Bandwidth of Invisible Cloak Realized with Split Ring Resonators

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Abstract. *Recently, an uniaxial multilayer cylinder cloak which does not scatter electromagnetic wave was proposed. The purpose of this paper is to analyze such a cloak and to characterize the achieved "invisibility" in mathematical terms by calculating the scattered field and equivalent blockage width – the structure width that is actually "perceived" by EM waves. For that purpose we have developed the G1DMULT algorithm for analyzing uniaxial multilayer structures of planar, circular – cylindrical and spherical shape. The results show that the analyzed metamaterial realization reduces the scattered field and is suitable only for excitation by a TM-polarized wave. Furthermore, due to the dispersion it works only in a narrow frequency band. It seems that analyzed structure is inferior to already known realizations of invisibility, e.g. by hard surfaces that can work for arbitrary polarization within larger frequency bandwidth, although suitable for only one direction of incoming wave.*

Keywords

Invisible cloaks, metamaterial realization, equivalent blockage width, Lorentz dispersion model, G1DMULT.

1. Introduction

The realization of structures that do not scatter electromagnetic field, i.e. structures that appear invisible for EM waves in outer space, is not a new concept. For example, hard surfaces [1] have been used in development of supporting struts for a feed of reflector antenna, to reduce blockage of EM field.

As another possible realization of invisibility, a so-called invisible cloak was recently described and constructed [3]. The considered structure consists of a conducting cylinder (e.g. copper cylinder) and a cloak made of metamaterial structures. The basic principle of the invisible cloak is to control the paths of electromagnetic waves within a material by introducing a prescribed spatial variation in the constitutive parameters based on coordinate transformations, which is obtained by using metamaterials. The coordinate transformations that are used for the cloak design affect the permittivity and permeability tensors ($\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$, respectively), making the needed materials spatially varying and anisotropic [2]:

$$\begin{aligned} r' &= \frac{b-a}{b} r + a \\ \varphi' &= \varphi \\ z' &= z \end{aligned} \tag{1}$$

where a and b represent the cloak inner and outer radius, respectively (the constitutive structure parameters are considered in the cylindrical coordinate system r-φ-z).

Note that no single component of permittivity and permeability tensors is constant, i.e. all of them are functions of radius, which implies a very complicated metamaterial design. However, the reported cloak [3] is intended to work properly only when illuminated by specific polarization (TM-polarization, i.e. with magnetic field polarized transverse to cylinder axis), Therefore, it is possible to make significant simplifications while only ε_z, μ_r and μ_θ components are then relevant (the cylinder axis is set in z-direction). The material is designed with the following properties, with only μ_r varying radially throughout the structure:

$$\begin{aligned} \varepsilon_z &= \left(\frac{b}{b-a}\right)^2 \\ \mu_r &= \left(\frac{r-a}{r}\right)^2 \\ \mu_\varphi &= 1 \end{aligned} \tag{2}$$

The unit cell, used in the realized metamaterial, is approximately cubic (Fig. 1) and contains split-ring resonators, while the layout of the cloak consisted of 10 concentric cylinders (Fig. 2), each of which was three unit cells tall. Dimensions of the split-ring resonators itself were varied for each layer in order to provide the desired magnetic permeability in radial direction. Although the simulations and experiments [3] had been successful, the quantification of invisibility in mathematical terms has not been given.

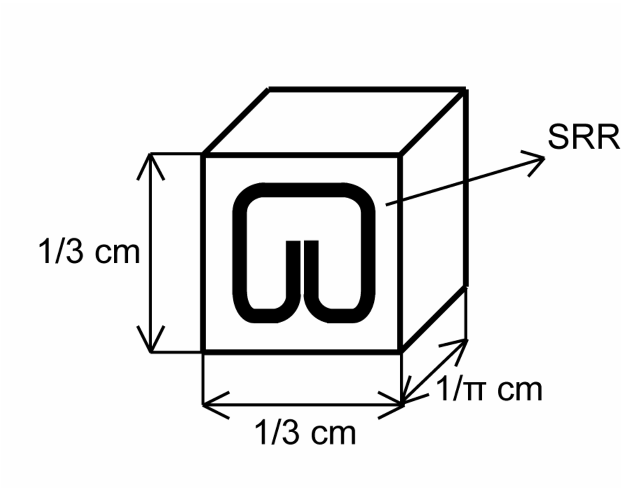

Fig.1. Metamaterial unit cell

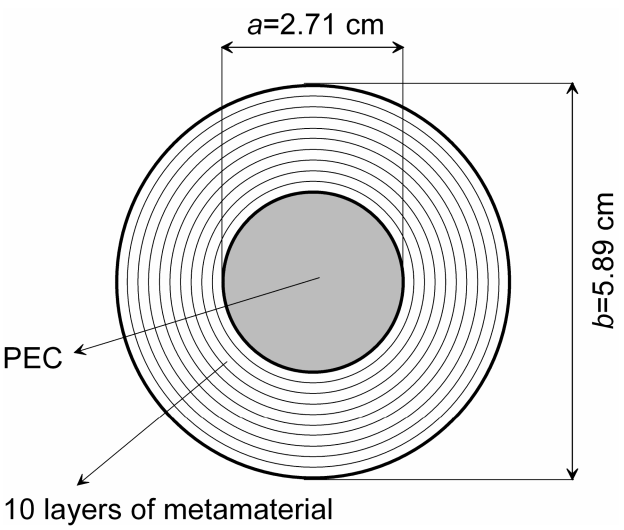

Fig. 2. Layout of the analyzed structure

In this paper we are going to provide the analysis of the mentioned structure, and to characterize it in mathematical terms by calculating the scattered field and the equivalent blockage width – the structure width that is "seen" by EM waves. Furthermore, we will provide the analysis of the frequency dependency of the invisibility for the real case – with dispersion included.

2. G1DMULT Algorithm

The G1DMULT algorithm [4] calculates the spectral-domain Green's functions in the same way for planar, circular-cylindrical and spherical geometries. The original algorithm has been modified to include radial anisotropy. The basic principle of the algorithm is as follows (e.g. for cylindrical geometry). For each value of spectral variables (m,k_z), the excited electromagnetic field has the same variation in ϕ- and z-directions as the corresponding current component, i.e. only the field dependence in r-direction is unknown. The algorithm is based on dividing the multilayer problem into equivalent sub-problems, one for each dielectric layer. The field components are given in the spectral domain, i.e. for the E_z component:

$$\tilde{E}_z(r,m,k_z) = \int_0^{2\pi}\int_0^{\infty} E_z(r,\varphi,z) e^{+jm\varphi} e^{+jk_z z} \, d\varphi dz \qquad (3)$$

The fields inside each anisotropic layer are of the form (e.g. the E_z component):

$$\tilde{E}_z^i(r,m,k_z) = a_m^i \, \mathrm{J}_{m'}(k_r^i r) + b_m^i \, \mathrm{H}_{m'}^{(2)}(k_r^i r) \qquad (4)$$

Here a_{nm}^i and b_{nm}^i are the unknown coefficients to be determined, while $\mathrm{J}_{m'}$ and $\mathrm{H}_{m'}^{(2)}$ are the Bessel functions of the first and Hankel functions of the second kind, for order m'. k_i is the wave number in i-th layer, calculated by the expression [5]:

$$k = \omega^2 \mu_\varphi \varepsilon_z \qquad (5)$$

Note that the order of Bessel/Hankel functions is not an integer and it is given by [5]:

$$m' = m\sqrt{\frac{\mu_\varphi}{\mu_r}}, \; m \in \mathbb{N} \qquad (6)$$

The equivalent sub-problems are connected (and the coefficients a_{nm}^i and b_{nm}^i are calculated) by forcing the continuity of tangential components of electric and magnetic fields, i.e. we have four unknowns per boundary. The original algorithm connects all equivalent sub-problems into a system of $4\cdot(N_{layer}\text{-}1)$ linear equations with the same number of unknowns (N_{layer} denotes the number of layers). More details about G1DMULT can be found in [4].

3. Results

In order to characterize the level of "invisibility" achieved by metamaterials, we have calculated, using the G1DMULT algorithm, the equivalent blockage width W_{eq} and the scattered electric field.

The equivalent blockage width is a complex-valued parameter (introduced in [7]) that represents the width of an ideal shadow which produces the same forward-scattered field as the cylinder that is being observed (in our case, a cloaking cylinder). Hence this parameter shows how wide the cylinder appears for electromagnetic waves, being a measure for the forward-scattered field.

We have analyzed the frequency dependency of the equivalent blockage width for the cloaking cylinder made of metamaterials, in a frequency range from 7 to 10 GHz.

The values of the cloak inner and outer radius values have been chosen with respect to constraints from the metamaterial unit cells design and layout requirements [3]. The structure is considered to be lossless. The radial variations of permeability $\mu_r(r)$ through the structure were approximated for the three cases with 1-, 4- and 10-step piecewise constant function representing the number of layers of metamaterial. The metamaterial constitutive parameters for each layer were calculated by Eq. 2 (the used radial coordinate r is the radius of the layer center).

At first, we have performed simulations for excitation by the TM-polarization, i.e. with the electric field polarized along the cylinder axis (the incident plane wave is assumed to propagate in the positive x-direction). The equivalent blockage width was calculated within a mentioned frequency range for each of the three cases. By implementing those three cases we have actually analyzed how the subtlety of the approximation of radial anisotropy influences the equivalent blockage width, i.e. "invisibility". The calculated equivalent blockage width and its frequency response for each case are shown in Fig. 3.

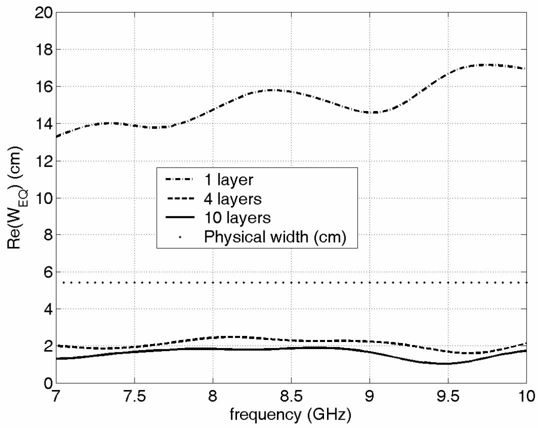

Fig. 3. Frequency response of equivalent blockage width

In addition, we have performed another simulation for the cloak excitation by the TE-polarization, i.e. with electric field polarized transverse with respect to cylinder axis. The cloak parameters have remained the same as in the previous simulation. For the TE-excitation the radial anisotropy of μ_r does not affect the value of calculated equivalent blockage width. That is because then the magnetic field is polarized along z-axis and thus constant in radial direction, while the electric field remains unchanged with respect to the number of layers, due to constant ε_z along the cloak.

Fig. 4 shows the comparison between the calculated equivalent blockage width for the TE- and TM-excitation, respectively. The TM-excitation is considered in the 10-layer case, as the good approximation of radial anisotropy of permeability, while the number of layers, i.e. approximations of μ_r, has no effect on TE-excitation.

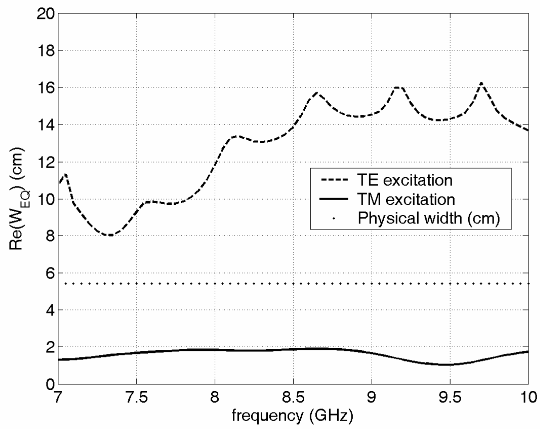

Fig. 4. Comparison of cloak effectiveness between TE- and TM-excitation

In a final simulation we have calculated the scattered electric field in the azimuthal plane both for the TM- and TE-excitation. The results are given in Fig. 5. Note that for the TE-excitation scattered field is significantly larger than for the TM-excitation, which was actually the expected result, since cloak is designed for TM-polarization.

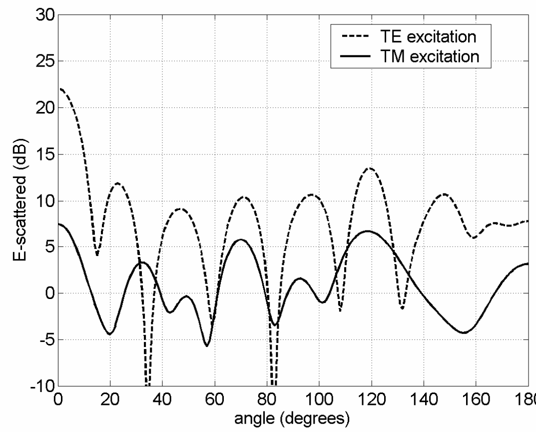

Fig. 5. Scattered field from cloaking cylinder for TE- and TM-excitation

4. Dispersion

The performed simulations show that the achieved invisibility is indeed unsuitable for TE-polarization, due to the simplifications in the metamaterial design. Meanwhile, it appears that the analyzed metamaterial structure still works for TM-polarization in a frequency range from 7 to 10 GHz, which however seems quite alright.

Actually, one more parameter hasn't been considered yet → dispersion. In reality, the magnetic permeability in metamaterials is also frequency dependent by the following expression (so-called Lorentz model [8]):

$$\mu_{eff} = 1 - \frac{f_{mp}^{\ 2} - f_0^{\ 2}}{f^2 - f_0^{\ 2} - j\gamma f} \tag{7}$$

Here f is the frequency of the signal, f_{mp} denotes the frequency at which μ_{eff}=0 for the lossless case (null-point of the function) and f_0 is the frequency at which μ_{eff} diverges (the pole of the function). Usually, f_{mp} amounts about 1.02 f_0 [9]. Furthermore, for simplicity the lossless case is being considered, i.e. $j\gamma f$=0.

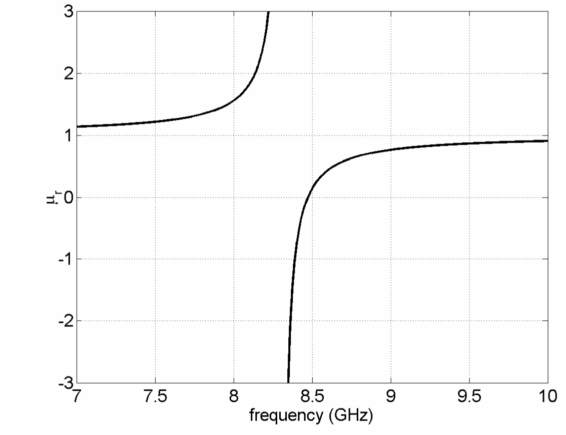

Fig. 6. Frequency dependency of μ_r

Note that the magnetic permeability increases proportionally with frequency (see Fig. 6) and thus affects the equivalent blockage width. In other words, if μ_r is designed to desired value in the structure for some frequency, by increasing frequency of the signal it changes. By changing μ_r finally the calculated equivalent blockage width also changes.

Figure 7 shows the frequency response of the equivalent blockage width for TM-excitation with 10 layers of metamaterial, with dispersion included. The response is shown in a narrow range around the central frequency of 8.5 GHz, for which the values of μ_r were designed. Note that, because of dispersion, the amount of the equivalent blockage width increases rapidly with frequency and the invisibility is achieved in a very small frequency range (about 0.24% of the central frequency).

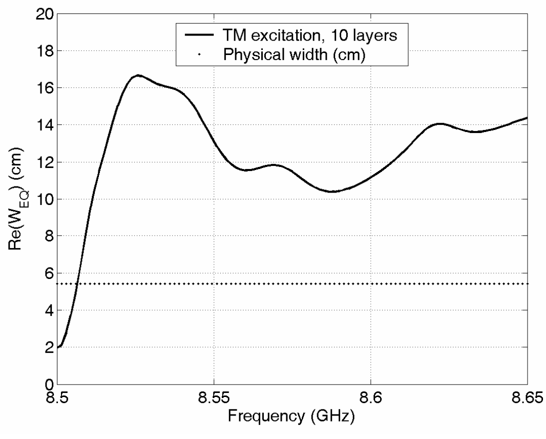

Fig. 7. Equivalent blockage width calculated for TM-excitation with dispersion included (central frequency is 8.5 GHz)

5. Conclusion

In this paper we have analyzed the invisible cloak realized as a multilayer cylindrical structure built from cells containing split-ring resonators. In order to characterize the level of the achieved "invisibility", we have calculated the equivalent blockage width and the scattered electric field in azimuthal plane.

Furthermore, the dispersion which occurs in reality was also taken into account. For the analyzed structure it is described by the Lorentz model. It was found that the achieved invisibility is, due to the dispersion, an extremely narrow-band phenomenon. The relative bandwidth where the invisibility is achieved is only about 0.24%. The results show that the metamaterial-based invisible cloaks still seem to be in their infant phase, and practical engineering realizations are yet to be seen.

References

[1] KILDAL, P. S. Artificially soft and hard surfaces in electromagnetics. *IEEE Transactions on Antennas and Propagation,* 1990, Vol. 38, No. 10, p. 1537-1544

[2] PENDRY, J. B., SCHURIG, D., SMITH, D. R. Controlling electromagnetic fields. *Science Magazine,* 2006, Vol. 312, p. 1780-1782

[3] SCHURIG, D., MOCK, J. J. , JUSTICE, B.J., CUMMER, S. A., PENDRY, J. B., STARR, A. F., SMITH, D. R. Metamaterial electromagnetic cloak at microwave frequencies. *Science Magazine,* 2006, Vol. 314, p. 977-979

[4] SIPUS, Z., KILDAL, P. S. , LEIJON, R., JOHANSSON, M. An algorithm for calculating Green's functions for planar, circular cylindrical and spherical multilayer substrates. *ACES Journal,* 1998, Vol. 13, p. 243-254

[5] IVSIC, B., SIPUS, Z. Analysis of uniaxial multilayer cylinders used for invisible cloak realization. In *Proceedings of 19th International Conference on Applied Electromagnetics and Communications*. Dubrovnik (Croatia), 2007, p. 193 - 196

[6] KILDAL, P. S., KISHK, A. SIPUS, Z. RF invisibility using metamaterials: Harry Potter's cloak or the Emperor's new clothes? In *IEEE Antennas and Propagation and URSI International Symposium Digest 2007 Honolulu.* Honolulu (Hawaii), 2007, paper no. 0593 on CD

[7] KILDAL, P. S., KISHK, A. A., TENGS, A. Reduction of forward scattering from cylindrical objects using hard surfaces. *IEEE Transactions on Antennas and Propagation,* 1996, Vol. 44, No. 11, p. 1509-1520

[8] ENGHETA N., ZIOLKOWSKY, R. (Editors) *Metamaterials: Physics and Engineering Explorations*. John Wiley and IEEE, 2006.

[9] HRABAR, S., BENIC L., BARTOLIC J. Simple Experimental Determination of Complex Permittivity or Complex Permeability of SNG Metamaterials. In *Proceedings on 36th European Microwave Conference 2006*. Manchester (U.K.), 2006, p. 1395 - 1398