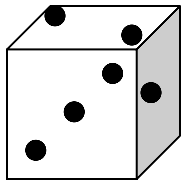
$$p({}_1^1\mathcal{S}^w) = p({}^1E_1^w) + p({}^1E_2^w) = 3/16$$

$$p({}_1^1\mathcal{S}^f) = p({}^1E_1^f) + p({}^1E_2^f) = 2/16$$

$$p({}_1^1\mathcal{S}^t) = p({}^1E_1^t) + p({}^1E_2^t) = 3/16$$

$$p({}_1^1\mathcal{S}^n) = p({}^1E_1^w) + p({}^1E_2^w) + p({}^1E_1^f) + p({}^1E_2^f) = 5/16\ .$$

Increasing the number of game levels can further complicate the game.



The probability at primary level (1) $p({}^1\mathcal{S}) = 1$ indicates a complete system of events.
The unconditional entropy of the primary game level (5) amounts to:

$$H({}^1\mathcal{S}) = H({}_1^1\mathcal{S}) = -\sum_{i=1}^{6} p({}^1E_i)\log p({}^1E_i) = 2.5\,bits\,(2.5849, 0.9671, 5.6568)\,.$$

Note maximal, relative (12) and average (13) values in parentheses.
The unconditional entropy of the primary game profile (7) amounts to:

$$H({}^1\mathcal{S}^{wft\,\prime}) = -p({}_1^1\mathcal{S}^w)\log p({}_1^1\mathcal{S}^w) - p({}_1^1\mathcal{S}^f)\log p({}_1^1\mathcal{S}^f) - p({}_1^1\mathcal{S}^t)\log p({}_1^1\mathcal{S}^t) =$$

$$= 1.5612\,bits\,(1.5849, 0.9580, 2.9511)$$

The conditional entropy (6) of the primary level with respect to the winning mode is:

$$H({}^1\mathcal{S}/{}_1^1\mathcal{S}^w) = -\frac{p({}^1E_1^w)}{p({}_1^1\mathcal{S}^w)}\log\frac{p({}^1E_1^w)}{p({}_1^1\mathcal{S}^w)} - \frac{p({}^1E_2^w)}{p({}_1^1\mathcal{S}^w)}\log\frac{p({}^1E_2^w)}{p({}_1^1\mathcal{S}^w)} = 0.9182 bits (1, 0.9182, 1.8898)\,.$$

The conditional entropy (6) of the primary level with respect to failing mode is:

$$H({}^1\mathcal{S}/{}_1^1\mathcal{S}^f) = -\frac{p({}^1E_1^f)}{p({}_1^1\mathcal{S}^c)}\log\frac{p({}^1E_1^f)}{p({}_1^1\mathcal{S}^c)} - \frac{p({}^1E_2^f)}{p({}_1^1\mathcal{S}^c)}\log\frac{p({}^1E_2^f)}{p({}_1^1\mathcal{S}^c)} = 1\,bits\ (1,1,2)\,.$$

The conditional entropy (6) of the primary level with respect to the transitive mode is:

$$H({}^1\mathcal{S}/{}_1^1\mathcal{S}^t) = -\frac{p({}^1E_1^t)}{p({}_1^1\mathcal{S}^t)}\log\frac{p({}^1E_1^t)}{p({}_1^1\mathcal{S}^t)} - \frac{p({}^1E_2^t)}{p({}_1^1\mathcal{S}^t)}\log\frac{p({}^1E_2^t)}{p({}_1^1\mathcal{S}^t)} = 0.9182\,bits\,(1, 0.9182, 1.8898)\,.$$

The conditional entropy (6) of the primary level with respect to the non-transitive mode is:

$$H({}^1\mathcal{S}/{}_1^1\mathcal{S}^n) = -\frac{p({}^1E_1^w)}{p({}_1^1\mathcal{S}^n)}\log\frac{p({}^1E_1^w)}{p({}_1^1\mathcal{S}^n)} - \frac{p({}^1E_2^w)}{p({}_1^1\mathcal{S}^n)}\log\frac{p({}^1E_2^w)}{p({}_1^1\mathcal{S}^n)} -$$

$$-\frac{p({}^1E_1^f)}{p({}_1^1\mathcal{S}^n)}\log\frac{p({}^1E_1^f)}{p({}_1^1\mathcal{S}^n)} - \frac{p({}^1E_2^f)}{p({}_1^1\mathcal{S}^n)}\log\frac{p({}^1E_2^f)}{p({}_1^1\mathcal{S}^n)} = 1.9219\ bits\ (2, 0.9609, 3.7892)$$

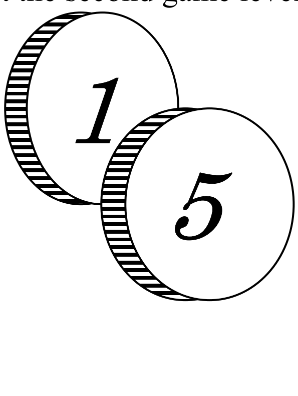
The primary uncertainty is more precisely expressed by the conditional entropy of the first game level with respect to the game profile (8) of winning, failing and transitive modes, because the knowledge of the game rules decreases the game uncertainty, as shown:

$$H({}^1\mathcal{S}/{}^1\mathcal{S}^{wft\,\prime}) = p({}_1^1\mathcal{S}^w)H({}^1\mathcal{S}/{}_1^1\mathcal{S}^w) + p({}_1^1\mathcal{S}^f)H({}^1\mathcal{S}/{}_1^1\mathcal{S}^f) + p({}_1^1\mathcal{S}^t)H({}^1\mathcal{S}/{}_1^1\mathcal{S}^t) =$$

$$= H({}^1S) - H({}^1S^{wft\,\prime}) = 0.9387\,bits\,(2.5849, 0.3631, 1.9168)$$

To the continuation of the game with two different unbalanced coins at the second game level, two systems states, corresponding to game requisites, are assigned:

$${}_1^2\mathcal{S} = \begin{pmatrix} {}_1^2E_1^w & {}_1^2E_1^f \\ 7/16 & 9/16 \end{pmatrix}, \qquad {}_2^2\mathcal{S} = \begin{pmatrix} {}_2^2E_1^w & {}_2^2E_1^f \\ 11/16 & 5/16 \end{pmatrix}.$$



To secondary probabilities of winning and failing are, respectively:

$$p({}_1^2\mathcal{S}^w) = p({}_1^2E_1^w) = 7/16\,, \qquad p({}_1^2\mathcal{S}^f) = p({}_1^2E_1^f) = 9/16$$

$$p({}_2^2\mathcal{S}^w) = p({}_2^2E_1^w) = 11/16\,, \qquad p({}_2^2S^f) = p({}_2^2E_1^f) = 5/16\ .$$

The secondary requisites are complete systems of events, since $p({}_1^2\mathcal{S})=1$ and $p({}_2^2\mathcal{S})=1$.

The entropy of the two independent secondary game requisites (5), regardless to the transitional character of the considered system, amount to:

$$H({}_1^2\mathcal{S})=-p({}_1^2E_1^w)\cdot\log p({}_1^2E_1^w)-p({}_1^2E_1^f)\cdot\log p({}_1^2E_1^f)=0.9886\,bits\,(1,0.9886,1.9843)$$

$$H({}_2^2\mathcal{S})=-p({}_2^2E_1^w)\cdot\log p({}_2^2E_1^w)-p({}_2^2E_1^f)\cdot\log p({}_2^2E_1^f)=0.8960\,bits\,(1,0.8960,1.8609)$$

The transitions from one level to the next are symbolically presented by transitive conditional subsystems of events appropriate to system states, as shown:

$$ {}_1^2\mathcal{S}\cap{}^1E_1^t=\begin{pmatrix} {}_1^2E_1^w\cap{}^1E_1^t & {}_1^2E_1^f\cap{}^1E_1^t \\ \dfrac{7\cdot 2}{16\cdot 16} & \dfrac{9\cdot 2}{16\cdot 16}\end{pmatrix},\quad {}_2^2\mathcal{S}\cap{}^1E_2^t=\begin{pmatrix} {}_2^2E_1^w\cap{}^1E_2^t & {}_2^2E_1^f\cap{}^1E_2^t \\ \dfrac{11\cdot 1}{16\cdot 16} & \dfrac{5\cdot 1}{16\cdot 16}\end{pmatrix}.$$

The probabilities of the game with secondary requisites (4) amount to:

$$p({}_1^2\mathcal{S}\cap{}^1E_1^t)=p({}^1E_1^t)=\frac{2}{16},\qquad p({}_2^2\mathcal{S}\cap{}^1E_2^t)=p({}^1E_2^t)=\frac{1}{16}.$$

The secondary game level is presented as a system of primary non-transitive events as well as of secondary events conditioned on primary transitive events, as follows:

$${}^2\mathcal{S}=\begin{pmatrix} {}^1E_1^w & {}^1E_2^w & {}_1^2E_1^w\cap{}^1E_1^t & {}_1^2E_1^f\cap{}^1E_1^t & {}_2^2E_1^w\cap{}^1E_2^t & {}_2^2E_1^f\cap{}^1E_2^t & {}^1E_1^f & {}^1E_2^f \\ 2/16 & 1/16 & 14/256 & 18/256 & 11/256 & 5/256 & 1/16 & 1/16\end{pmatrix}.$$

The secondary system of events is a complete system of events since $p({}^2\mathcal{S})=1$.

The unconditional entropy of the secondary level (5) is calculated as follows:

$$H({}^2\mathcal{S})=\sum_{all\,E\in{}^2\mathcal{S}}p(E)\log p(E)=H({}^1\mathcal{S})+H({}^2\mathcal{S}/{}^1\mathcal{S})=2.8591\,bits\,(3,0.9530,7.2560).$$

The assessment of the game uncertainty by considering entropy of independent systems of events corresponding to a die and to a coin is quite insufficient for the proposed two-level game. Such a situation is pertinent to observers, which are acquainted with only the game with a die or only with a coin and do not know anything about the agreed rules of the game. However, the uncertainty of a simultaneous game with a die and with a coin can be expressed by the entropy of two independent systems pertaining to both levels (additivity of entropy):

$$H({}^1\mathcal{S}\ {}^2\mathcal{S})=H_{\max}({}^1\mathcal{S})+H_{\max}({}^2\mathcal{S})=\log 6+\log 2=2.5849+1=3.5849\,bits$$

The amount $H({}^1\mathcal{S}\ {}^2\mathcal{S})$ is the upper limit of entropy for all games with a die and a coin.

The conditional entropy of the secondary level with respect to the primary level (9) is:

$$H({}^2\mathcal{S}/{}^1\mathcal{S})=\sum_{j=1}^{{}^1N^t}p({}^1E_j^t)\cdot H({}_j^2\mathcal{S})=H({}^2\mathcal{S})-H({}^1\mathcal{S})=0.3951\,bits\,(3,0.1197,1.2826).$$

The secondary game profile in this example is ${}^2\mathcal{S}^{wft}{}'=\begin{pmatrix} {}_1^1\mathcal{S}_1^w & {}^1E_1^t & {}^1E_2^t & {}_1^1\mathcal{S}_1^f \\ 3/16 & 2/16 & 1/16 & 2/16\end{pmatrix}.$

The unconditional entropy of the secondary game profile (7) amounts to:

$$H({}^2\mathcal{S}^{wft}{}')=-p({}_1^1\mathcal{S}^w)\log p({}_1^1\mathcal{S}^w)-\sum_{i=1}^{2}p({}^1E_i^t)\log p({}^1E_i^t)-p({}_1^1\mathcal{S}^f)\log p({}_1^1\mathcal{S}^f)=$$

$$=1.9056\ bits\ (2,0.9528,3.7467).$$

The uncertainty of the secondary functional level can be expressed in this example by the secondary conditional entropy with respect to the secondary game profile (8), as shown:

$$H\left({}^{2}\mathcal{S}/{}^{2}\mathcal{S}^{wft\,\prime}\right)= p({}_{1}^{1}\mathcal{S}^{w})H({}_{1}^{1}\mathcal{S}^{w}) + \sum_{j=1}^{2} p({}^{1}E_{j}^{t})H({}_{j}^{2}\mathcal{S}) + p({}_{1}^{1}\mathcal{S}^{f})H({}_{1}^{1}\mathcal{S}^{f}) = .$$

$$= H\left({}^{2}\mathcal{S}\right) - H\left({}^{2}\mathcal{S}^{wft\,\prime}\right) = 0.9535 \ \ bits\,(3, 0.3178, 1.9366)$$

The proposed two-level game with two gambling devices is less uncertain that each of the simple games with a coin or solely with a die, due to the implementation of more complex rules of games, which reduce uncertainties. The increments in unconditional uncertainties of functional levels and game profiles due to transition, are calculated as shown:

$$H({}^{2}\mathcal{S}) - H({}^{1}\mathcal{S}) = \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t}) \cdot H({}_{j}^{2}\mathcal{S}) = 0.3591\,bits$$

$$H({}^{2}\mathcal{S}^{wft\,\prime}) - H({}^{1}\mathcal{S}^{wft\,\prime}) = p({}_{1}^{1}\mathcal{S}^{t}) \cdot H({}^{1}\mathcal{S}) = 0.3443\,bits\,.$$

The game can be played in different ways, e.g. starting with a coin and continuing with a die, or using other requisites, such as cards, roulett etc.

The change in the conditional uncertainty due to transition from the primary to the secondary level (11), may consist, in some cases of complex rules, less than zero, as shown:

$$H\left({}^{2}\mathcal{S}/{}^{2}\mathcal{S}^{wft\,\prime}\right) - H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{wft\,\prime}) = \left[H({}^{2}\mathcal{S}) - H({}^{1}\mathcal{S})\right] - \left[H({}^{2}\mathcal{S}^{wft\,\prime}) - H({}^{1}\mathcal{S}^{wft\,\prime})\right] = 0.0148 \ \ bits$$

Gamblers may agree that there are not the probabilities the only quality, which make the games more entertaining. Some players may find subjectively more joy in playing with more gambling devices in several game sequences. However, these subjective views do not effect the objective uncertainties due to a number of events and a distribution of their probabilities.

4. Conclusion

The note demonstrates the feasibility of event oriented system analysis applied to multi-level systems. The note also proves that the unconditional entropy in case of transition to a higher system level must increase. Moreover, it is demonstrated that the increment is quantifiable. Simultaneously, the conditional entropy of more complex systems is reducing due to implementation of rules. However, the aim of the example of multi-level hazardous games, not employing game theory and information theory but ordinary probability and entropy concept in probability, was not only to demonstrate how can be assessed the level of entertainment conveyed by playing more complex hazardous games. The note attempts to add impetus for evaluation of more complex probabilistic models in science and engineering. Event oriented analysis of systems without transitive potentials are elaborated elsewhere [Žiha 2001]. The uncertainty modeling presented herein is hopefully applicable to systems with transitive potentials followed by redistribution of capabilities and demands in case of component failures on current operational levels.

In a naturally “fair” game, all the participants use the same requisites and cannot modify the probabilities of elementary events. However, scientist and engineers are motivated to use their knowledge and experience to maximize their chances against nature. The designers can intervene on the physical properties of planned objects and artificially change the probabilities employing event oriented system analysis, in order to design system behavior to everyone's satisfaction. System complexity may provoke considerable increase of computational efforts. The embarrassment due to problem size can hopefully be managed by problem partitioning pertinent to the event oriented system analysis.

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