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CONFIGURATION-DEPENDENT INTERPOLATION IN 3D BEAM ELEMENTS OF ARBITRARY ORDER

Gordan Jelenić

University of Rijeka
Department of Civil Engineering
Viktora Cara Emina 5, Rijeka
Republic of Croatia
gordan.jelenic@gradri.hr

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ABSTRACT

In non-linear 3D beam theory with rotational degrees of freedom [1], a configuration-dependent interpolation of the rotational degrees of freedom appears to be indispensable when tackling the problem of providing a result which is independent of the choice of the beam reference line [2] or invariant to rigid-body rotation [3]. For two-noded beam elements, the two problems turn out to complement each other and the attempts to solve them independently led to the same resulting interpolation for the rotational degrees of freedom [4].

An important consequence of the invariance with respect to the choice of the beam reference line is also a configuration-dependent interpolation of the position vector to the beam centroidal axis – a very useful result which considerably contributes to accuracy of the solution and completely eliminates the problem of shear locking. In contrast, in order to avoid this anomaly in the formulations with the standard polynomial interpolation for positions, the use of reduced integration is mandatory. Furthermore, the configuration-dependent interpolation of the position vector obtained in is in fact identical to the interpolation applied to the rotational degrees of freedom and such an interpolation is called the helicoidal interpolation [2]. In spite of it being configuration-dependent, this interpolation inherits the completeness properties of the standard Lagrangian interpolation. In linear analysis, the resulting linear helicoidal interpolation coincides with the theoretical solution for the Timoshenko beam element with constant moment loading.

In this work, the relationships between the two different aspects of invariance is further analysed with a view to providing a methodology for developing higher-order interpolation. To this end it has to be noted that the work given in [2] is limited to two-noded element and a generalisation of this approach to higher-order elements does not appear to be immediately obvious. In this context, an alternative approach is proposed based on the the facts that (i) according to [2] the configuration-dependent interpolation of positions and rotations is one and the same and (ii) while no higher-order generalisation of the helicoidal interpolation is immediately applicable, such a generalisation exists for the rotational degrees of freedom alone [3,5].

Combination of these two premises leads to a higher-order configuration-dependent interpolation for both the position and the rotation field variable. In this work we analyse why, in contrast to two-noded elements, in linear analysis such an interpolation does not coincide with the theoretical result. A simple intervention which corrects this anomaly is presented by studying the behaviour of a linearised problem [6]. A more complete account of this behaviour is gven in a companion cnontribution [7].

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