

# Optimal Design of Low-Sensitivity, Low-Power 2<sup>nd</sup>-Order BP Filters

Drazen Jurisic
Neven Mijat
Faculty of Electrical Engineering and Computing,
University of Zagreb,
Unska 3, HR-10 000 Zagreb, Croatia,
e-mail: drazen.jurisic@fer.hr, neven.mijat@fer.hr

Abstract— In this paper we present an optimal design procedure for 2<sup>nd</sup>-order BP filters with a single operational amplifier. We compare two known design procedures and investigate how a combination of two design methods yields an optimal 2<sup>nd</sup>-order BP filter. The new design provides a filter with low sensitivity to passive components. We present this straightforward design procedure in the form of a tabulated step-by-step design framework and demonstrate it with a 2<sup>nd</sup>-order Chebyshev filter. The resulting sensitivity is investigated using the Schoeffler sensitivity measure.

Keywords: Lossy LP-BP transformation, impedance tapering, low-sensitivity filters, 2<sup>nd</sup>-order BP filters.

# I. INTRODUCTION

In two recent publications [1], [2] the design of lowsensitivity, single-amplifier 2<sup>nd</sup>-order band-pass (BP) filters has been investigated. According to the classification given in [3] the 2<sup>nd</sup>-order BP filter under consideration is referred to as a class-4 (Sallen & Key [4]) BP filter of type A. Partial results on design techniques using component substitution and using impedance tapering (first introduced in [5]) have been published in [1] and [2], respectively. In this paper we summarize both results and additionally investigate which part of each design method will provide an optimum design procedure. The purpose of this paper is to find the connection between these two design approaches and to establish a new and efficient design procedure which besides, design simplicity, provides filters with exceptionally low sensitivity to component tolerances. As a result, a novel step-by-step filter design procedure is obtained.

# II. TWO APPROACHES TO BP FILTER DESIGN

# A. Approach 1: Lossy LP-BP Transformation Carried Out by The Component Substitution

In [1] the design of a 2<sup>nd</sup>-order BP filter using a lossy LP-BP transformation carried out by component substitution has been reported in detail. In this paragraph only the most important steps will be presented. In order to design a 2<sup>nd</sup>-order BP filter we start with the 1<sup>st</sup>-order LP passive-RC filter circuit shown in Fig. 1.

The voltage transfer function T(S) for this circuit is given by

George S. Moschytz School of Engineering, Bar-Ilan University, IL-52 900 Ramat-Gan, Israel, e-mail: moschytz@isi.ee.ethz.ch

$$T(S) = \frac{V_{out}}{V_1} = \frac{(RC)^{-1}}{S + (RC)^{-1}} = \frac{k_0 \cdot \gamma}{S + \gamma} = \frac{k_0 \cdot (-s_1)}{S - s_1}.$$
 (1)

This function has a single negative real pole  $s_1 = -\gamma$ , where  $\gamma = (RC)^{-1}$ , and the pass-band gain  $k_0 = 1$ . If we apply the standard LP-BP frequency transformation

$$S = \frac{s^2 + \omega_0^2}{Bs} \,, \tag{2}$$

to the LP prototype transfer function (1) we obtain the 2<sup>nd</sup>-order BP transfer function, given by

$$T_{BP}(s) = \frac{k_0 B \gamma s}{s^2 + B \gamma s + \omega_0^2} = \frac{k_0 (\omega_0 / q_p) s}{s^2 + (\omega_0 / q_p) s + \omega_0^2}, \quad (3)$$

with 
$$q_n = \omega_0 / (B\gamma)$$
. (4)

In order to prepare an LP prototype filter for transformation to BP, we perform a predistortion, i.e. a transformation of the LP transfer function (also referred as " $\delta$ -shift")

$$S=p-\delta$$
. (5)

Applying (5) to (1) results in a new transfer function  $T_1(p)$ , having a new shifted real pole  $p_1$  in the complex p-plane. The pole  $s_1$  is shifted by an amount  $\delta$  to the right, parallel to the real axis. The new LP filter prototype transfer function is

$$T_1(p) = \frac{k_0 \gamma}{p + \gamma - \delta} = \frac{K_0 \Gamma}{p + \Gamma} = \frac{K_0 \cdot (-p_1)}{p - p_1},$$
 (6a)

where the new real pole  $p_1$ = $-\Gamma$ , and the pass-band gain  $K_0$  are given by

$$\Gamma = \gamma - \delta$$
, and  $K_0 = k_0 \gamma / \Gamma$ . (6b)

In order to shift the real pole to the right, we must modify the 1<sup>st</sup>-order passive-RC LP circuit in Fig. 1. By adding an operational amplifier we provide a positive feedback loop, enabling the pole shift. The transfer function (6a) is realized using the 1<sup>st</sup>-order active-RC circuit, with an operational amplifier, shown in Fig. 2.

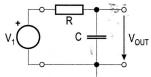


Fig. 1. 1st-order passive RC LP circuit.

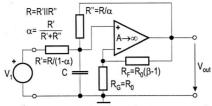


Fig. 2. 1st-order modified active-RC LP prototype circuit.

The voltage transfer function of the circuit in Fig. 2 is given by

$$T_{1}(p) = \frac{V_{out}}{V_{1}} = \frac{(1-\alpha)\beta}{p \cdot RC + 1 - \alpha\beta} = \frac{(1-\alpha)\beta \cdot (RC)^{-1}}{p + (1-\alpha\beta) \cdot (RC)^{-1}}, \quad (7)$$

where the gain  $\beta$  represents the positive gain

$$\beta = 1 + R_F / R_G, \tag{8}$$

and the positive feedback constant  $\alpha$  is given by

$$\alpha = R'/(R''+R'). \tag{9}$$

Using  $\gamma = (RC)^{-1}$  from (1), we can rewrite the voltage transfer function  $T_1(p)$  in (7) into the form

$$T_1(p) = \frac{V_{out}}{V_1} = \frac{(1-\alpha)\beta}{p/\gamma + 1 - \alpha\beta} = \frac{(1-\alpha)\beta\gamma}{p + (1-\alpha\beta)\gamma} = \frac{K_0\Gamma}{p + \Gamma}. (10)$$

Note that the shift  $\delta$  in (6) is realized in (10) by  $\alpha$ ,  $\beta$  and  $\gamma$ , therefore we use the notation  $\delta = \alpha \beta \gamma$ . Furthermore the application of (5) to (2), results in a new "lossy"-transformation in the variable p, given by

$$p = \frac{s^2 + \omega_0^2}{Rs} + \delta \,. \tag{11}$$

If the "lossy" LP-BP transformation (11), were to applied to the predistorted transfer function  $T_1(p)$  in (10), it would produce the same BP filter transfer function (3), as the conventional LP-BP transformation (2), applied to the original LP transfer function T(S) in (1), because the  $\delta$ -shift cancels out. Thus the transformation of variable S to S in (2) produces the same result as the transformation of variable p to S given in (11).

The poles of the prewarped transfer function  $T_1(p)$  can even be in the right-half plane since the "lossy" LP-BP transformation in (11) maps the poles back into the left-half s-plane. Thus, they do not produce stability problems, because the initial  $\delta$ -shift is cancelled out.

Dividing both sides of the lossy-transformation in (11) by  $\gamma$ , we obtain the " $\gamma$ -normalized" lossy-transformation in the form

$$\frac{p}{\gamma} = \frac{s^2 + \omega_0^2}{B\gamma s} + \frac{\delta}{\gamma} \,. \tag{12}$$

In order to physically realize the "lossy" transformation defined in (11), we introduce the following network transformation which substitutes each resistor of the LP prototype filter by a series RC circuit, and each capacitor by a parallel RC circuit, as shown in Fig. 3.

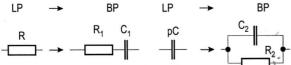


Fig. 3. RC LP-BP network transformation.

It can be represented in the form of a substitution given by

$$R = R_1 + 1/(sC_1), \ pC = 1/R_2 + sC_2.$$
 (13)

Thus, we replace the expression  $p \cdot RC$  in (7) [or  $p/\gamma$  in (10)] by a product of the admittance of a parallel RC circuit with the impedance of a series RC circuit, i.e.,

$$p \cdot RC = \left(R_1 + \frac{1}{sC_1}\right) \left(\frac{1}{R_2} + sC_2\right),$$
 (14)

which we can readily rewrite in the form

$$p \cdot RC = \frac{s^2 + 1/(R_1 C_1 R_2 C_2)}{s \cdot 1/(R_1 C_2)} + \frac{R_1}{R_2} + \frac{C_2}{C_1}.$$
 (15)

Since the left side of the component substitution (15) has the value  $p \cdot RC$  instead of p, and keeping in mind that  $\gamma = (RC)^{-1}$ , we conclude that the substitution given by (15) has the same form as the normalized transformation in (12) with

$$\omega_0^2 = \frac{1}{R_1 C_1 R_2 C_2}, \ B_{sub} \cdot \gamma = \frac{1}{R_1 C_2}, \ \frac{\delta_{sub}}{\gamma} = \frac{R_1}{R_2} + \frac{C_2}{C_1}. \ (16)$$

Note that we have added index "sub" to B and  $\delta$ . This emphasizes that (16) denotes the component replacement—i.e. a symbolic substitution. Generally, we are free to choose various sets of values of components  $R_1$ ,  $R_2$ ,  $C_1$ , and  $C_2$  such that values  $B_{sub}$  and  $\delta_{sub}$  can differ from given numeric values of B and  $\delta$ , whereas the chosen components  $R_1$ — $C_2$  still realize the given transfer function. The element substitution in Fig. 3 to the circuit in Fig. 2 gives the  $2^{nd}$ -order BP-A filter shown in Fig. 4.

The voltage transfer function T(s) for the circuit in Fig. 4 is obtained by applying (12) (with "sub") to (10)

$$T(s) = \frac{V_{out}}{V_1} = \frac{(1 - \alpha)\beta B_{sub}\gamma \cdot s}{s^2 + B_{sub}(\delta_{sub} + \gamma - \alpha\beta\gamma) \cdot s + \omega_0^2}.$$
 (17)

If we compare (17) with (3) we have

$$q_{p} = \frac{\omega_{0} / B_{sub}}{\delta_{sub} + \gamma - \alpha\beta\gamma}, \ k_{0} = \frac{(1 - \alpha)\beta\gamma}{\delta_{sub} + \gamma - \alpha\beta\gamma}, \tag{18}$$

and if we introduce (16) into (18) the transfer function parameters as a function of filter components are given by

$$\omega_{0} = \frac{1}{\sqrt{R_{1}C_{1}R_{2}C_{2}}}, q_{p} = \frac{\sqrt{R_{1}C_{1}R_{2}C_{2}}}{R_{1}C_{1} + R_{2}C_{2} + R_{2}C_{1}(1-\alpha\beta)}, k_{0} = q_{p}(1-\alpha)\beta\sqrt{\frac{R_{2}C_{1}}{R_{1}C_{2}}}$$

B. Approach 2: Design Procedure Using Impedance Tapering

The concept of impedance tapering [5] can be applied to the passive network of the  $2^{nd}$ -order BP filter type A as in Fig. 4 [2], introducing the impedance ratios r and  $\rho$  given by

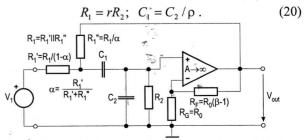


Fig. 4. 2<sup>nd</sup>-order BP- A filter constructed by RC impedance substitution.

We choose these arbitrary impedance-scaling factors to be  $r \ge 1$  and  $\rho \ge 1$  in order to reduce the filter characteristic's sensitivity to component tolerances of the circuit. According to [2] for the 2<sup>nd</sup>-order allpole class-4 BP filter of type-A, ideal impedance scaling  $\rho = r >> 1$  of the first L-section provides circuits with minimum sensitivity to the component tolerances. We will use this result in building our novel design procedure.

# C. A Novel Design Method: Combination of Two Approaches

The goal of the research in this paper is to find a connection between the element substitution design as in [1] (approach 1) and the low-sensitivity design using impedance tapering as in [2] (approach 2). As a result we present the rules for the new design procedure capable of producing filters in the same straightforward way as in approach 1, and at the same time having the same minimum sensitivity as in approach 2.

If we apply the general component scaling factors in (20) to (16), we have

$$\omega_0 = (R_2 C_2)^{-1} \sqrt{\rho/r}$$
,  $B_{sub} \gamma = \omega_0 / \sqrt{r\rho}$ ,  $\delta_{sub} / \gamma = r + \rho$ . (21)

Since in (16) and (21) we have 3 equations with 4 unknowns (i.e. components  $R_1$ ,  $R_2$ ,  $C_1$ , and  $C_2$ ); we can choose one component freely (e.g.  $C_2$ ). It is convenient to calculate the capacitance ratio  $\rho=C_2/C_1$  or the resistor ratio  $r=R_1/R_2$ , from (21). They are given by

$$\rho = \frac{1}{\gamma} \left( \frac{\delta_{sub}}{2} \pm \sqrt{\Delta} \right); r = \frac{1}{\gamma} \left( \frac{\delta_{sub}}{2} \mp \sqrt{\Delta} \right); \Delta = \left( \frac{\delta_{sub}}{2} \right)^2 - \left( \frac{\omega_0}{B_{sub}} \right)^2.$$

The value of the constant  $\delta_{sub}$  in (22) has a lower bound. Since the expression under the square root (i.e. a discriminant  $\Delta$ ) may not be negative, there exists a realizability constraint on the value of the parameter  $\delta_{sub}$  given by

$$\delta_{sub} \ge 2\omega_0 / B_{sub} = \delta_{\min}. \tag{23}$$

Note that  $\delta_{sub} = \delta_{min}$  when  $\Delta = 0$ . If the constraint given by (23) is satisfied, the component substitution in Fig. 3 exists.

To investigate all possibilities, in our new design method we distinguish between two main cases, which can occur in (17) and (18).

1) Case I: Full  $\delta$  Cancellation:  $\delta = \delta_{\text{sub}} \Rightarrow B = B_{\text{sub}}$  The shift constant  $\delta$  in the pre-distorted transfer function  $T_1(p)$  in (10), is fully cancelled by the application of the component transformation in Fig. 3. In this case we restrict the choice of the value of the product  $\alpha\beta\gamma = \delta$  to be:

$$\alpha\beta\gamma = \delta = \delta_{sub} = (r + \rho)\gamma. \tag{24}$$

In this case  $B=B_{sub}$ . Introducing (24) into (18), the values of pole Q-factor,  $q_p$ , and pass-band gain  $k_0$  are given by:

$$q_p = \frac{\omega_0}{B\gamma}, \ k_0 = (1-\alpha)\beta; \text{ or } K_0 = \frac{(1-\alpha)\beta}{1-\alpha\beta},$$
 (25)

where  $K_0$  follows from (10). We can rewrite  $\delta_{min}$  in (23) into the form

$$\delta_{\min} = 2\omega_0 / B = 2\gamma \cdot \omega_0 / (B\gamma) = 2\gamma q_p. \tag{26}$$

From (24) and (25) the gain  $\beta$  and the attenuation  $\alpha$  are

$$\beta = k_0 + \delta/\gamma$$
; or  $\beta = K_0(1 - \delta/\gamma) + \delta/\gamma$ ,  $\alpha = \delta/(\beta\gamma)$ . (27)

We distinguish between case I.a in which  $\rho=r$ , and case I.b in which  $r\neq \rho$ . In the case I.a from (22) and (23) it can readily be seen that  $\delta=\delta_{\min}$ , and from (21), (24) and (26) we have

$$\omega_0 = (R_2 C_2)^{-1}; \rho = r = q_p = \delta_{\min}/(2\gamma).$$
 (28)

In the case I.b we see from (22) and (23) that  $\delta > \delta_{\min}$ , therefore we choose  $\delta > \delta_{\min}$  and calculate  $\rho$  and r from (22) with  $\omega_0/B = \delta_{\min}/2 = q_p \gamma$  [see (26)]. Finally, in the case I.a we choose, e.g.  $C_2$  and from (28) with  $\omega_0$  we calculate  $R_2$ , whereas in the case I.b we have to first use equation (21) and with  $\omega_0$ ,  $\rho$  and r calculate  $R_2$ .

2) Case II: Partial  $\delta$  Cancellation:  $\delta \neq \delta_{\text{sub}} \Rightarrow B \neq B_{\text{sub}}$  The shift constant  $\delta$  in the pre-distorted transfer function  $T_1(p)$  is generally not cancelled by the application of the component transformation in Fig. 3. This "general" case usually happens when a designer picks a known BP active filter structure and calculates its elements by comparing the corresponding transfer function parameters with the parameters of the chosen structure. In the case II the gain  $\beta$  and attenuation  $\alpha$  should be calculated using more general expressions in (19). With (20) applied to (19) we can readily calculate

$$\beta = 1 + r + \rho + (k_0 - 1)q_p^{-1}\sqrt{r\rho}, \ \alpha = 1 - (k_0 / \beta) \cdot q_1^{-1} \cdot \sqrt{r\rho}.$$
 (29)

We choose r,  $\rho$ ,  $C_2$  and using first equation in (21) with given pole frequency  $\omega_0$ , calculate  $R_2$  and then the other component values. Note that  $\alpha$  and  $\beta$  realize both the  $\delta$ -shift and the pass-band gain  $k_0$ , at the same time.

### III. EXAMPLES

As an illustration of the proposed design procedure, we consider a practical example, in which, for simplicity and design generality, all transfer function parameters and component values are normalized.

Consider the normalized  $2^{\rm nd}$ -order BP filter with center frequency  $\omega_0=1$ , pole Q factor,  $q_p=5$ , and pass-band gain  $k_0=1$ . This simple BP filter is constructed applying the LP-BP transform (2) to the  $1^{\rm st}$ -order filter, obtained from tables for Chebyshev filters with pass-band ripple  $R_p=0.5$ dB, having the single negative real pole  $\gamma=2.862775$  [3]. The bandwidth is  $B=0.06986\approx0.07$  and center frequency  $\omega_0=1$  thus we obtain the  $2^{\rm nd}$ -order BP filter with pole Q factor,  $q_p=\omega_0/(B\gamma)=5$ . The cut-off frequencies of the BP filter are the ones where the filter's magnitude reaches the minimum specified ripple value in the pass-band -0.5dB. The corresponding magnitudes are shown in Fig. 5.

In order to analyze the shift-constant  $\delta$  influence on the sensitivities to component tolerances, and to find its optimal

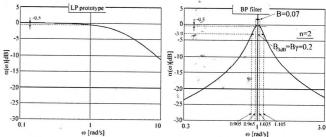


Fig. 5. LP-BP transformation of Chebyshev transfer function.

TABLE I NORMALIZED COMPONENT VALUES IN DESIGN EXAMPLES OF  $2^{\rm ND}$ -ORDER BP FILTER TYPE A AS IN FIG. 4.

No.	Case	δ/γ	r	ρ	$R_1$	$R_2$	$C_1$	$C_2$	α	β
1.	I.a)	10	5	5	5	1	0.2	1	0.9091	11
2	I.b)	15	1.910	13.09	5	2.618	0.0764	1	0.9375	16
3.		15	13.09	1.910	5	0.382	0.524	1	0.9375	16
4.		20	1.34	18.66	5	3.732	0.0536	1	0.9524	21
5.		20	18.66	1.34	5	0.268	0.7464	1	0.9524	21
6.	II)	2	1	1	1	1	1	1	0.9333	3
7.		20	10	10	10	1	0.1	1	0.9048	21

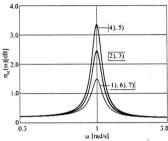


Fig. 6. Schoeffler's sensitivities for realizations in Table I.

value in the "lossy" transformation, five different realizations corresponding to the case I and to three values of  $\delta$  are analyzed as circuit examples no. 1 to 5 in Table I. The case-I.a design can be carried out by the step-by-step design procedure, which is outlined in Table II. For the case I.b (to design circuits no. 2-5) we do not present design steps because they are somewhat more complicated and do not produce minimum sensitivity filters. Consider filter no. 1 for which we selected  $r=\rho=q_p=5$  and which has, using (10) and the data in Table I, the value of  $\delta = \alpha \beta \gamma = 28.627$  in  $\delta$ -shifted prototype. In the substitution using (16) it has the value of  $\delta_{\text{sub}} = \gamma (R_1/R_2 + C_2/C_1) = 28.627$  and therefore  $\delta = \delta_{\text{sub}}$ . Thus we have full  $\delta$ -cancellation in that case (the same is true for all examples of the case I.b). Two filter examples are calculated corresponding to the "general" case II and to two arbitrary values of  $\delta$  (circuits no. 6 and 7 in Table I). The step-by-step design procedure for the circuits no. 6 and 7 is presented in [2]. For the circuit no. 6 we selected  $r=\rho=1$  and can calculate  $\delta=8.016>\delta_{\text{sub}}=5.73$ , whereas for the circuit no. 7 we selected  $r=\rho=10$  and have  $\delta=54.4 < \delta_{\text{sub}}=57.3$ . Therefore, in the case II there is  $\delta \neq \delta_{\text{sub}}$  and the component substitution causes partial δ-cancellation.

In all examples in Table I, an overall sensitivity analysis was performed with the relative changes of the resistors and capacitors assumed to be uncorrelated random variables, with a zero-mean Gaussian distribution and 1% standard deviation. The standard deviation  $\sigma_{\alpha}(\omega)[dB]$  (which is related to the Schoeffler's sensitivities) of the variation of the logarithmic gain  $\Delta\alpha=8.68588\cdot\Delta|T(\omega)|/|T(\omega)|$  [dB], with respect to all passive elements, was calculated. For the filters in Table I the corresponding standard deviations  $\sigma_{\alpha}(\omega)$  are shown in Fig. 6.

Observing the sensitivity curves in Fig. 6 we conclude that the ideally impedance tapered filters (having  $r=\rho$ ) no. 1, 6 and 7 have minimum sensitivity. This coincides with the analytical sensitivity expressions given in [2] and [5]. It is apparent from there that only keeping  $\rho=r>1$  which results in increasing the  $\delta$ -shift value in  $\delta=(r+\rho)\gamma=2\gamma\rho$  reduces the filter sensitivity to the passive components.

 $\label{eq:table_interpolation} TABLE~II$  Step-By-Step Design Procedure for  $2^{\text{\tiny ND}}\text{-}Order~BP~Filter.$ 

Step:	Equations:
i) LP prototype	$\gamma$ or $q_p = \omega_0/(B\gamma)$ and $k_0 = 1^a$
ii) Min. δ	$\delta = \delta_{\min} = 2\gamma q_p = 2\omega_0/B$
iii) New LP prototype	$\Gamma = \gamma - \delta$ , $K_0 = k_0 \gamma / \Gamma$
iv) Calculate the pa-	$\beta = K_0(1-\delta/\gamma) + \delta/\gamma$ or $\beta = k_0 + \delta/\gamma$ ,
rameters	$\alpha = \delta/(\beta \gamma)$ and $\rho = r = q_p$
v) Calculate the ele-	Choose: $C_2=1$ , $R_G=1$ . Calculate:
ments of the BP filter	$R_2 = (\omega_0 C_2)^{-1}$ ; $C_1 = C_2/\rho$ ; $R_1 = r \cdot R_2$ ;
	$R'_1 = R_1/(1-\alpha); R''_1 = R_1/\alpha; R_F = R_G(\beta-1)$

a we wish to realize the filter with 0-dB maximum gain.

Filter no. 1, which has min. sensitivity in the case I, was designed by the simplest procedure in which  $\delta = \delta_{\min}$  (case I.a) has minimum component spread as well. Other more complicated designs in examples no. 2-5, having larger values for  $\delta$ , do not provide low-sensitivity filters for  $r \neq \rho$ .

Filter examples no. 6 and 7 designed using the general design procedure (referred to as case II) use  $\rho=r$ , and thus provide filters with reduced sensitivity. According to their  $\delta$ -values, the filter no. 6 has slightly higher, whereas the filter no. 7 has slightly lower sensitivity than the filter no. 1. The design procedure in the case I.a is preferable over the design procedure in the case II, because of its *simplicity*. It is outlined in Table II.

This simple design procedure is accomplished by choosing  $\rho = r = q_p$  and thus  $\delta = \delta_{\min} = 2\gamma q_p$ . This choice is reasonable because the sensitivity of the filter is proportional to the pole Q-factor value  $q_p$ , and the decrease in sensitivity obtained by application of impedance tapering is also larger when the scaling factors  $\rho = r$  increase. Therefore, this design is appropriate for filters having both smaller and larger pole Q-factor values (using this circuit we realize medium pole Q-factors  $2 \le q_p \le 20$  as shown in [3]).

#### IV. CONCLUSIONS

In recent publications [1], [2] two different approaches of designing single-amplifier 2<sup>nd</sup>-order BP-A filters have been presented. In this paper we exploit the best of both approaches and combine them into a new design procedure. The new and efficient design procedure is straightforward and provides filters having low sensitivity to component tolerances. A novel step-by-step filter design procedure is presented in table form.

# REFERENCES

- [1] D. Jurišić and N. Mijat, "Use of "Lossy" LP-BP-Transformation in Active Second-order BP Filter Design Procedure," In Proc. of ISPA 2001, (Pula, Croatia), pp. 491-495, June 19-21, 2001.
- [2] D. Jurišić, G. S. Moschytz and N. Mijat, "Low-sensitivity active-RC high- and band-pass second-order Sallen and Key allpole filters," In Proc. of ISCAS 2002, (Phoenix, Arizona-USA), Vol. 4, pp. 241-244, May 26-29, 2002.
- [3] G. S. Moschytz and P. Horn, Active Filter Design Handbook. Chichester, U.K.: Wiley 1981.
- [4] R. P. Sallen and E. L. Key, "A practical Method of Designing RC Active Filters," IRE Transactions on Circuit Theory, vol. CT-2, pp. 78-85, 1955.
- [5] G. S. Moschytz, "Low-Sensitivity, Low-Power, Active-RC Allpole Filters Using Impedance Tapering," IEEE Transactions on Circuits and Systems II, vol. CAS-46, No. 8, pp. 1009-1026, Aug. 1999.