Protective Predictive Control of Electrical Drives with Elastic Transmission

Mario Vašak, Nedjeljko Perić
Faculty of Electrical Engineering and Computing
University of Zagreb
Zagreb, Croatia
Email: mario.vasak@fer.hr, nedjeljko.peric@fer.hr

***Abstract*— In this paper we propose a protective predictive control scheme for motor drives with elastic transmission that are subject to physical and safety constraints on their variables. Namely, we extend the classical LQR controller with a safety set obtained using invariant sets methodology. The easy implementation of this set on-line allows the correction of the LQR control signal in order to suppress the violation of constraints and thus avoid possibly dangerous situations during the drive transients. The added protective algorithm can be also used with any other drive control scheme to correct its outputted control signal.**

***Keywords*— electrical drive with elastic transmission, LQR, constraints, the maximum controlled invariant set**

I. Introduction

Quality control of electrical drives is a necessity in many manufacturing processes and many services. The reasons for that are: the outcomes quality, power savings, drive safety, prolongation of the exploitable life of the drive etc. There is a significant number of applications where the rotor of the drive and the load rotational mass are coupled with a shaft whose torsional characteristics may not be neglected during the drive control system synthesis. In such situations a satisfactory control cannot be attained using classical cascade control system with PI controllers solely due to the occurring low-damped torsional vibrations of the shaft. To suppress the vibrations the control algorithm should be more sophisticated.

Electrical drives with elastic transmission can be well controlled using polynomial controllers [1], state-space controllers [2], [3] or controllers based on neural networks and fuzzy logic [4], usually coupled with observers for hard-to-measure drive variables. However, the synthesis of such controllers does not take into account the physical and safety constraints on variables in the drive, e.g. limitations of the available motor torque and the maximal allowable torsion of the elastic shaft. Not only that disregarding those limitations in operation may outwear the drive, but may also lead to unpredicted and possibly dangerous behavior of the control system.

In order to cope with the mentioned problems in this paper the standard drive control algorithm is extended. This extension is actually a protection algorithm which guarantees that the electrical drive system does not violate the constraints imposed on its variables. The electrical drive with elastic transmission, including the load torque effect, is modeled using a linear state-space model with the states: rotor and load speed, shaft torsional angle, the drive and the load torque. This model is extended with the reference speed of the load rotational mass. Based on the model we design a classical linear quadratic tracking controller (LQR) with a linear observer of the model states.

The existing physical and safety constraints for the drive at hand form bounded polytopic sets of the admissible model states and inputs. Guarantees on imposed constraints satisfaction are introduced by off-line computing the largest controlled invariant set [5] within the set of admissible states. In simple words, controlled invariant set is a set of system states where for each state in it there exists a feasible control input that keeps the successor state in that set. Furthermore, based on the computed controlled invariant set it is straightforward for a certain state to compute on-line the interval of feasible control inputs that allow keeping of the state in the set forever according to the model and input constraints. The mentioned computations allow implementation of a drive controller with an additional protection of the control system. Namely, the control signal obtained for a certain drive state by the drive controller (LQR or some other) is checked whether it matches the interval of feasible control inputs for that state. If the computed input lies outside the interval, it is replaced with the closest value from the interval and otherwise is left unchanged. The presented comparison between the drive control algorithm without protection (LQR) and with protection indicates that practically all violations of the constraints are suppressed for any transient due to the reference speed change. The facts that the considered protection scheme is rather easy to compute and implement, and that it can be also combined with any other drive control scheme, are also very motivating.

The paper is organized as follows. After this introductory section, Section II outlines the state-space model of the drive with an elastic shaft between its rotor and the load mass. Section III presents the

978-1-4244-1742-1/08/$25.00 © 2008 IEEE

drive control system design and Section IV gives a comparison of the electrical drive system behavior with and without the proposed controlled invariant set based protection. The paper contributions are summarized in the concluding Section V.

II. State-space model of the drive with elastic transmission

The electrical drive with an elastic transmission between the rotor and the load mass is modeled as a two-mass elastic system [6], see Fig. 1.

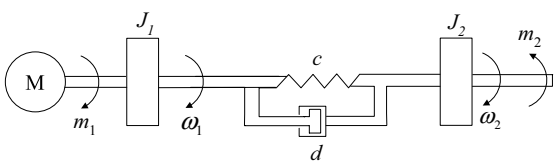

Fig. 1. Functional scheme of the considered electrical drive

It is assumed that the inner current control loop (the motor torque control loop) can be modeled as a first-order lag system, such that the following relation holds between the reference torque signal $m_{1,R}$ and the actual torque m_1:

$$\dot{m}_1 = \frac{1}{T_i}(m_{1,R} - m_1), \tag{1}$$

where T_i is the equivalent time constant of the current control loop. The variables involved in the mechanical part of the drive, speed of the rotor ω_1, speed of the load mass ω_2 , the shaft torsion angle ψ and the load torque m_2, are connected with the following relations:

$$\begin{aligned} \dot{\omega}_1 &= \frac{1}{J_1}\left(m_1 - c\psi - d(\omega_1 - \omega_2)\right), && (2)\\ \dot{\omega}_2 &= \frac{1}{J_2}\left(c\psi + d(\omega_1 - \omega_2) - m_2\right), && (3)\\ \dot{\psi} &= \omega_1 - \omega_2, && (4) \end{aligned}$$

where c is the shaft stiffness, d is the damping coefficient, J_1 and J_2 are, respectively, the rotor and the load mass moments of inertia. Normalization parameters M_n, Ω_n and Ψ_n are introduced (see Table I) together with the following normalized variables: $\bar{\omega}_i = \omega_i/\Omega_n$, $\bar{m}_i = m_i/M_n$, $\bar{m}_{1,R} = m_{1,R}/M_n$, $\bar{\psi} = \psi/\Psi_n$ $(i = 1, 2)$. The normalization of variables is used due to numerical reasons in further computations with the model and due to easier presentation of the results. Parameters M_n and Ω_n are also the rated torque and speed of the drive, respectively.

The following model with normalized variables is obtained:

TABLE I

Parameters for the considered electrical drive

Symbol	Value
M_n	14.8 Nm
Ω_n	210 $\frac{\text{rad}}{\text{s}}$
Ψ_n	$\frac{5\pi}{180}$ rad
$T_{M1} = \frac{M_n}{\Omega_n J_1}$	0.147 s
$T_{M2} = \frac{M_n}{\Omega_n J_2}$	0.241 s
$T_\psi = \frac{\Psi_n}{\Omega_n}$	0.42 ms
$\bar{c} = \frac{c\Psi_n}{M_n}$	900
$\bar{d} = \frac{d\Omega_n}{M_n}$	0.7

$$\begin{aligned} \dot{\bar{\omega}}_1 &= \frac{1}{T_{M1}}\left(\bar{m}_1 - \bar{c}\bar{\psi} - \bar{d}\bar{\omega}_1 + \bar{d}\bar{\omega}_2\right), && (5)\\ \dot{\bar{\omega}}_2 &= \frac{1}{T_{M2}}\left(\bar{c}\bar{\psi} + \bar{d}\bar{\omega}_1 - \bar{d}\bar{\omega}_2 - \bar{m}_2\right), && (6)\\ \dot{\bar{\psi}} &= \frac{1}{T_\psi}\left(\bar{\omega}_1 - \bar{\omega}_2\right), && (7)\\ \dot{\bar{m}}_1 &= \frac{1}{T_i}\left(\bar{m}_{1,R} - \bar{m}_1\right), && (8) \end{aligned}$$

where T_{M1}, T_{M2}, T_ψ, $\bar{c}$ and $\bar{d}$ are given in Table I. The given parameters stem from the experimental setup of the soft-coupled electrical drive described in [7].

The predictive reference tracking control law computation also requires information about the evolution of the load torque and the reference signal $\bar{\omega}_{2,R} = \omega_{2,R}/\Omega_n$. Since we do not know their behavior in the future, we simply presume they remain unchanged:

$$\begin{aligned} \dot{\bar{m}}_2 &= 0, && (9)\\ \dot{\bar{\omega}}_{2,R} &= 0. && (10) \end{aligned}$$

Using the notation

$$x = \begin{bmatrix} \bar{\omega}_1 & \bar{\omega}_2 & \bar{\psi} & \bar{m}_2 & \bar{\omega}_{2,R} \end{bmatrix}^T, \quad u = \bar{m}_{1,R} \tag{11}$$

we can simply rewrite (5)–(10) as

$$\dot{x} = Ax + Bu, \tag{12}$$

where the structure of A and B can be deduced from (5)–(11). We denote the reference tracking error as

$$y = \bar{\omega}_{2,R} - \bar{\omega}_2 = Cx, \tag{13}$$

where $C = [0 \ \ -1 \ \ 0 \ \ 0 \ \ 1]$. For the considered soft-coupled drive an appropriate sampling time is $T = 5$ ms [1]. Discretizing the process (12) leads to the discrete-time state-space model

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k), && (14)\\ y(k) &= Cx(k), && (15) \end{aligned}$$

where $x(k)$, $u(k)$ and $y(k)$ are short notations for $x(kT)$, $u(kT)$ and $y(kT)$, respectively. Model (14)–(15) is the starting point for the computation of the predictive drive control strategy which is detailed in the sequel.

III. Design of the drive control system

The goal of the drive control system is to provide good load speed tracking of the reference speed. However, during its operation the control system should respect the physical and safety constraints on the electrical drive variables. Since the produced motor torque cannot be larger than its maximum available value considering the drive at hand (for example 120% of the rated torque), we pose the physical constraints

$$|\bar{m}_1| \leq 1.2, \tag{16}$$

$$|\bar{m}_{1,R}| \leq 1.2. \tag{17}$$

Constraint (17) incorporates also the physical limits of the drive inverter (maximum available inverter current rate). We introduce safety limits on the speeds of the rotor and the load mass (the boundary values are for example 110% of the rated speed):

$$|\bar{\omega}_1| \leq 1.1, \tag{18}$$

$$|\bar{\omega}_2| \leq 1.1. \tag{19}$$

Very important for long-life of the mechanical parts of the drive is to prevent excessive shaft twist from the steady-state value $\bar{\psi}_0 = \frac{\bar{m}_2 M_n}{c\Psi_n}$ that may occur during drive transients. Thus the constraint

$$|\bar{\psi} - \bar{\psi}_0| \leq 3 \tag{20}$$

is introduced, which forbids the shaft to twist more than 15° above and below the steady-state value. Finally it is assumed that the reference speed is always less or equal than rated and that the load torque is not greater than 110% of the rated torque:

$$|\bar{\omega}_{2,R}| \leq 1, \tag{21}$$

$$|\bar{m}_2| \leq 1.1. \tag{22}$$

Constraints on the drive variables are described with the following polytopes

$$\mathcal{P}^x = \{x | C^x x \leq C^1\} \subset \mathbb{R}^6, \tag{23}$$

$$\mathcal{P}^u = \{u | C^u u \leq C^2\} \subset \mathbb{R}^1, \tag{24}$$

where matrices C^x and C^1 follow from (11), (16) and (18)-(22), while C^u and C^2 follow from (11) and (17).

In the first step of the drive control system synthesis the existing constraints (16)-(22) are disregarded and and an LQR-based full-state feedback controller [8] for the motor drive presented in the previous section is designed. Besides the tracking error penalization, in the corresponding LQR cost function introduced are also the deviation of the normalized torsion angle $\bar{\psi}$ from the steady state value $\bar{\psi}_0$, as well as the deviation of the normalized reference torque $\bar{m}_{1,R}$ from $\bar{m}_2$. The cost function has the form

$$\begin{aligned} J = & \sum_{k=0}^{\infty} \big(1000(\bar{\omega}_{2,R}(k) - \bar{\omega}_2(k))^2 + \\ & +5(\bar{\psi}(k) - \bar{\psi}_0(k))^2 + (\bar{m}_{1,R}(k) - \bar{m}_2(k))^2\big) = \\ = & \sum_{k=0}^{\infty} \left(x^T(k)Qx(k) + u^T(k)Ru(k) + 2x^T(k)Nu(k)\right). \end{aligned} \tag{25}$$

The coefficients 1000, 5 and 1 by the three penalization terms in (25) are chosen to enforce fast speed tracking with low torsional vibrations. Note that due to (9) $\bar{\psi}_0(k)$ and $\bar{m}_2(k)$ are actually constant during predictions based on the model (14), and that analog reasoning applies to $\bar{\omega}_{2,R}(k)$ due to (10):

$$\begin{aligned} &\bar{\psi}_0(k) = \bar{\psi}_0(0),\ \bar{m}_2(k) = \bar{m}_2(0),\ \forall\ k > 0, \\ &\bar{\omega}_{2,R}(k) = \bar{\omega}_{2,R}(0),\ \forall\ k > 0. \end{aligned} \tag{26}$$

The minimization of J over $u(k) = \bar{m}_{1,R}(k)$ subject to the dynamics (14) leads to the control law

$$u(0) = K_{LQR}x(0) \tag{27}$$

with

$$K_{LQR} = [-33.91\ 15.77\ -0.56\ -0.84\ 3.33\ 18.14]. \tag{28}$$

The dominant poles of the closed-loop control system are placed at 0.96 and 0.86 ± j0.47 in the complex z-plane.

Since LQR control requires feedback information from all states, its implementation requires synthesis of an observer used for estimation of hard-to-measure drive states, see e.g. [2] or [9]. In our problem setup we assume that the drive states $\bar{\omega}_1$, $\bar{\omega}_2$, $\bar{\psi}$ and $\bar{m}_1$ can be measured, while the load torque $\bar{m}_2$ needs to be estimated. We employ a full-state linear current observer [10]:

$$\begin{aligned} &\hat{z}(k|k-1) = A_o\hat{z}(k-1|k-1) + B_o u(k-1), \\ &\hat{z}(k|k) = \hat{z}(k|k-1) + K_o(y_m(k) - C_o\hat{z}(k|k-1)), \end{aligned} \tag{29}$$

where

$$\begin{aligned} z &= [\bar{\omega}_1\ \bar{\omega}_2\ \bar{\psi}\ \bar{m}_1\ \bar{m}_2]^T, \\ y_m &= [\bar{\omega}_1\ \bar{\omega}_2\ \bar{\psi}\ \bar{m}_1]^T, \end{aligned} \tag{30}$$

$\hat{z}(k_1|k_2)$ is the estimate of z in step k_1 based on information until the moment k_2, A_o and B_o are simply formed of the corresponding rows of A_d and B_d in (14), C_o extracts values corresponding to y_m from z and K_o is selected such that the observer dominant pole is placed at 0.5 in the complex z-plane. The state feedback $x(k)$ for LQR is taken from $\hat{z}(k|k)$ and from given $\bar{\omega}_{2,R}(k)$.

The control strategy designed so far does not take into account the constraints (16)–(22) which can be very harmful for the drive. In the second design step additionally the maximum controlled invariant set $\mathcal{I}$ inside the polytope $\mathcal{P}^x$ for the system (14) subject to constraint (17) is computed off-line. The set $\mathcal{I}$ contains all the states for which there exists an admissible control input according to (17) such that the successor state is also in $\mathcal{I}$ and is the largest of all the sets in $\mathcal{P}^x$ with that property. In other words, $\mathcal{I}$ contains all the states from which there exist transients in the future that do not violate given constraints. The computational scheme for obtaining $\mathcal{I}$ is the following [5]:

1) Initialize $\mathcal{T} := \mathcal{P}^x$;
2) Compute $\mathcal{T}^+ := \{x |\ \exists u : [x\ u]^T \in \mathcal{P}_\mathcal{T}\} \cap \mathcal{P}^x$ where $\mathcal{P}_\mathcal{T} = \{[x\ u]^T |\ A_d x + B_d u \in \mathcal{T},\ u \in \mathcal{P}^u\}$;

3) If $\mathcal{T}^+ = \mathcal{T}$ set $\mathcal{I} := \mathcal{T}$, else $\mathcal{T} := \mathcal{T}^+$, and goto step 2.

Convergence of the given procedure occurs in our problem setup, but unfortunately for a general linear model with constraints there is no guarantee that it ends in finite time. Finally, for on-line implementation the polytopic set

$$\begin{aligned}\mathcal{P}_{\mathcal{I}} &= \{[x\ u]^T |\ A_d x + B_d u \in \mathcal{I},\ u \in \mathcal{P}^u\} = \\ &= \{[x\ u]^T | H_{\mathcal{I}} x + L_{\mathcal{I}} u \leq K_{\mathcal{I}}\}\end{aligned} \tag{31}$$

is needed. For any measured $x \in \mathcal{I}$ the interval $\mathcal{U}_{\mathcal{I}}(x) = \{u | L_{\mathcal{I}} u \leq K_{\mathcal{I}} - H_{\mathcal{I}} x\}$ is determined. It contains all feasible control inputs u that can be applied to the drive in order for the transient that starts in x not to violate the constraints in any future time.

We also introduce some additional measures to guarantee certain level of robustness in constraints satisfaction for this control scheme. We assume that any of $\bar{\omega}_1$, $\bar{\omega}_2$, $\bar{\psi}$, $\bar{m}_1$ and $\bar{m}_2$ may deviate by ± 0.01 due to measurement noise or estimation error. We then account for that such that the set $\mathcal{I}$ is made smaller by implementing the so-called Minkowski difference [11]:

$$\tilde{\mathcal{I}} := \mathcal{I} \ominus \mathcal{W} = \{x \in \mathcal{I} |\ x + w \in \mathcal{I}\ \forall w \in \mathcal{W}\}, \tag{32}$$

where $\mathcal{W}$ is a lower-dimensional hypercube defined as:

$$\mathcal{W} = [-0.01, 0.01]^5 \times \{0\}. \tag{33}$$

The overall control algorithm performed at each time-step on-line is as follows:

1) Compute $u' = K_{LQR} x$;
2) Compute $\mathcal{U}_{\tilde{\mathcal{I}}}(x)$ from off-line computed matrices $H_{\tilde{\mathcal{I}}}$, $L_{\tilde{\mathcal{I}}}$, $K_{\tilde{\mathcal{I}}}$;
3) If $u' \in \mathcal{U}_{\tilde{\mathcal{I}}}(x)$ implement it, otherwise implement the edge-value of $\mathcal{U}_{\tilde{\mathcal{I}}}(x)$ that is closer to u'.

The steps 1) and 3) of the listed on-line procedure require negligible amount of processing time compared to the step 2). Implementation of the step 2) on a DSP is analyzed in the sequel.

The obtained polytope $\mathcal{P}_{\tilde{\mathcal{I}}}$ in our problem setup consists of 288 constraints and for measured x we thus need $288 \cdot 6 = 1728$ multiple-accumulate instructions to compute the feasible interval $\mathcal{U}_{\tilde{\mathcal{I}}}(x)$ defined by its boundary values. If it is required that the algorithm for computing u ends in one tenth of the sampling time, i.e. in 0.5 ms, and if we assume that a multiple-accumulate instruction takes a single instruction cycle on a DSP, the DSP clock frequency should be greater than 1728/0.5 ms = 3.46 MHz.

It should be noted that the step 1) of the on-line procedure is not bound to LQR or any other specific control strategy, i.e. the proposed protective predictive control scheme can accompany also polynomial or neuro-fuzzy drive control strategies with a mandatory state-observer.

IV. Simulation results

In this section we present the simulation results obtained on the considered drive model, using LQR only and LQR extended with the derived protective predictive control strategy based on invariant sets, see Fig. 2. The normalized reference signal and load torque have rectangle shapes of different frequencies, both with the amplitude equal 0.5. Without using the computed protection (from 0 to 8.5 s) it may be observed that in the transient due to reference change significant violation of the safety constraint on the shaft twist ψ may occur. However with the proposed predictive protection (from 8.5 s on) such behavior is suppressed and the constraints violation during the transients prevented. The transients in both modes, with and without protection, last comparably long which indicates that the introduced protection scheme does not deteriorate the drive control system performance regarding the reference speed tracking.

Another testing of the protective predictive control scheme is performed in Fig. 3. Here the speed reference abruptly changes from the full rated speed to the full rated reversed speed. The load torque abruptly changes between -0.4 and 0.4 asynchronously with the speed. Again the protective predictive strategy eliminates almost all constraints violations. The only non-eliminated violation happens at 19.01 s when the unpredictable load torque change induced abrupt change in the allowed torsion angle interval.

Fig. 4 gives another interesting insight into the performance of the protective algorithm since it indicates how often is the protection activated for the transients given in Fig. 3. Level 0 indicates that LQR signal is outputted by the controller, level 1 indicates activation of protection and level 2 indicates that violation occurs. It is obvious that for the presented large and abrupt reference and load torque changes the protection scheme is rather often activated, i.e. rather often limits the LQR computed control signal in order to avoid constraints violation.

V. Conclusion

In this work we propose a protective predictive control for motor drives with elastic transmission. We extend the classical LQR controller plus observer structure with a safety set obtained using invariant sets methodology. Implementation of this set on-line allows us to correct the LQR control signal in order to suppress the violation of safety constraints on the drive variables. The implementation of this protective control, although presented in a combination with an LQR controller, is not limited only to such type of controllers and could be realized in a DSP.

Acknowledgment

This work was supported by the Ministry of Science, Education and Sports of the Republic of Croatia under grant No. 036-0361621-3012.

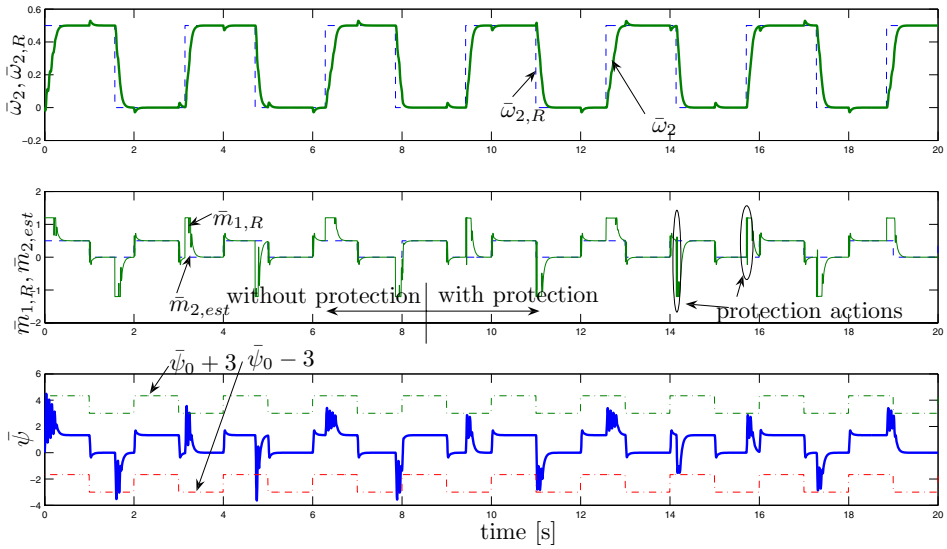

Fig. 2. Illustration of the control system operation without and with the proposed protection

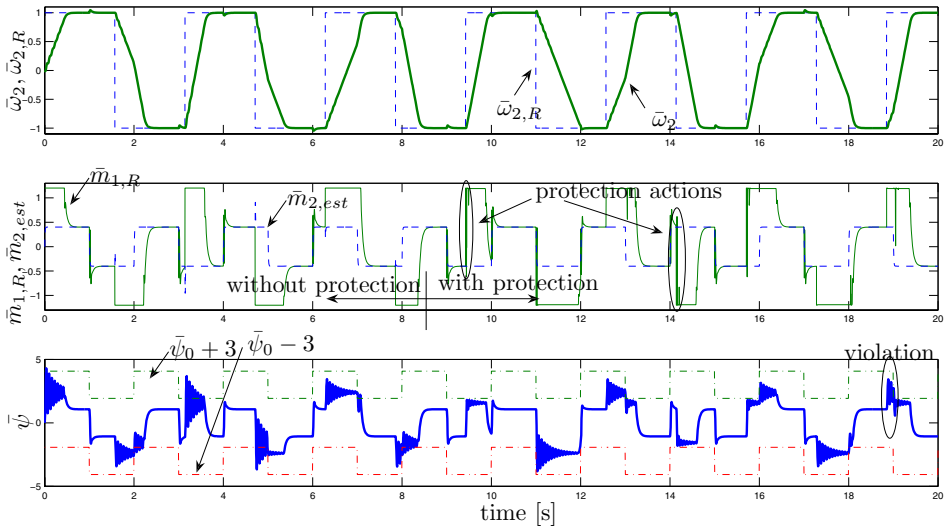

Fig. 3. Illustration of the control system operation without and with the proposed protection

References

[1] J. Deur and N. Perić. Design of polynomial speed controller for electrical drives with elastic transmission. In *Conference Record of the 8th European Conference on Power Electronics and Applications*, pages 1–10, Lausanne, Switzerland, September 1999.

[2] J. Ji and S. Sul. Kalman filter and lq based speed controller for torsional vibration suppression in a 2-mass motor drive system. *IEEE Transactions on Industrial Electronics*, 42(6):564–571, 1995.

[3] J. Deur, T. Koledić, and N. Perić. Optimization of speed control system for electrical drives with elastic coupling. In *Proceedings of the 1998 IEEE International Conference on Control Applications*, pages 319–325, Trieste, Italy, September 1998.

[4] T. Orlowska-Kowalska and K. Szabat. Control of the drive system with stiff and elastic couplings using adaptive neuro-fuzzy approach. *IEEE Transactions on Industrial Electronics*, 54(1):228–240, 2007.

[5] F. Blanchini. Set Invarinace in Control – A Survey. *Automatica*, 35(11):1747–1767, 1999.

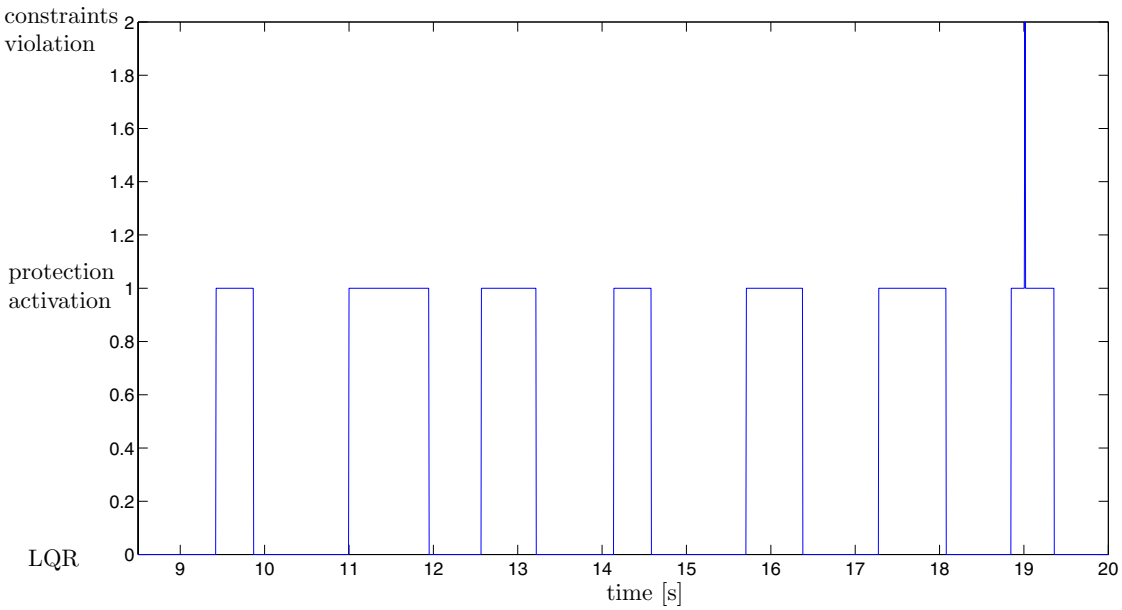

Fig. 4. Illustration of the frequency of protection activation for the transients in Fig. 3

[6] W. Leonhard. *Control of Electrical Drives*. Springer, Berlin, 2001.

[7] J. Deur, A. Božić, and N. Perić. Control of electrical drives with elastic transmission, friction and backlash – experimental system. *Automatika, Journal for Control, Measurement, Electronics, Computing and Communications*, 40(3–4):129–137, 1999.

[8] F. Lewis. *Optimal Control*. John Wiley and Sons, New York, 1986.

[9] T. Orlowska-Kowalska and K. Szabat. Neural-network application for mechanical variables estimation of a two-mass drive system. *IEEE Transactions on Industrial Electronics*, 54(3):1352–1364, 2007.

[10] J. O'Reilly. *Observers for Linear Systems*. Academic Press, Burlington, 1983.

[11] S. V. Raković, E. C. Kerrigan, D. Q. Mayne, and J. Lygeros. Reachability Analysis of Discrete-Time Systems With Disturbances. *IEEE Transactions on Automatic Control*, 51(4):546–561, 2006.