Redistribution, Horizontal Inequity, and Reranking: How to Measure Them Properly

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Redistribution, horizontal inequity and reranking: how to measure them properly

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Abstract The decomposition of the redistributive effect of an income tax into vertical, horizontal and reranking contributions according to the model of Aronson, Johnson and Lambert (1994), henceforth AJL, is revisited. When close equals groups are used, rather than the exact equals groups upon which the model is predicated, problems arise. A new measurement system is proposed, in which three distinct forms of reranking are disentangled and the vertical and horizontal contributions are redefined. Other approaches to measuring equity in tax systems are set in context. Findings are applied to Croatian data, and recommendations for users of the AJL methodology are given.

JEL classification: H23.
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1. Introduction

The decomposition of the redistributive effect of an income tax system into vertical, horizontal and reranking components has been much studied of late. A sequence of related papers, those of Aronson, Johnson and Lambert (1994), Aronson and Lambert (1994), and Aronson, Lambert and Trippeer (1999) lay out the appropriate (Gini-based) measurement theory for a model in which the pre-tax income distribution can be partitioned into exact pre-tax equals groups, and the tax system does not rerank such groups, nor does it (obviously) rerank the members within any group, in the transition from pre-tax to post-tax income. The methodology has, though, usually been applied in the context of *close equals groups.* See, for example, Wagstaff *et al.* (1999) and, most recently, Hyun and Lim (2005).

The disparity between the model of the Aronson *et al.* papers and the domain in which the result is commonly applied is accounted for by the lack (or sparseness) of exact equals in typical real-world data sets, notwithstanding that the typical tax system does cause both rank reversals and the unequal treatment of equals (the former in most samples and the latter at the population level if not in one's sample). In van der Ven, Creedy and Lambert (2001) it is shown that an arbitrary specification of close equals groups can lead to misleading results, and a procedure to define the close equals groups optimally in terms of class width is suggested. Problems with the decomposition remain, though, as these authors note, "when reranking occurs within groups of close equals or if entire groups are reranked" (*ibid.*, p. 385). Unfortunately these forms of reranking are all too common in typical microdata.

The prevalence and implications of both *reranking within close equals groups* and *reranking of entire close equals groups* had not occurred to one of us (Lambert, during the writing of the Aronson *et al* papers), but became obvious to the other (Urban) as soon as an empirical study of redistributive effect in Croatia was begun. In the present paper, we explain fully the modifications which must be made to the measurement system of the Aronson *et al* papers when close equals groups are invoked by the practitioner, and when he or she finds one or both of the unenvisaged forms of reranking to be present in the sample data. In the process of thus tidying things up for empirical analysts working in the area, we have also been able to finally reconcile a number of approaches to measuring equity in tax systems which co-exist in the literature but appear to offer slightly differing recipes to the practitioner (principally those of Atkinson 1980, Plotnick 1981 and Kakwani 1984, but with connections also to work of King 1983 and Jenkins 1994), using a consistent theoretical framework.

The structure of the paper is as follows. In Section 2, we explain the three forms of reranking which a tax system can induce between and within close equals groups, using an illustrative data set. In Section 3, we carefully delineate a number of transformations between income vectors, appropriately ordered, which can be used to capture the vertical, horizontal and reranking stances of a tax system, using the hypothetical data of Section 2 to illustrate. This leads to a fully articulated measurement system, in which three distinct forms of reranking are disentangled. Moreover, the various approaches to measuring horizontal and vertical equity in tax systems in previous literature, which we have referred to above, are all accounted for, drawn together and reconciled in the new measurement system. In Section 4, we apply the findings to Croatian data, and make recommendations for future empirical studies. Section 5 concludes.

2. Reranking within, and of, entire close equals groups

In Table 1, the columns X and N present simulated but realistic-looking data on pre-tax and post-tax incomes. The first thing a researcher will need to do when he or she has gathered the data is to sort both columns together, by increasing order of pre-tax income. Already here the first problem arises. It may happen that there is specific amount of income, for example a minimum wage, received by many taxpayers. In our sample this happens for income units {#6,#7,#8,#9} and {#21,#22,#23}. We will call these income units *pre-tax exact equals.*

How should we arrange the post-tax incomes N of pre-tax exact equals? A simple way is to sort the income units in increasing order of post-tax income within each group of pre-tax exact equals. This has already been done in Table 1.[1]

[Table 1 about here]

In reality, income units having exactly the same pre-tax income as each other will form a tiny part of the sample, or may be absent. Pre-tax income is usually assessed in terms of $1/100^{th}$ of a currency unit and most taxpayers will have different pre-tax incomes. Thus, the second step will be to form artificial groups of income units which have similar incomes. We call such income units *close pre-tax equals,* and use bands to allocate income units to groups. In Table 1, we have used a bandwidth of $50. The lower (upper) limit is $50 ($99.99) for the first group, $100 ($149.99), and so on. We obtain in this way six close equals groups with the following memberships: G_1 = {#1,#2,#3}, G_2 = {#4,…,#9}, G_3 = {#10,#11}, G_4 = {#12,…,#17}, G_5 = {#18,#19}, G_6 = {#20,…,#26}.

We are ready to proceed with an intuitive analysis of reranking. Column r_X in Table 1 shows the pre-tax income ranking of income units. Higher income means a higher rank. Column r_N shows the ranking of these same income units after the tax has been subtracted. The two scales are different and this means that the tax induces reranking. Observe the most drastic examples: the rank of income unit #19 goes down 8 points to 11^{th} place after tax; at the other extreme, income unit #11 moves up by 6 points to 17^{th} place after tax.

The process of reranking that takes us from r_X to r_N can be broken down into three distinct phases: within-group reranking, entire-group reranking, and a third form of reranking which we associate with the model of Aronson, Johnson and Lambert (1994), henceforth *AJL*. In the column of Table 1 labeled K_1, the income units are ordered as in the beginning, before tax, and in the final column, labeled K_2, they are lined up in ascending order of post-tax income.[2] We can go from K_1 to K_2 in three steps, as follows. First, note that in K_1, the income units already fall into close equals groups, shown graphically by the boxes drawn around the groupings in that column. Now consider the following three steps, starting from K_1:

1. Introduce reordering of income units *within groups* only, such that post-tax incomes increase monotonically in each group. In particular, income unit #4 moves up 4 places in G_2. There are also changes in G_4, G_5 and G_6 but none are necessary in G_1 and G_3 By this, we obtain the column K_3. The transition $K_1 \rightarrow K_3$ measures *within-groups reranking.*

[1] Spreadsheet calculators and statistical packages offer the possibility of ordering two columns together, first by one column (X), and then by the second column (N), in a single action. This must be done before going further. If the building of N were left to accident, quite arbitrary orderings of N could emerge.

[2] For example, income unit #5, placed third lowest in column K_2, has post-tax income rank $r_N = 3$; income units #19 and #11, which have $r_N = 11$ and $r_N = 17$ respectively, appear 11^{th} and 17^{th} in column K_2 (etc).

2. Now reorder whole groups, keeping the order of income units within each group unchanged, in such a way that the mean post-tax income rises monotonically from one group to the next. In the case of our data, this necessitates G_3 and G_4 changing places, since the mean post-tax income for G_3 is 162.39 and the mean post-tax income for G_4 is only 160.45. No other changes are needed. Column K_4 is the result; the transition $K_3 \rightarrow K_4$ represents *entire-groups reranking.*

3. Keeping everything else the same in K_4, we now introduce the final change, to K_2, which is of a new kind, in that some income units 'jump' or escape from their original group to take up new positions elsewhere. Now all income units are now lined up in ascending order of their post-tax income, regardless of the group memberships. This final transition, from K_4 to K_2, which involves shuffles of members across groups into their post-tax positions, is the only type of reranking that can take place within the confines of AJL's tax model. We will therefore say that the transition $K_4 \rightarrow K_2$ describes *AJL reranking*.

In summary, we have identified three phases of reranking here. The original (pre-tax) ranking r_X is that of column K_1. The post-tax ranking r_N is that of column K_2. Three types of reranking, respectively the within-groups, entire-groups and AJL forms of reranking, are captured by the three steps of the sequence which took us from K_1 to K_2: $K_1 \rightarrow K_3 \rightarrow K_4 \rightarrow K_2$. All of these stages are involved in the redistributive process, as we shall shortly explain in greater detail.

3. Measurement system

In what follows we allude repeatedly to the illustrative data of Table 1 above, but our arguments will apply equally to any data set and should be sufficient to guide the analyst in possession of real-world microdata.

First, we order the income units by pre-tax income level and, among exact pre-tax equals, by post-tax income level. Call this ordering criterion O_1. We used this ordering in Table 1 above to define the columns X, N and K_1. Table 2 below also contains this income data, as the two columns X and N_1. The column for X is non-decreasing; the Lorenz curve for pre-tax income, call it L_X, is computed using this ordering.

[Table 2 about here]

For each row of Table 2, the N_1-value is the actual post-tax income of a person with pre-tax income given by the X-value at the start of that row. The vector N_1 is not necessarily increasing. Indeed, it will exhibit decreases precisely when the tax system causes reranking (*i.e.* if, as in this data set, there are people with higher pre-tax income and lower post-tax income than others). Let C_1 be the post-tax concentration curve with respect to the ordering O_1. C_1 will not equal the post-tax Lorenz curve if N_1 is not increasing. Kakwani (1984) recognized this. For him, the transformation $L_X \rightarrow C_1$ determined the vertical characteristic of the tax system (see also Kakwani, 1986, p. 83-84 on this). For other quite different conceptions of the vertical action of the tax system, see on.

Let O_2 be the ordering of income units by post-tax income level (shown in column K_2 of Table 1). N_2, the increasing vector of post-tax incomes so ordered, is shown as a column in Table 2, and L_N is the Lorenz curve for post-tax incomes, computed from N_2. However, to the extent that the tax system causes any reranking, there are N_2-values which do not correspond to the X-values at the start of the relevant rows. The transition $N_1 \rightarrow N_2$ unscrambles *all* reranking caused by the tax system. Atkinson (1980), Plotnick (1981) and Kakwani (1984, 1986) measure this reranking, which we call *APK-reranking* in the sequel, by the transition $C_1 \rightarrow L_N$.

The mapping $X \rightarrow N_2$, associating each N_2-value with the X-value at the start of the same row in Table 2, assigns to people at various positions in the pre-tax income parade the post-tax income value appearing at that same position in the post-tax parade. This mapping features in the work of King (1983) and Jenkins (1988, 1994), who characterize horizontal inequity by the 'distance' (somehow defined) between N_2 and N_1. See also Jenkins and Lambert (1999) and Lambert (2001, p. 247) on this.

Next, the analyst must select close pre-tax equals groups. In the case of the data in Table 2, we have used income ranges \$50-\$99.99, \$100-\$149.99, \$150-\$199.99, \$200-\$249.99, \$250-\$299.99, \$300-\$349.99 to define the groups. Obviously the pre-tax means are increasing from each group to the next. Now order the income units by post-tax income within each group, and order the groups by the pre-tax group means; call this ordering criterion O_3. Column K_3 of Table 1 shows the income units ordered according to criterion O_3. Table 2 shows the post-tax income vector as N_3 for this situation; C_3 is the concentration curve for post-tax income according to the ordering O_3. N_3 will differ from N_1 (and C_3 from C_1) to the extent that, as in our data set, within any close equals group there are people with higher pre-tax income and lower post-tax income than others in that group. The transformation $N_1 \rightarrow N_3$ unscrambles such rerankings within groups. But N_3 is not itself an increasing vector if, again as here, there are people in lower pre-tax income ranges [close equals groups] who have higher post-tax incomes than others in higher pre-tax income ranges [close equals groups]. The transition $C_1 \rightarrow C_3$ measures (only) the within-group reranking caused by the tax system; it does not unscramble whole-group rerankings.

Now consider the ordering, call it O_4, in which the groups are lined up in order of their *post-tax* means, and within the groups, the income units are ordered by their post-tax incomes. Within each close equals group, the ordering of persons is the same as for O_3, but the groups come in a different order to the extent that the tax system reranks entire groups [making people in a close equals group with a lower pre-tax mean on average better off than their counterparts in a close equals group with a higher pre-tax mean]. In this situation, overlap can be seen between the post-tax income ranges occupied by the respective pre-tax close equals groups. In Table 1, column K_4 shows the income units ordered in this way. Let N_4 be the post-tax income vector so ordered, and let C_4 be the concentration curve for post-tax income with respect to O_4. As shown in Lambert and

Aronson (1993), the degree of such overlap is measurable on the Lorenz diagram by the transition $C_4 \rightarrow L_N$, and in the decomposition of the post-tax Gini coefficient across pre-tax equals groups, by the residual term.

In Aronson, Johnson and Lambert (1994), henceforth AJL, the degree of tax-induced overlap of the post-tax income ranges of distinct pre-tax equals groups was measured by this residual, and characterized as a form of reranking, and shown to be a negative component in the redistributive effect of the tax system (itself measured by the transformation $L_X \rightarrow L_N$). We call the transition $C_4 \rightarrow L_N$ the *AJL-reranking* of the tax system in all that follows.

Before moving on to the vertical and horizontal effects of the tax system, consider the transformation $N_3 \rightarrow N_4$. Post-tax income values are ordered from smallest to largest within each close equals group in both vectors, but the groups themselves are re-ordered in this transformation (by post-tax mean instead of pre-tax mean). We could capture by the transformation $C_3 \rightarrow C_4$ the effect of the tax system in reranking entire groups (having purged already within-group rerankings), but overlap clearly remains. AJL did not note the possibility that a tax system might engender entire-group reranking, because their model admitted only exact equals groups and they assumed that the tax on average had a marginal rate of less than unity (however, see their footnote 1, page 264).

AJL decompose the redistributive effect of the tax system into vertical, horizontal and AJL-reranking components, in a model in which the population is partitioned into exact equals groups and the post-tax income *on average* in an equals group increases with pre-tax income level (i.e. from one equals group to the next adjacent one). The vertical effect in this model measures, as a transformation in the Lorenz diagram, the effect of the tax *on average* across such groups. The horizontal effect captures the unequal tax treatment of exact equals, also as a transformation in the Lorenz diagram. To obtain these two transformations, an artificial post-tax income distribution is constructed in which pre-tax (exact) equals have a common post-tax income, equal to the average *actual* post-tax income for the group.

In the current scenario, with the population partitioned into *close* rather than *exact* equals groups, we can replicate these constructions by also computing a counterfactual post-tax income distribution, one in which the operation of the tax system within each close equals group has been smoothed, and use this smoothed distribution to express the vertical and horizontal stances of the tax system. The way to do it is to make the counterfactual tax schedule distributionally neutral within groups - proportional, that is - rather than equalizing the incomes of close equals, which would involve rich-to-poor redistribution within each group, thus introducing a hypothetical and quite unjustified vertical effect within groups.

This procedure accords with proposals of Camarero *et al.* (1993), Pazos et al. (1995) and Lambert and Ramos (1997), and strictly, it captures *an estimate only* for true classical horizontal inequity which is, of course, predicated on the tax treatment of *exact* equals. The horizontal measure derived from the analyst’s choice of close equals group and the resultant smoothing may be negative in some circumstances, and the vertical effect correspondingly understated (see on).

Thus for each close equals group, let *g* be the share of total pretax income taken in tax, as shown in Table 2. Ordering the income units according to O_1 and forming the close equals groups as before, let N_5 be the post-tax income vector that would obtain if, counterfactually, each income unit had its pretax income X reduced by the fraction g particular to its group, and denote by C_5 the concentration curve for N_5. If whole groups are reranked by the actual tax system, they will be reranked also by the smoothed tax system (due to the way the tax rate g is constructed, group by group). Thus the transformation $L_X \rightarrow C_5$ has a feature which AJL did not recognize – whole-group reranking (but again see their footnote 1, page 264) – and it measures the full vertical effect of the tax system given the close equals groups selected by the analyst.

For N_6, which is a rearrangement of N_5, we re-order the groups by post-tax means; within the groups, the income units take the values and order from N_5. By this transition ($N_5 \rightarrow N_6$), any whole-group rerankings caused by the counterfactual smoothed tax system are unscrambled (in the same way that they were for the actual tax system by the transformation $N_3 \rightarrow N_4$). As then, rerankings of individuals between groups are left in place (as evidenced, for example, in the middle part of the vector N_6). C_6 is the corresponding concentration curve. The vertical effect of the tax system as perceived by AJL, and appropriate to their measurement system as we shall see, is realized by the transformation $L_X \rightarrow C_6$.

Let N_S be the vector obtained when either N_5 or N_6 is rearranged in increasing order, and let L_{NS} be the Lorenz curve for post-tax income induced by the counterfactual tax system, derived from N_S. The redistributive effect of the counterfactual system is given by the transformation $L_X \rightarrow L_{NS}$. The APK-reranking caused by the counterfactual tax system is seen in the transformation $N_5 \rightarrow N_S$ (and $C_5 \rightarrow L_{NS}$), whilst the AJL-reranking in the counterfactual tax system is measured by the transformation $N_6 \rightarrow N_S$ (and $C_6 \rightarrow L_{NS}$).

Finally, we come to the horizontal effect of the tax system. In Jenkins and Lambert (1999), two distinct, basic approaches to capturing the horizontal inequity (HI) in a tax system are identified and explored. Approach 1 focuses upon inequality of post-tax income among (exact) equals; approach 2 rests upon the person-by-person departures of actual post-tax incomes from those generated by a reference schedule, itself constructed counterfactually to be HI-free within equals groups. Approach 1 is typified by the methodology of AJL, whilst the work of King (1983) and Jenkins (1994) follows approach 2.[3]

With close rather than exact equals groups, the AJL methodology would capture HI within each group by the inequality of actual post-tax incomes relative to that of the smoothed ones, and overall as an aggregate of these within-group effects. Lambert and Ramos (1997) characterize as *pseudo-horizontal inequity* the process whereby "it is as if

[3] Two further approaches are in fact also enumerated by Jenkins and Lambert. These apply the same reasoning to tax liabilities instead of post-tax incomes

the tax acts to increase inequality within close equals groups" *(page 29).* N_3 and N_5, representing respectively the actual and counterfactual (smoothed) tax systems, are identical in their ordering of close equals groups, as are N_4 and N_6. Either of the transformations $C_6 \rightarrow C_4$ and $C_5 \rightarrow C_3$ can be used to capture the pseudo-horizontal effect of the tax system; this effect does not depend on the ordering of close equals groups.

However the underlying transitions $N_6 \rightarrow N_4$ and $N_5 \rightarrow N_3$ break the link between peoples' actual and smoothed post-tax incomes, because for N_3 and N_4, post-tax incomes have been reranked non-decreasingly within groups (compare N_3 with N_1 to see this), whilst for N_5 and N_6 the smoothed values, being derived directly from X, are (also) non-decreasing but *not* reranked. Hence for a person-by-person comparison of actual and smoothed post-tax incomes, the transition $N_5 \rightarrow N_1$ is appropriate. This gives rise to a second possibility for specifying the horizontal effect, namely, *via* the transformation $C_5 \rightarrow C_1$. The two horizontal effects are related, as we shall see, and both can be negative, in which case the corresponding vertical effects will be understated.[4]

We have now characterized a number of tax effects, by transformations between Lorenz and concentration curves, the latter constructed using various orderings of income units. Writing G for the Gini coefficient for a Lorenz curve L (hence G_X, G_N and G_{NS} in the present case), and D for the equivalent index in the case of a concentration curve (hence the concentration coefficients D_1, D_3, D_4, D_5, D_6), we may now arrive at numerical measures of the strength of the relevant effects of the tax system, as in Table 3 below (where they are written in their order in which they were presented, and with the underlying orderings of income units also defined for convenience).

ORDERING	CHARACTERIZATION
O_1	by pre-tax income, and among exact pre-tax equals, by post-tax income
O_2	by post-tax income
O_3	by post-tax income within groups, and the groups by pre-tax means
O_4	by post-tax income within groups, and the groups by post-tax means

TRANSFORMATION	CHARACTERIZATION	INDEX MEASURE
$L_X \rightarrow C_1$	Kakwani vertical effect	$V^K = G_X - D_1$
$C_1 \rightarrow L_N$	APK-reranking	$R^{APK} = G_N - D_1$
$C_1 \rightarrow C_3$	within-group reranking	$R^{WG} = D_3 - D_1$
$C_4 \rightarrow L_N$	AJL-reranking	$R^{AJL} = G_N - D_4$
$C_3 \rightarrow C_4$	entire group reranking	$R^{EG} = D_4 - D_3$
$L_X \rightarrow L_N$	redistributive effect	$RE = G_X - G_N$

[4] A simple example shows this rather starkly. In a population of four persons, let the pre-tax incomes be 114, 126, 194, 206 and let the corresponding post-tax values be 80, 80, 100, 100. Form two close equals groups, {114, 126} and {194,206}. The tax creates complete equality within each group. The counterfactual tax rates are $g = ⅓$ and $g = ½$ respectively, and hence the smoothed incomes become 76, 84, 97, 103. Inequality is unambiguously higher under the counterfactual tax than under the actual tax, both within each close equals group and overall. This leads to negative measures for (pseudo-)horizontal inequity, and understated vertical effects.

$L_X \rightarrow C_5$	full vertical effect	$V = G_X - D_5$
$L_X \rightarrow C_6$	AJL vertical effect	$V^{AJL} = G_X - D_6$
$L_X \rightarrow L_{NS}$	redistributive effect of smoothed tax	$RE_S = G_X - G_{NS}$
$C_5 \rightarrow C_6$	entire group reranking of smoothed tax	$R_S^{EG} = D_6 - D_5$
$C_5 \rightarrow L_{NS}$	APK-reranking of smoothed tax	$R_S^{APK} = G_{NS} - D_5$
$C_6 \rightarrow L_{NS}$	AJL-reranking of smoothed tax	$R_S^{AJL} = G_{NS} - D_6$
$C_6 \rightarrow C_4$ or $C_5 \rightarrow C_3$	AJL–type pseudo-horizontal effect	$H^{AJL} = D_4 - D_6 = D_3 - D_5$
$C_5 \rightarrow C_1$	second type of horizontal effect	$H = D_1 - D_5$

TABLE 3. CONCEPTS AND MEASURES

A number of relationships stem immediately from Table 3. *First*, we have:

(1) $RE = V^K - R^{APK}$

as in Kakwani (1984, 1986);

(2) $RE = V^{AJL} - H^{AJL} - R^{AJL}$

as in AJL; and

(3) $R^{APK} = R^{AJL} + R^{WG} + R^{EG}$

showing that AJL did not account for all possible reranking in their study (namely, their model did not allow for within-group reranking or entire group reranking, which are both accommodated in the Atkinson-Plotnick-Kakwani measurement system).

Second, in respect of the counterfactual smoothed tax system, we have analogous results:

(4) $RE_S = V^{AJL} - R_S^{AJL}$

which is the analogue of (1) and (2); and

(5) $R_S^{APK} = R_S^{AJL} + R_S^{EG}$

which is the analogue of (3) (noting that the smoothed tax does not rerank income units within close equals groups). Furthermore, rearranging $H = D_4 - D_6 = D_3 - D_5$, we have that $D_4 - D_3 = D_6 - D_5$. This means that the entire group reranking of the actual tax system is the same as that of the smoothed version:

(6) $R^{EG} = R_S^{EG}$

Third, notice that what we have termed the 'full vertical effect' of the tax system, namely V, does not figure in any of the above relationships, and nor does the 'second type of horizontal effect', H. Using (6) along with the definition of V, we have

(7) $V = V^{AJL} + R^{EG}$

affirming that the AJL conception of the vertical effect of the (smoothed) tax system does not recognize any whole group reranking that may be taking place. Also, directly from the definitions in Table 3, we have

(8) $H = H^{AJL} - R^{WG}$

exposing the within-group ranking difference that distinguishes H^{AJL} from H. Finally, from (2), (3), (7) and (8) comes:

(9) $RE = V - H - R^{APK}$

which is a decomposition of redistributive effect into vertical, horizontal and reranking components that is *identical in form* to AJL's (in equation (2)), but with *different component measures*. Naturally, (9) reduces to exactly the AJL form for a tax system such as AJL's in the case of exact equals groups and no entire group reranking.

Summarizing, our careful modeling has exposed three alternative decompositions of redistributive effect into vertical, horizontal and reranking components. Two of these come from existing literature, namely:

(1) $\mathbf{RE = V^{K} - R^{APK}}$

which is due to Kakwani (1984) and lacks a pure horizontal term, and:

(2) $\mathbf{RE = V^{AJL} - H^{AJL} - R^{AJL}}$

which extends the AJL methodology to the close equals group scenario but does not capture within-group and entire-group rerankings (if these occur). The third such decomposition:

(9) $\mathbf{RE = V - H - R^{APK}}$

which is new, rests upon the 'full vertical effect' and the 'type 2 horizontal effect', both of which we constructed *ab initio* for the close equals model. In this decomposition, as in Kakwani's, the Atkinson-Plotnick-Kakwani measure R^{APK} is the measure of reranking, and it captures *all* forms of reranking (recall (3)). Moreover, although the vertical and horizontal contributions V and H may be understated, as already explained, their difference, V-H, most certainly is not, since it equals RE - R^{APK} which does not depend on the close equals group construction. Of course, from (1), it must be that

(10) $V^{K} = V - H$

showing that the missing horizontal term in Kakwani's decomposition has to be netted out of his vertical contribution. In the case that H is negative (as in our example in footnote 4), V is correspondingly understated but Kakwani's V^{K}, which combines both terms, is independent of close equals groupings.

In sum, we have here developed three vertical measures (V^{K}, V^{AJL} and V), two horizontal measures (H^{AJL} and H) and four reranking measures (R^{APK}, R^{AJL}, R^{WG} and R^{EG}), all of which describe facets of an *actual* tax system – and yet another array of measures for the counterfactual (smoothed) tax system. What is the practitioner to make of all this? Which indices should be presented to the policy analyst?

Equation (9) specifies the only decomposition of redistributive effect into vertical, horizontal and reranking components which takes account of all three forms of reranking identified in Section 2 of the paper. It also rests upon vertical and horizontal effects which we constructed *ab initio* for the close equals model, whereas AJL's decomposition of redistributive effect, in (2), neglects important (and empirically prevalent) sources of reranking when applied in close equals group scenarios. Not only is R^{AJL} inadequate as a measure of reranking in that setting (recall (3)), but also V^{AJL} and H^{AJL} are inadequate as measures of vertical and horizontal redistribution too - because, in contrast with the corresponding effects of decomposition (9), they take no account of entire-group and within-group rerankings (recall (7) and (8)).

Table 4 shows the numerical values of all measures for the illustrative data of Tables 1 and 2. For the recommended decomposition, in (9), vertical redistribution accounts for 126.7% of actual redistributive effect, with losses of 4.9% and 21.8% attributable to horizontal inequity and reranking respectively. Kakwani's decomposition, in (1), also shows a 21.8% loss of redistributive effect attributable to reranking, but this decomposition does not rely upon close equals group modeling and thereby lacks a horizontal component. For AJL's decomposition, in (2), vertical redistribution accounts for 125.6% of redistributive effect, with losses of 13.7% and 11.9% attributed to horizontal inequity and reranking respectively. The reranking effect is less for AJL because their model neglects within-group and entire-group rerankings. Indeed, from (3), AJL-reranking accounts for only 54.7% of the reranking induced by this (hypothetical) tax system. Of the remainder, 40.4% occurs within groups, and 4.9% comes from entire-group rerankings.

(1) $RE = V^{K} - R^{APK}$
$.045139 = .054986 - .009847$

(2) $RE = V^{AJL} - H^{AJL} - R^{AJL}$
$.045139 = .056720 - .006191 - .005390$

(3) $R^{APK} = R^{AJL} + R^{WG} + R^{EG}$
$.009847 = .005390 + .003974 + .000483$

(4) $RE_{S} = V^{AJL} - R_{S}^{AJL}$
$.054381 = .056720 - .002339$

(5) $R_{S}^{APK} = R_{S}^{AJL} + R_{S}^{EG}$
$.002822 = .002339 + .000483$

(6) $R^{EG} = R_{S}^{EG}$
$.000483 = .000483$

(7) $V = V^{AJL} + R^{EG}$
$.057203 = .056720 + .000483$

(8) $H = H^{AJL} - R^{WG}$
$.002217 = .006191 - .003974$

(9) $RE = V - H - R^{APK}$
$.045139 = .057203 - .002217 - .009847$

(10) $V^{K} = V - H$
$.054986 \quad .057203 - .002217$

TABLE 4. REDISTRIBUTIVE EFFECT AND ITS VERTICAL, HORIZONTAL AND RERANKING CONTRIBUTIONS FOR THE ILLUSTRATIVE DATA OF TABLES 1 AND 2

It is important to stress that *all of* these results (except that of Kakwani) are conditioned by the choice of close equals groups, which in this illustrative case were defined by a $50 bandwidth. The choice of bandwidth also determines the counterfactual (smoothed) tax system (whose own characteristics for our illustration can also be found in Table 4). As we have pointed out, in some applications and for some choices of bandwidth, negative horizontal effects and understated vertical effects may be expected.

Finally, before turning to our Croatian data, we briefly consider the methodology used in previous empirical studies which have adapted the AJL approach to close equals groups. Typically, the vertical component of redistributive effect has been computed by some form of averaging within close equals groups, just as AJL do for the exact equals groups of their model.

Thus, Wagstaff *et al.* (1999) and Hyun and Lim (2005) compute their vertical component of redistributive effect, let us call it V^{WAG}, from the Kawani (1977) progressivity index $K_{\overline{T}}$ calculated for the averaged tax, call it $\overline{T}_k$, across members of each close equals group k, with no pre-tax income averaging.[5] That is, V^{WAG} measures the redistributive stance of a tax which is *lump sum* (*i.e.* highly regressive) within each close equals group. The reranking term, R^{WAG}, is computed by these authors from the concentration curve for post-tax incomes when lined up by first by their pre-tax close equals groups and then, within those groups, by post-tax income. This concentration curve is our C_3; R^{WAG} in fact equals $G_N - D_3$ in our notation. The horizontal contribution, H^{WAG}, is obtained as the residual (i.e. by subtraction), ensuring an exact decomposition:[6]

(11) $RE = V^{WAG} - H^{WAG} - R^{WAG}$

From Table 3, we can in fact express R^{WAG} in terms of familiar indices:

(12) $R^{WAG} = R^{APK} - R^{WG} = R^{AJL} + R^{EG}$

Hence R^{WAG} involves only two of the three forms of reranking known to be present in general when close equals groups are invoked.

In van der Ven, Creedy and Lambert (2001), henceforth VCL, the averaging of both pre-tax incomes and post-tax incomes (equivalently, of taxes) within close equals groups is advocated in order to compute the vertical contribution to RE, call this V^{VCL}. Hence V^{VCL} expresses the redistributive effect of a hypothetical tax function that would transform *average* pre-tax income into *average* post-tax income in each close equals group (unlike V^{WAG}).[7] Reranking is measured as R^{AJL} by VCL, and, as in the other studies, the horizontal effect, call it H^{VCL}, may then be obtained as the residual (i.e. by subtraction): $H^{VCL} = RE - V^{VCL} - R^{AJL}$. The VCL procedure is consistent with economic theory, as can be demonstrated in terms of the decompositions $G_X = G_{B,x} + \sum_{k=1}^{K} a_{k,x} G_{k,x}$ and $G_N = G_{B,n} + \sum_{k=1}^{K} a_{k,n} G_{k,n} + E_n$ of the pre- and post-tax Gini coefficients across close equals groups.[8] The notation here is, perhaps, self-explanatory: the components are the

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[5] This approach was also adopted by Doorslaer *et al.* (1999) in the context of health care finances.

[6] It must be said that, had H^{WAG} been obtained directly, from the formula published in Aronson *et al.* (1994) rather than as a residual, namely as $H^{WAG} = \sum_k a_{k,n} G_{k,n}$, where $G_{k,n}$ is the Gini coefficient for post-tax income in close equals group k and $a_{k,n}$ is the product of that group's population share and post-tax income share, then the decomposition would have failed. See ahead.

[7] The VCL approach has also been adopted in the recent paper of Wagstaff (2004).

[8] For more on the Gini decomposition in general, see Lambert and Aronson (1993) and Lambert and Decoster (2004).

between-groups, within-groups and overlap terms (the first of these involving averaging, and the last being zero for the pre-tax income distribution, because of the way close equals groups are constructed). Subtracting one decomposition from the other, we arrive at $RE = (G_{B,x} - G_{B,n})$ - $\left(\sum_{k=1}^{K} a_{k,n}G_{k,n} - \sum_{k=1}^{K} a_{k,x}G_{k,x} \right)$ - E_n which is VCL's equation (4) (with a minor change of notation). In this, $E_n = G_N - D_4 = R^{AJL}$ and $(G_{B,x} - G_{B,n}) = V^{VCL}$. Hence $H^{VCL} = \left(\sum_{k=1}^{K} a_{k,n}G_{k,n} - \sum_{k=1}^{K} a_{k,x}G_{k,x} \right)$, and the decomposition

(13) $RE = V^{VCL} - H^{VCL} - R^{AJL}$.

is exact when all terms are calculated directly.[9]

As this discussion has shown, more than one attempt has been made in empirical literature to adapt the AJL system of measurement to the close equals group situation. The resulting vertical effects V^{WAG} and V^{VCL} have different rationales than both V^{AJL} and V (involving forms of averaging rather than smoothing); the reranking effects, $R^{AJL} + R^{EG}$ (from (12)) and R^{AJL} respectively, do not capture all three forms of reranking we have identified; and the (residual) horizontal terms, H^{WAG} and H^{VCL} , which differ in design very substantially from our H and H^{AJL}, have somewhat fraught interpretations.[10]

Table 5 shows all of these vertical and horizontal estimates, for our illustrative dataset. We find small differences between V^{WAG}, V^{VCL}, V^{AJL} and V, and small differences between H^{VCL} and H^{AJL}, but the H of our equation (9) differs considerably from the other horizontal measures, and H^{WAG} is untrustworthy, as we have explained.[11] . Equation (9) is our recommended one, in which *all* forms of reranking are included in the reranking term, R^{APK}, and the vertical and horizontal terms have their own rationales, arising from

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[9] Despite this distinction, it is nevertheless true of both the WAG and VCL procedures that, if the groups were of *exact* equals (so that $G_{k,x} = 0$ for all k), then the counterfactual tax function would become precisely the one of AJL's model, and the respective decompositions would both reduce to AJL's. We can now understand why decomposition (11) would fail for close equals groups, were H^{WAG} to be calculated directly. Subtract the decomposition of G_N across close equals groups from G_X : $RE = [G_X - G_{B,n}] - \sum_k a_{k,n}G_{k,n} - R^{AJL}$. The middle term on the right is H^{WAG} as evaluated directly (see footnote 6), but the other two terms are certainly not V^{WAG} and R^{WAG} (the latter of which equals $R^{AJL} + R^{EG}$ from (12)). V^{WAG} and $[G_X - G_{B,n}]$ are, respectively, the redistributive effects of a tax which is lump sum within each close equals group, and one which completely equalizes the incomes of close equals.

[10] Just consider the VCL decomposition of RE_S for the counterfactual smoothed tax system. The "horizontal" term is $H_S^{VCL} = \sum_k (a_{k,n} - a_{k,x})G_{k,x}$ since the smoothed tax is distributionally neutral in each close equals group. But this counterfactual tax is HI-free by construction. Hence H_S^{VCL}, which is clearly non-zero in general, can hardly be regarded as a horizontal contribution to RE_S. The "horizontal" measure H^{WAG} fares little better. From (4), (6) and (7), $RE_S = V^{AJL} - R_S^{AJL} = [V - R_S^{EG}] - R_S^{AJL} = V - R_S^{WAG}$ (with no horizontal term). Yet also $RE_S = V_S^{WAG} - H_S^{WAG} - R_S^{WAG}$ in which H_S^{WAG} is certainly not zero.

[11] Indeed, were H^{WAG} to have been calculated directly for our illustrative dataset (see footnote 6), rather than obtained as the residual term needed to make (11) hold, then (11) would *not* have held: by direct calculation, $H^{WAG} = 0.011006$, much larger than any of the other horizontal terms (H^{VCL}, H^{AJL} and H), and violating (11).

the introduction of a counterfactual smooth tax function. Moreover, V and H can each be computed directly using the appropriate software (see on).

(2)	RE	=	V^{AJL}	–	H^{AJL}	–	R^{AJL}
	.045139	=	.056720	-	.006191	-	.005390
(9)	RE	=	V	–	H	–	R^{APK}
	.045139	=	.057203	-	.002217	-	.009847
(11)	RE	=	V^{WAG}	–	H^{WAG}	–	R^{WAG}
	.045139	=	.055034	-	.004022	-	.005873
(13)	RE	=	V^{VCL}	–	H^{VCL}	–	R^{AJL}
	.045139	=	.056806	-	.006277	-	.005390

TABLE 5. WAG AND VCL DECOMPOSITIONS, COMPARED WITH (2) AND (9), FOR THE ILLUSTRATIVE DATA OF TABLES 1 AND 2

4. Empirical application

As part of a research project on redistributional aspects of the Croatian personal income tax (henceforth PIT) and social security contribution (henceforth SSC), databases for personal income in 1997, 2001, and 2003 have been compiled.[12] They are 5% representative samples from the respective populations of PIT payers, containing, for each taxpayer: gross income by source, social security contributions paid by the employer and employee (these are imputed), personal income tax paid, and the amounts of allowances and deductions. The following analysis draws upon these databases. Many other results of the research project are given in Čok and Urban (2005).

The analysis of redistributive effect undertaken as part of that project showed that unequal treatments and rerankings caused by the PIT *alone* had a very limited effect, for example causing in 2003 a loss of under 3% of redistributive power. This was contrary to expectations, since the government had introduced 18 new PIT deductions between 2000 and 2002 – and additionally, the personal allowance for dependants had been raised. However, the loss of RE is somewhat higher if both the PIT and SSC are taken into consideration, as the results presented below will show. This is undoubtedly due to the fact that the SSC is not levied upon pensioners, who make up a significant portion of the observed population.

Pre-tax income X is defined as gross income before the PIT and SSC are imposed. Personal taxes T are given by the sum of the PIT and the SSC. We used income intervals ranging (on an annual basis) from 500 HRK to 5,000 HRK to define pre-tax close equals groups. The current exchange rate for the Croatian currency (the kuna) is EUR ≈ HRK

[12] These microdata were compiled with permission, help and support from the Tax Administration.

7.30 and USD ≈ 5.71 HRK. The average net monthly salary in Croatia is 4,000 HRK. Values for pre- and post-tax inequality and redistributive effect are not conditioned by the choice of bandwidth for close equals. We show these values for 2003, along with the vertical and reranking components of the Kakwani decomposition (which are also unaffected by the banding) in Table 6.

G_X	0.494241
G_N	0.431964
RE	0.062277
V^K	0.070020
R^{APK}	0.007743
V^K (% of RE)	112.4
R^{APK} (% of RE)	12.4

TABLE 6. MEASURES FOR 2003 NOT CONDITIONED BY CHOICE OF BANDWIDTH

Vertical redistribution, as measured by Kakwani's (1984) V^K, accounted for 112.4% of actual redistributive effect in 2003, with a 12.4% loss due to APK-reranking. When a bandwidth is selected in order to separate out the pure horizontal effect H from V^K (recall equations (9)-(10)), results now depend on the chosen bandwidth. Table 7 shows the relevant values, for various choices of the bandwidth, and expressed as percentages of either RE or R^{APK}. [13] Figures 1 and 2 show the same information graphically.

Bandwidth in HRK	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000
V (% of RE)	112.44	112.45	112.39	112.46	112.41	112.33	112.33	112.29	112.17	112.37
H (% of RE)	0.00	0.02	-0.04	0.03	-0.03	-0.11	-0.10	-0.15	-0.27	-0.06
V^{AJL} (% of RE)	112.11	112.27	112.32	112.36	112.40	112.32	112.33	112.28	112.16	112.37
H^{AJL} (% of RE)	0.42	0.81	1.19	1.53	1.89	2.20	2.49	2.78	3.04	3.35
R^{AJL} (% of RE)	11.69	11.46	11.13	10.83	10.51	10.12	9.84	9.50	9.12	9.01
R^{AJL} (% of R^{APK})	94.03	92.15	89.52	87.11	84.54	81.41	79.11	76.43	73.39	72.49
R^{EG} (% of R^{APK})	2.66	1.48	0.60	0.78	0.06	0.05	0.05	0.04	0.04	0.03
R^{WG} (% of R^{APK})	3.31	6.37	9.88	12.11	15.40	18.54	20.84	23.53	26.57	27.47
H^{AJL} (% of R^{APK})	3.35	6.51	9.57	12.33	15.19	17.68	20.05	22.35	24.43	26.96

TABLE 7. MEASURES CONDITIONED BY THE CHOICE OF BANDWIDTH

[13] We chose to calculate the decomposition components in MATLAB, using procedures which were purpose-built. Details are available on request from one of us (Urban).

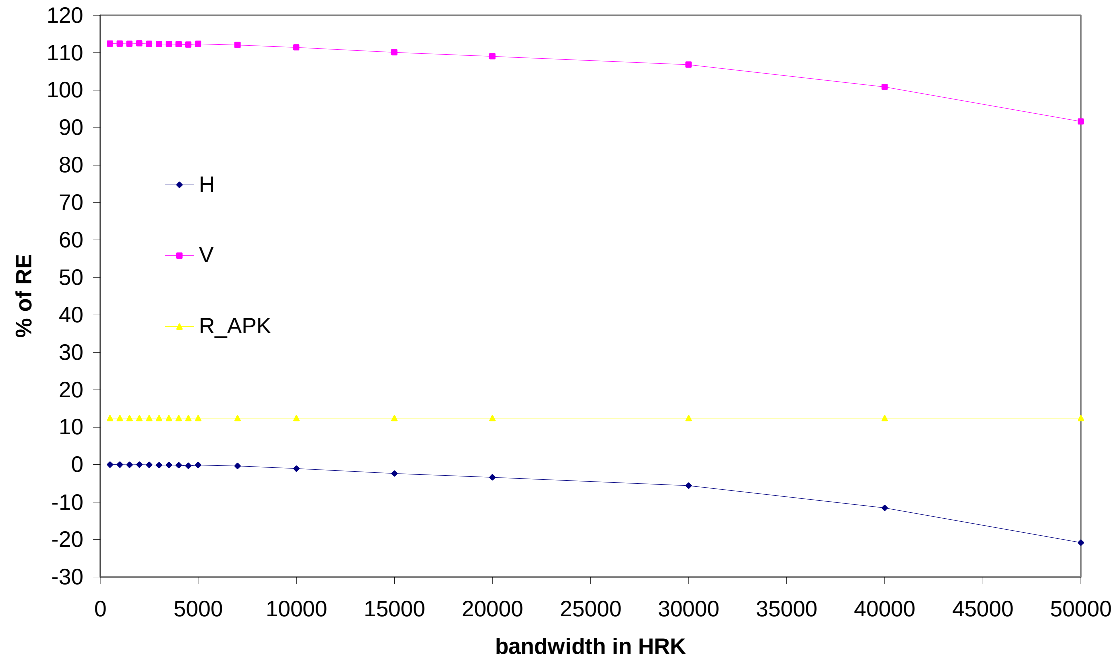


FIGURE 1 : VERTICAL, HORIZONTAL AND RERANKING EFFECTS (AS %'S OF RE) PLOTTED OVER A LARGE RANGE OF BANDWIDTHS FOR 2003

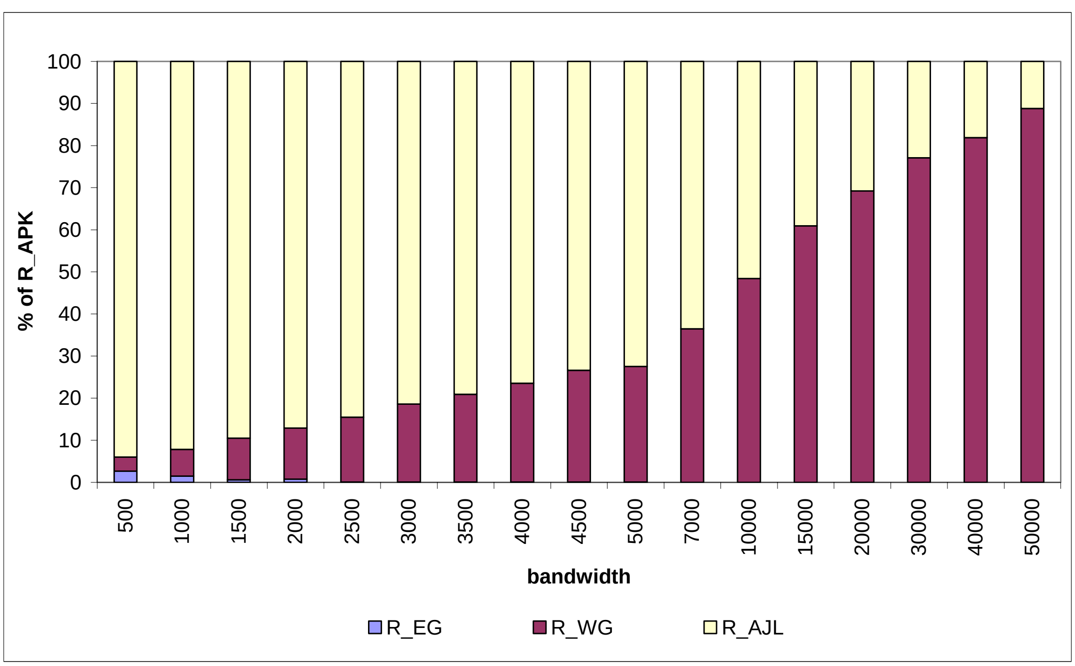


FIGURE 2 : COMPOSITION OF R^{APK} IN 2003 FOR DIFFERENT BANDWIDTHS

Notice that H is almost negligible as a percentage of RE for small bandwidths. The question of an optimal bandwidth for estimating the components in our decomposition (9) arises. For very large bandwidths, as Figure 1 shows, H is seriously underestimated, becoming large and negative. In van der Ven *et al.* (2001), it is suggested that the "logical" choice of bandwidth would be the one that maximizes the vertical component of redistributive effect. However, this conclusion is linked to the VCL decomposition and not our (9), and would, if taken literally, involve maximizing V^{VCL} and not V. In Figure 3, we show all four vertical measures, V, V^{AJL}, V^{WAG} and V^{VCL}, plotted against bandwidth for 2003. Clearly, the numerical differences can be very significant. From our Table 7, a bandwidth of approximately 2000 HRK would maximize V as a percentage of RE. Then $H \approx$ 0.03% of RE. Another finding is that AJL-reranking accounts only for a part of total reranking, and this part decreases from 94.03 to 72.49% as the bandwidth is raised. Evidently, as the bandwidth is widened, within-groups reranking rises;[14] and as is to be expected, entire-groups reranking falls.

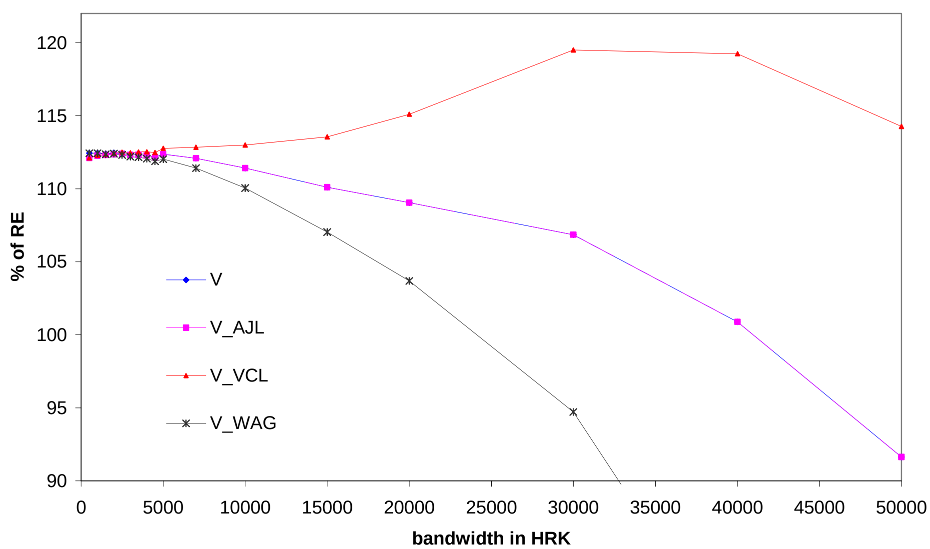
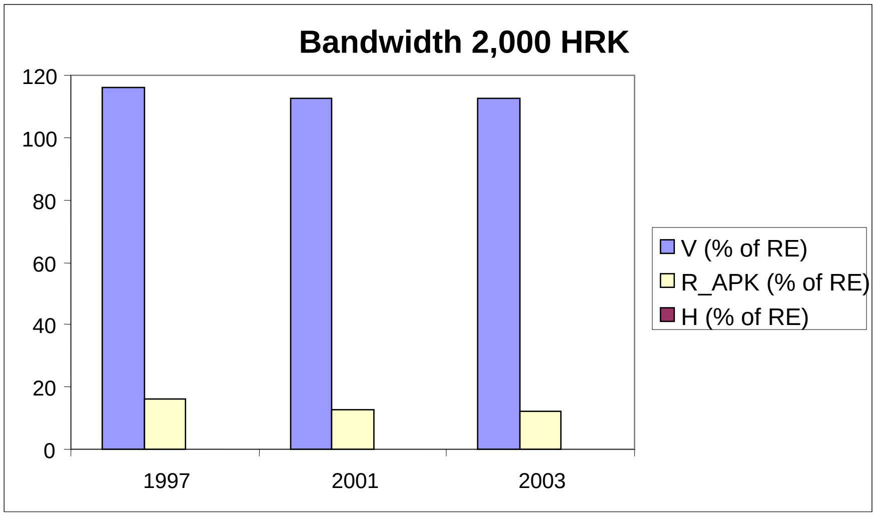


FIGURE 3 : V, V^{AJL}, V^{WAG} AND V^{VCL} PLOTTED AGAINST BANDWIDTH FOR 2003

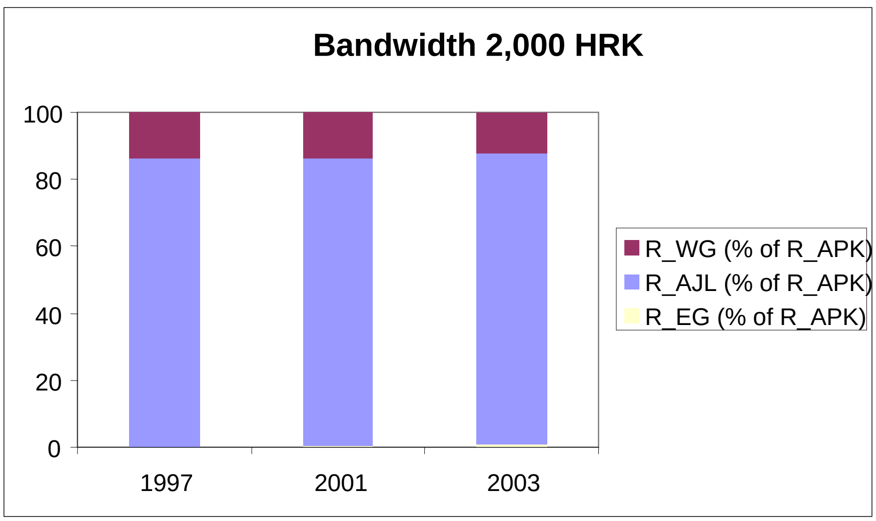
In Figure 4 we show the changing compositions of RE and R^{APK} over the period 1997-2003 covered by our data, for a bandwidth of 2000 HRK.[15] It is clear that the contribution of reranking R^{APK} to redistributive effect decreases slightly in importance through time, and that, within R^{APK}, there is a downward trend in R^{AJL}.

[14] From equation (8), we know the relationship between H^{AJL} and R^{WG}. Since H is very small, we may conclude that AJL's horizontal effect actually 'hides' the within-groups reranking within itself.

[15] The median values of pre-tax income were approximately 23,900 HRK, 26,900 HRK and 29,500 HRK in the years 1997, 2001, 2003.



(A) COMPOSITION OF *RE*



(B) COMPOSITION OF R^{APK}

FIGURE 4 : COMPOSITION OF *RE* AND R^{APK} IN 2003

We cannot let it pass without comment that the empirical values of H for the Croatian tax system throughout the period 1997-2003 are exceedingly small, and approach zero as the bandwidth is reduced to zero (reflecting the lack of exact equals in our samples). On the other hand, even in the limit, we can still distinguish three kinds of reranking (refer back to Figure 2). The 'no reranking' approach to capturing tax inequity arose at least in part in response to the sparsity or lack of exact equals in sample data (if not in the population). Does Kakwani's decomposition $RE = V^K - R^{APK}$ tell the whole story for such data, then?

We conclude our examination of the redistributive properties of the Croatian PIT and SSC with a deeper consideration of the horizontal effect *H*. If a researcher gets such a small value for *H*, how can this be used in the meaningful description of tax system properties? One answer, perhaps, has been anticipated already in footnote 7: what would have been counted as HI before, when following the AJL approach, is now, in the fully developed measurement system, actually within-groups reranking. The tiny magnitudes of *H* may, of course, be particular to the Croatian data; for our illustrative data set, *H* amounted to 4.9% of *RE*. The main difference between the illustrative data and real empirical data lies in the number of groups. In Table 2, the whole population was divided into only 6 groups, with consequently large intervals; in the empirical case, we have small intervals and many groups.

As an aside, we remark here that the horizontal effect according to our other measure, H^{AJL}, and those that would be obtained by following the WAG and VCL adaptations of the AJL methodology, also become very small as the bandwidth shrinks, although they are very different, and also different from H, for larger bandwidths. See Figure 5.

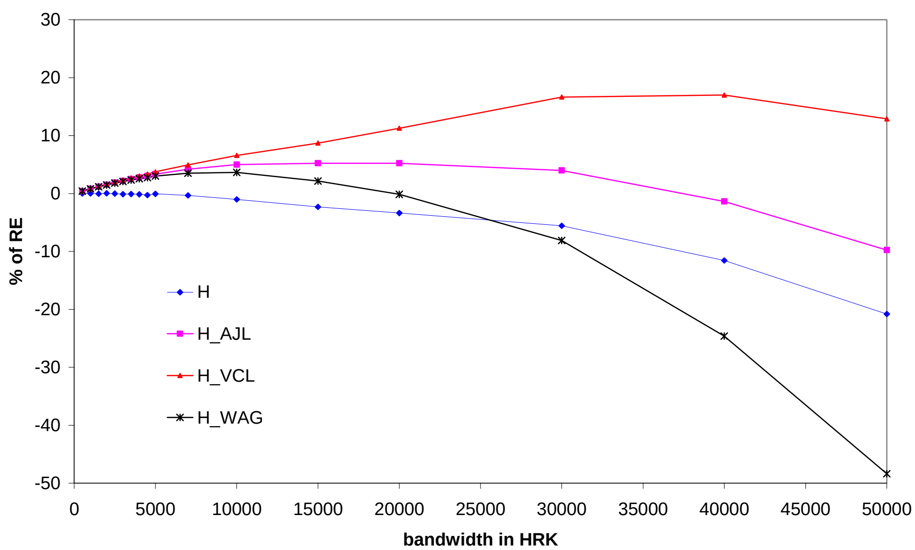


FIGURE 5 : H, H^{AJL}, H^{WAG} AND H^{VCL} IN TERMS OF THE BANDWIDTH IN 2003

It seems intuitive that, the larger the bandwidth, the greater will be the *absolute* deviations of post-tax incomes from the smoothed values. Although this is not (quite) what H measures, a small modification can make it so. If the concentration curves C_1 and C_5 cross, perhaps many times,[16] then there are areas for which C_1 is above C_5, and vice versa. These areas may 'cancel' each other, leading to a net value for H that may be very small and either positive or negative.[17] Suppose we break *H* into two parts, H^P being the

[16] The number of crossings may theoretically be equal to K-1, where K is number of groups.

[17] This effect cannot be observed with exact equals. By the way the vector N_5 is constructed, the counterfactual incomes in each group would be equal, as would the pre-tax incomes. Hence C_1 would lie below (or on) C_5 at all percentiles in this case.

sum of all areas where the horizontal effect is positive, and H^N being the sum of all areas where the horizontal effect is negative, such that $H = H^P + H^N$. Now define H^T as

(12) $H^T = H^P + abs\{H^N\} = H^P - H^N$

Referring back to the original rationale for H (in terms of person-by-person distances between post-tax incomes and reference values), we can see that H^T measures 'total HI' across members of equals groups in terms of *absolute* deviations of post-tax incomes from the counterfactual ones, which of course cannot cancel out.

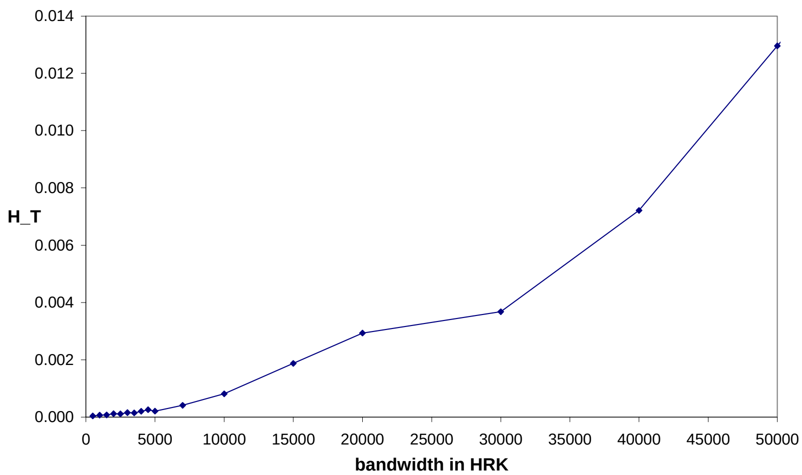


FIGURE 6 : "TOTAL HI", H^T, VERSUS BANDWIDTH IN 2003

As Figure 6 shows, the values of H^T for Croatia in 2003 indeed increase as the bandwidth is raised, exactly as the intuition outlined above would predict; and these values are decidedly non-negligible for small bandwidths (e.g. $H^T \approx 0.18\%$ of *RE* for a bandwidth of 2000 HRK, six times as big as *H* itself). Hence it could be misleading to interpret zero or near-zero values of H as "absence of horizontal inequity".[18]

5. Conclusions

Following the impetus provided by the papers of Aronson, Johnson and Lambert (1994) and Aronson and Lambert (1994), a number of authors have undertaken decompositions of redistributive effect into vertical, horizontal and reranking components. The 1994 papers laid out the appropriate (Gini-based) measurement theory for a model in which the pre-tax income distribution can be partitioned into exact pre-tax equals groups. The model assumes that the income tax system does not rerank equals groups; obviously it could not induce rerankings *within* equals groups (since all members of each group started equal). The methodology has, though, usually been applied in the context of *close equals groups,* not least because of the sparsity or absence of exact equals in typical sample data. When close equals groups are specified, typically it is found that reranking

[18] When we repeated the calculations for Table 2, using not 6 but 3 groups, entire-group reranking disappeared and H turned negative, but H^T increased. Its value rose to -11.7% of RE.

does occur within groups, and also entire groups may be reranked. The AJL methodology falls down in such a case, and needs to be rather carefully adapted.

In this paper, we have explained the careful modifications which must be made to the AJL measurement system when close equals groups are invoked by the practitioner. In the process, we have been able to finally reconcile a number of approaches to measuring equity in tax systems which have co-existed in the literature up until now, but have not been fitted into a unified framework. We refer principally to the approaches of Atkinson (1980), Plotnick (1981) and Kakwani (1984, 1986), but have also pointed to connections with the approaches of King (1983) and Jenkins (1988, 1994).

The adaptations of the AJL measurement system we were required to make for close equals group analysis were several. First, any tax system can induce three different forms of reranking between and within close equals groups, only one of which is possible for groups of exact equals. The adaptation had to identify and disentangle all three forms of reranking. Second, the vertical stance of the tax is characterized by AJL in terms of *the averaged tax,* where the averaging is done post-tax within each group of exact pre-tax equals. For the close equals group scenario, we needed to formulate a different counterfactual tax, one which "smoothed" the actual effect of the tax within each close equals group, and use this to express the vertical stance of the actual tax. Finally, in the AJL measurement system, horizontal inequity is captured as post-tax inequality introduced among pretax equals. Whilst it was possible to adapt this directly, and measure horizontal inequity in terms of pre- and post-tax inequality comparisons group by group, a far more suitable adaptation for us, the one which allowed us to complete our measurement system, rested upon person-by-person comparisons of actual and smoothed post-tax incomes within close equals groups.

We explained the new measurement system fully by reference to the artificial dataset introduced in Section 2 to illustrate the three different forms of reranking that can be caused by a tax system when close equals groups are specified. Our recommendation for future users of the AJL approach is this. Having selected a bandwidth for close equals, decompose redistributive effect according to (9), $RE = V - H - R^{APK}$, and use (3), $R^{APK} = R^{AJL} + R^{WG} + R^{EG}$, to ascertain the percentage contributions of each form of reranking.[19] Now repeat the calculations for different bandwidths, and either focus upon the one that maximizes V, or, better, produce plots that show the dependences of the various contributions upon the bandwidth selected. We have one further suggestion. This stems from our detailed analysis of the vertical, horizontal and reranking characteristics of the Croatian direct tax system using the new methodology. In the face of vanishingly small values for H, the analyst may have recourse to the "total horizontal inequity" measure H^T, which we have also defined, to check whether canceling (of deviations of actual post-tax incomes from counterfactual ones, in close equals groups) is rendering the H value extremely small.

[19] Once the terms in (9) and (3) have all been evaluated, V^K can be computed from (10), and V^{AJL} and H^{AJL} from (7) and (8) respectively. Hence the terms in decompositions (9) and (3) contain all the information needed to arrive at the Kakwani (1984) and AJL decompositions given in (1) and (2), should these be desired.

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unit	X	r_X	N	r_N
#1	57.13	1	47.27	1
#2	61.87	2	51.42	2
#3	68.61	3	61.75	4
#4	100.36	4	93.50	7
#5	121.59	5	55.68	3
#6	125.50	6	77.44	5
#7	125.50	7	88.50	6
#8	125.50	8	100.40	8
#9	125.50	9	112.95	9
#10	160.61	10	151.59	12
#11	189.49	11	173.18	17
#12	213.41	12	170.73	16
#13	213.43	13	128.49	10
#14	217.81	14	152.47	13
#15	219.42	15	153.59	14
#16	232.44	16	162.71	15
#17	247.40	17	194.81	22
#18	264.12	18	184.88	19
#19	278.30	19	149.40	11
#20	309.37	20	185.62	20
#21	321.17	21	180.58	18
#22	321.17	22	192.70	21
#23	321.17	23	200.34	25
#24	326.91	24	196.15	23
#25	333.37	25	200.02	24
#26	338.36	26	203.02	26

K_1	→	K_3	→	K_4	→	K_2
#1		#1		#1		#1
#2		#2		#2		#2
#3		#3		#3		**#5**
#4		#5		#5		**#3**
#5		#6		#6		#6
#6		#7		#7		#7
#7		#4		#4		#4
#8		#8		#8		#8
#9		#9		#9		#9
#10		#10		#13		#13
#11		#11		#14		**#19**
#12		#13		#15		**#10**
#13		#14		#16		#14
#14		#15		#12		#15
#15		#16		#17		#16
#16		#12		#10		#12
#17		#17		#11		**#11**
#18		#19		#19		**#21**
#19		#18		#18		#18
#20		#21		#21		#20
#21		#20		#20		#22
#22		#22		#22		**#17**
#23		#24		#24		#24
#24		#25		#25		#25
#25		#23		#23		#23
#26		#26		#26		#26

Between groups reranking affected these income units:
#4, #5, #6, #7; #12, #13, #14, #15, #16; #18, #19; #20, #21, #23, #24, #25
Entire groups reranking affected these income units:
#10, #11, #12, #13, #14, #15, #16, #17
AJL reranking affected these income units:
#3, #5, #10, #11, #17, #19, #21

TABLE 1. INTRODUCING RERANKING IN STAGES

p	X	L_X	N_1	C_1	N_2	L_N	N_3	C_3	N_4	C_4	g	N_5	C_5	N_6	C_6	N_S	L_{NS}
0.0385	57.13	0.0105	47.27	0.0129	47.27	0.0129	47.27	0.0129	47.27	0.0129	0.1448	48.86	0.0133	48.86	0.0133	48.86	0.0133
0.0769	61.87	0.0220	51.42	0.0269	51.42	0.0269	51.42	0.0269	51.42	0.0269	0.1448	52.91	0.0277	52.91	0.0277	52.91	0.0277
0.1154	68.61	0.0346	61.75	0.0437	55.68	0.0421	61.75	0.0437	61.75	0.0437	0.1448	58.67	0.0437	58.67	0.0437	58.67	0.0437
0.1538	100.36	0.0531	93.50	0.0692	61.75	0.0589	55.68	0.0589	55.68	0.0589	0.2700	73.26	0.0637	73.26	0.0637	73.26	0.0637
0.1923	121.59	0.0756	55.68	0.0844	77.44	0.0800	77.44	0.0800	77.44	0.0800	0.2700	88.76	0.0879	88.76	0.0879	88.76	0.0879
0.2308	125.50	0.0987	77.44	0.1055	88.50	0.1041	88.50	0.1041	88.50	0.1041	0.2700	91.61	0.1129	91.61	0.1129	91.61	0.1129
0.2692	125.50	0.1219	88.50	0.1296	93.50	0.1296	93.50	0.1296	93.50	0.1296	0.2700	91.61	0.1378	91.61	0.1378	91.61	0.1378
0.3077	125.50	0.1450	100.40	0.1570	100.40	0.1570	100.40	0.1570	100.40	0.1570	0.2700	91.61	0.1628	91.61	0.1628	91.61	0.1628
0.3462	125.50	0.1682	112.95	0.1878	112.95	0.1878	112.95	0.1878	112.95	0.1878	0.2700	91.61	0.1878	91.61	0.1878	91.61	0.1878
0.3846	160.61	0.1978	151.59	0.2291	128.49	0.2228	151.59	0.2291	128.49	0.2228	0.0724	148.99	0.2284	152.89	0.2294	148.99	0.2284
0.4231	189.49	0.2328	173.18	0.2763	149.40	0.2635	173.18	0.2763	152.47	0.2643	0.0724	175.78	0.2763	152.90	0.2711	152.89	0.2700
0.4615	213.41	0.2722	170.73	0.3228	151.59	0.3048	128.49	0.3113	153.59	0.3062	0.2836	152.89	0.3179	156.04	0.3136	152.90	0.3117
0.5000	213.43	0.3116	128.49	0.3578	152.47	0.3464	152.47	0.3528	162.71	0.3505	0.2836	152.90	0.3596	157.20	0.3565	156.04	0.3542
0.5385	217.81	0.3517	152.47	0.3994	153.59	0.3882	153.59	0.3947	170.73	0.3971	0.2836	156.04	0.4021	166.52	0.4019	157.20	0.3971
0.5769	219.42	0.3922	153.59	0.4412	162.71	0.4326	162.71	0.4390	194.81	0.4502	0.2836	157.20	0.4450	177.24	0.4502	162.77	0.4414
0.6154	232.44	0.4351	162.71	0.4856	170.73	0.4791	170.73	0.4856	151.59	0.4915	0.2836	166.52	0.4904	148.99	0.4908	166.52	0.4868
0.6538	247.40	0.4808	194.81	0.5387	173.18	0.5263	194.81	0.5387	173.18	0.5387	0.2836	177.24	0.5387	175.78	0.5387	171.51	0.5336
0.6923	264.12	0.5295	184.88	0.5891	180.58	0.5755	149.40	0.5794	149.40	0.5794	0.3837	162.77	0.5830	162.77	0.5830	175.78	0.5815
0.7308	278.30	0.5809	149.40	0.6298	184.88	0.6259	184.88	0.6298	184.88	0.6298	0.3837	171.51	0.6298	171.51	0.6298	177.24	0.6298
0.7692	309.37	0.6379	185.62	0.6804	185.62	0.6765	180.58	0.6790	180.58	0.6790	0.4020	185.01	0.6802	185.01	0.6802	185.01	0.6802
0.8077	321.17	0.6972	180.58	0.7296	192.70	0.7290	185.62	0.7296	185.62	0.7296	0.4020	192.07	0.7325	192.07	0.7325	192.07	0.7325
0.8462	321.17	0.7565	192.70	0.7821	194.81	0.7821	192.70	0.7821	192.70	0.7821	0.4020	192.07	0.7849	192.07	0.7849	192.07	0.7849
0.8846	321.17	0.8157	200.34	0.8367	196.15	0.8356	196.15	0.8356	196.15	0.8356	0.4020	192.07	0.8372	192.07	0.8372	192.07	0.8372
0.9231	326.91	0.8761	196.15	0.8902	200.02	0.8901	200.02	0.8901	200.02	0.8901	0.4020	195.50	0.8905	195.50	0.8905	195.50	0.8905
0.9615	333.37	0.9376	200.02	0.9447	200.34	0.9447	200.34	0.9447	200.34	0.9447	0.4020	199.36	0.9449	199.36	0.9449	199.36	0.9449
1.0000	338.36	1.0000	203.02	1.0000	203.02	1.0000	203.02	1.0000	203.02	1.0000	0.4020	202.35	1.0000	202.35	1.0000	202.35	1.0000

TABLE 2. ORDERINGS, CONCENTRATION CURVES AND LORENZ CURVES FOR THE ILLUSTRATIVE DATA OF TABLE 1