

# Analysis of Spherical Lens Antennas – Comparison of Three Analysis Methods

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**Abstract.** *The paper presents a comparison of three approaches for analyzing spherical lens antennas – spectral-domain method, ray-tracing method, and time-domain finite integration technique (FIT) implemented in commercial software CST Microwave Studio®. The spectral-domain method takes advantage of transforming the three-dimensional problem into a set of one-dimensional problems. The spherical lens antenna is rigorously taken into account by implementing the Green's functions for the multilayer spherical structures. The ray-tracing method is a general analysis method that can be applied to all kinds of lenses with general (non-symmetric) shape. Finally, FIT implemented in commercial software is general purpose full-wave method. The results of the analysis are compared with measured radiation pattern of a 12 layer Luneburg lens antenna.*

## Keywords

Spherical lens antennas, Spectral-domain analysis, Ray-tracing method.

## 1. Introduction

There is a growing interest in finding solutions to allow Internet connection for traveling users. Mobility under consideration is either by a car, a bus, a plane, a high-speed train, or a ship. To reduce the environment effects one solution is to consider a radio connection between a geostationary satellite and the moving vehicle (e.g. a high speed train).

Spherical lens antennas are attractive antennas for such moving vehicles as each pencil-beam, originating from one feed antenna, can be scanned by moving the feed around the surface of the lens. The antenna system consists of two main parts: of the feeding antenna and of the lens antenna for focusing the energy. The feeding antenna than can be any type of antenna (horns, dipoles, microstrip patches, and even arrays of antenna elements) and the spherical lens that collimates incident divergent energy to prevent it from spreading in undesired directions. These antennas are inherently wide-band antennas and the feed,

not the lens, typically limit bandwidth. Therefore, there is a growing interest for spherical lens antennas for applications in satellite and mobile communication systems at higher microwave and millimeter wave frequencies where lens dimensions become small enough for integration.

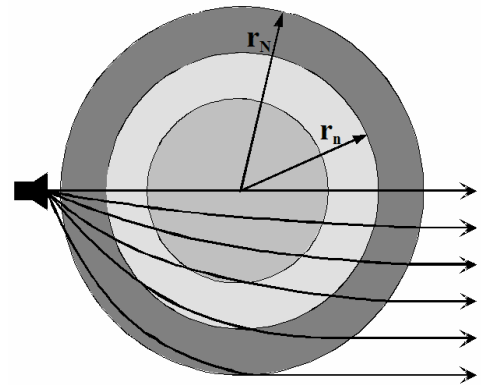


Fig. 1. Spherical multiplayer lens antenna fed by horn

The purpose of the paper is to compare three methods for analyzing spherical lens antennas. One based on the spectral domain approach (full-wave method applicable to symmetric structures), second one on ray-tracing method (generally applicable method that can have problems when lens dimensions become small in regards to wavelength) and finally third FIT method implemented in commercial software as a general purpose full-wave method.

## 2. Ray-tracing approach

Fig. 1 shows typical multiplayer spherical lens antenna with  $N$  layers that can have different permittivity and permeability. Each layer is defined by material permittivity ( $\epsilon_{r,n}$ ), permeability ( $\mu_{r,n}$ ) and inner ( $r_n$ ) and outer radius ( $r_N$ ).

Geometrical optics (GO) is an approximate high-frequency method for determining wave propagation for incident, reflected, and refracted fields. Solving of problems by its principles can be called ray tracing as we, in fact, trace ray propagation throughout media that can have different refraction indexes. When entering new media, the Snell's law determines new ray direction. When

the refractive-index profile is graded the trajectory of a ray is determined by the eikonal equation. Light intensity, in GO, is governed by the conservation of energy. Intensity  $S$  at one point is correlated to intensity at some reference point  $S_0$  (if we assume that rays travel within a tube) by following equation

$$\frac{S(s)}{S_0(0)} = \frac{dA_0}{dA}. \quad (1)$$

In this equation we assume that  $S$  and  $S_0$  are constant throughout the cross-sectional areas  $dA_0$  and  $dA$ , respectively. As intensity is related to field by square, we arrive at equation.

$$\frac{|\mathbf{E}|}{|\mathbf{E}_0|} = \sqrt{\frac{dA_0}{dA}}. \quad (2)$$

Equation is not accurate for electromagnetic waves of lower frequencies, as we haven't taken in account two significant properties phase and polarization. By taking them into consideration we arrive at [1]:

$$\mathbf{E}(s) = \mathbf{E}_0(0)e^{j\Phi_0(0)} \sqrt{\frac{dA_0}{dA}} e^{-j\beta s}. \quad (3)$$

This equation is not valid along the caustics and is not very accurate near them, so it should not be used in those regions. At the same time, it correctly predicts +90 phase jumps each time caustic is crossed in the direction of propagation. When analyzing spherical lens antennas, the output E-field distribution is obtained by following equation [2].

$$F(r, \phi) = \left( \frac{\sin \psi}{r} \frac{d\psi}{dr} \right)^{1/2} E_p(\psi, \phi) T(\psi, \phi) e^{-j\chi}. \quad (4)$$

Terms in brackets and under square root are related to change in cross-sectional areas,  $E_p$  signifies angular source,  $T$  is total transmission coefficient, and  $\chi$  is phase delay of each ray until radiating aperture. Following the equivalence principle, these aperture distributions can be used to calculate the radiated field of the antenna from

$$E(u, \phi) = \frac{j \exp(-jkR)}{\lambda R} \int_0^{2\pi} \int_0^{r_m} F(r, \phi') \exp\{j[-\chi + rk \sin \theta \cos(\phi - \phi')]\} r dr d\phi' \quad (5)$$

where  $r_m$  is the effective aperture radius. It should be noted that equivalent aperture plane is placed in the middle of lens antenna [2].

### 3. Spectral domain approach

The solution procedure makes use of the Fourier transformation technique. The feeding horn is replaced with equivalent aperture currents by using free-space equivalent principle [1]. In the case of dipole feeding, the current on dipole is used as excitation. The equivalent or

physical current excitations of electric and magnetic type are Fourier transformed. Since the problem is described in spherical coordinate system, we use the vector Legendre transformation in  $\theta$  and  $\phi$  directions, defined by [3], [4].

By the vector Legendre transformation, the 3D excitations are transformed into harmonic current shells. If the source is infinitely thin in  $r$  direction, we get one discrete current shell per source, otherwise we get a continuous distribution of current shells in  $r$ -direction. The E- and H-fields induced by the harmonic current sources have the same harmonic variations in  $\theta$  and  $\phi$  as the source. Therefore, only the field variation in the direction perpendicular to the boundaries is unknown, and we have a harmonic one-dimensional (1D) field problem. In this way the spectral domain problem is interpreted as a 1D spatial domain problem consisting of 1D multilayer structure and harmonic 1D sources in the form of current shells.

The harmonic 1D problem is solved by making use of the equivalent problems, one for each layer. The unknowns are the tangential E- and H- fields at the layer boundaries. Since the variation of the E- and H-fields in the direction tangential to the boundaries is harmonic with known periodicity, we only need to determine the complex field amplitudes at the interfaces, i.e., we have four unknowns per boundary. The developed algorithm connects all equivalent subproblems into a system of  $4(N_{\text{layer}}-1)$  linear equations with the same number of unknowns ( $N_{\text{layer}}$  denotes the number of layers). Once the amplitudes of the tangential fields have been determined, it is easy to determine the field amplitudes anywhere in the multilayer structure by applying the homogeneous region equivalent of the layer inside which we want to determine the field value. The core problem in the formulation is to calculate the E- and H- fields due to a harmonic current shell in a homogeneous region. The algorithm that calculates the field distribution in presence of the multilayer structure is called G1DMULT, and it is described in details in [4]. Additional details on applied numerical techniques for circumventing numerical stability problems in analyzing structures of large sizes in terms of wavelength can be found in [5].

### 4. Results

Three analysis methods are compared by calculating the radiation pattern of a 12-shell Luneburg lens antenna. For the spectral domain approach, two feeding type models were used – small horn antenna (so-called Huygens source) and rectangular horn antenna with proper phase variations at the aperture. In CST rectangular horn feed by waveguide has been constructed while in GO approach we have used the approximation of the horn radiation pattern – it is approximated as  $\cos(\theta)^{1.7}$  in the E-plane and as  $\cos(\theta)^{2.1}$  in the H-plane (the horn radiation pattern is obtained using CST Microwave Studio®). This approximation was very good for degrees of  $\pm 60^\circ$  that are the relevant for illuminating the spherical lens. All calculated radiation

pattern results were compared with measurements of a 12-shell Luneburg lens antenna [6]. The lens parameters are given in Table 1. The operating frequency is 10 GHz. The outer radius of the lens antenna is 15.9 cm, therefore the antenna is approximately  $10\lambda$  in diameter. The primary feed antenna (the rectangular horn with opening  $2.0 \times 3.0$  cm) is placed 18.2 cm from the center of the lens, which corresponds to maximum directivity. All methods returned good matching with measurements, but with different computation requirements. Both geometry optics and spectral domain are extremely fast with calculations typically taking around, or even less than a second on a typical PC, while FIT technique implemented in CST takes considerable longer time and has much greater memory requirements. Actual times vary on many parameters, but typically calculations took over thirty minutes on the same PC.

Shell	Radius (cm)	Permittivity
1	3.755	1.93 - j0.0003
2	5.520	1.74 - j0.0003
3	7.410	1.71 - j0.0003
4	8.380	1.65 - j0.0003
5	9.415	1.63 - j0.0003
6	9.940	1.56 - j0.0003
7	11.840	1.54 - j0.0003
8	12.500	1.50 - j0.0003
9	13.400	1.40 - j0.0003
10	14.270	1.28 - j0.0003
11	15.150	1.20 - j0.0003
12	15.900	1.10 - j0.0003

Tab. 1. Spherical lens geometry and electrical characteristics.

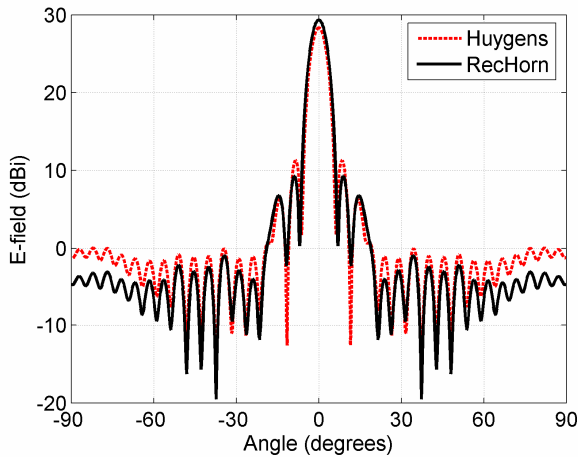


Fig. 2. Comparison of different feed models in G1DMULT in E-plane

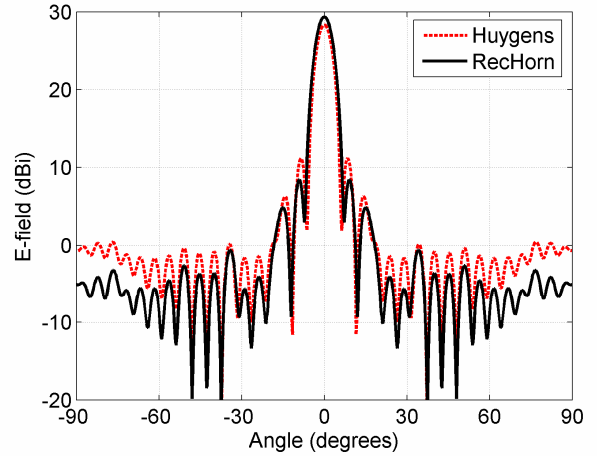


Fig. 3. Comparison of different feed models in G1DMULT in H-plane

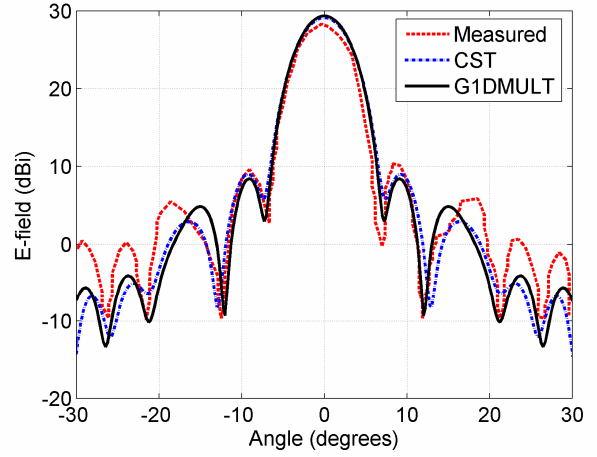


Fig. 4. Comparison of G1DMULT (RecHorn feed) in H-plane with measurements and CST. Matching is very good.

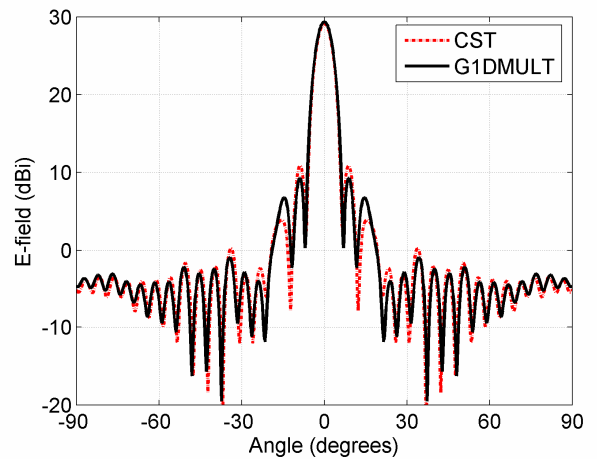


Fig. 5. Comparison of G1DMULT and CST in E-plane.

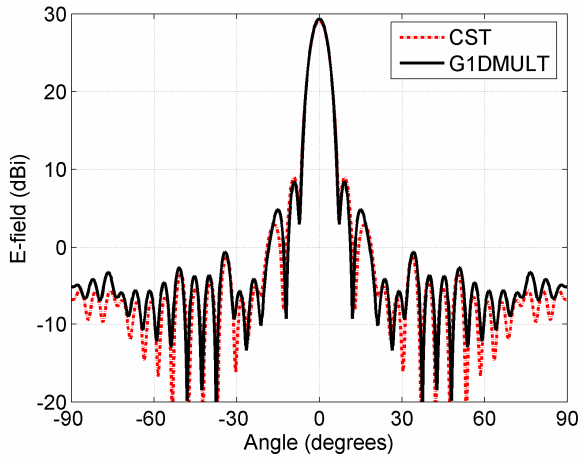


Fig. 6. Comparison of G1DMULT and CST in H-plane.

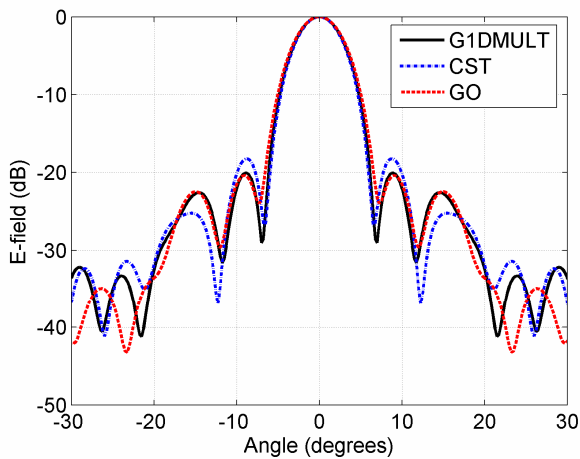


Fig. 7. Comparison of G1DMULT, CST, and GO in E-plane. Normalized to 0 dB as GO approach, in current version, doesn't compute directivity

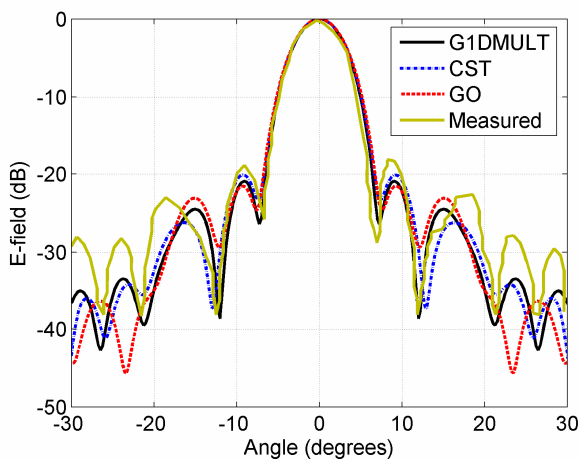


Fig. 8. Comparison of G1DMULT, CST, GO, and measurements in H-plane. Normalized to 0 dB as GO approach, in current version, doesn't compute directivity

## 5. Conclusions

This paper presents three methods for analyzing spherical lens antennas and compares them to measurements. One takes advantage of the spectral-domain approach, other method is based on simple rules of geometry optics, or, more popularly, ray tracing, and final method is FIT technique implemented in commercial software CST Microwave Studio<sup>®</sup>. Matching between G1DMULT, CST and measurements is very good, while geometrical optics results match well to other methods only for angles close to main beam. G1DMULT and GO techniques are both extremely fast, so both are suitable for global optimization techniques but each has its limitations. G1DMULT can work only with symmetrical structures and GO, as a high-frequency approach, has problems with structures that are comparable in size to working wavelengths and generally doesn't match to measurements as good as full-wave methods.

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