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DRAFT: ON THE ASSESSMENT OF REDUNDANCY OF SHIP STRUCTURAL COMPONENTS

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ABSTRACT

This paper firstly sums up some of the views on structural redundancy with particular emphasis on ship and marine structures. Next, it places the engineering decision process in the event space and, consequently, applies the representation of operational modes by systems of events. Furthermore, the paper takes some of the relations from the entropy concept in information theory. The entropy concept in probability theory is employed in the paper to redefine the structural redundancy in terms of conditional entropy of operational modes. The redundancy modeling is presented by systems of operational modes in which some of the transitive events may lead to new operational states. Finally, a ship substructure example of a stiffened panel with a girder is elaborated. The conclusion supports the thesis that the efficient structural redundancy can be comprehended as the most uniform distribution of the operational modes probabilities. Moreover, the efficient structural redundancy can be maximized.

1 INTRODUCTION

A majority of the engineering objects intended to be economical and reliable in service act at least as 'fail-safe' systems also described as damage tolerant systems or redundant systems. The traditional probabilistic system analysis of lasting interest in engineering of ship and marine structures based on physical and/or technical components of a system ([1], [2], [3]), may be extended by an event oriented system analysis (EOSA) [4]. When an engineering object, operating in an uncertain environment, is subjected to event-oriented system analysis, the system redundancy relates to the conditional entropy of the probability distribution of operational modes [5]. Event oriented system analysis in the paper is applied in order to assess the probabilities and uncertainties of different functional states and modes of action. This paper argues possible benefits of representing ship structural components by systems of events. It describes how the conditional entropy of operational subsystem of events can model important property of ship structures, the redundancy. Redundancy is recognizable property of the complex engineering objects such as ship and marine structures ([6], [7], [8]). It can be considered as the capacity of a system to operate even when some of the physical components have failed; therefore it is a desired capability of all structures that tend to be reliable in service. Redundancy is usually related to system reliability and the optimization of system redundancy has been recognized as an effective method in reliability improvement ([9], [10], [11]). The redundancy can be quantified in deterministic [12], semi-probabilistic terms [13] and probabilistic terms. The main disadvantage of deterministic measures of redundancy is that they do not take into consideration the statistical uncertainties of the system. The semi-probabilistic methods in redundancy assessment benefit from first order reliability analysis ([1], [2], [3]). The most widely used probabilistic measure of system redundancy is based on the conditional probability of systems survival given if one failure occurs ([14], [15]).

A rational and objective probabilistic evaluation of redundancy includes statistical uncertainties of the considered system through an event-oriented system analysis [4]. In the EOSA the system uncertainty analysis is based on the concept of entropy as defined firstly in information theory ([16], [17]) and lately applied to probability theory ([21], [22]). The conditional entropy of failure modes provides insight in system's robustness [5] and on other of operational modes provides insight in system's redundancy ([18], [19]). A probabilistic model of geometrically over-determinate structures with transitive events provides an analogy to time-variant redundant systems in engineering.

Determination of the overall residual system capacity (residual strength) alone is not sufficient to assess the system redundancy. It is necessary to consider the distribution of the residual strength among the remaining non-failed components since the remaining components of redundant systems, after some component has failed, may be able to sustain applied loads. In event-oriented system analysis the redundancy is viewed as a capability of a system to continue operations by performing different random operational modes of given probabilities in case of random failures of components. Performing another operational mode, the system may be brought on equal or on the reduced operational capacity. The operational capacities can be viewed either as the probabilities of operational modes or as the physical or technical working capacities of a damaged system.

This paper briefly describes practical modeling of the reliability accompanied by redundancy of ship structures by an event-oriented system analysis [20]. The redundancy analysis in the example demonstrates that the conditional entropy of redundant structures is quantifiable and useful in understanding of behavior of ship structural systems in damaged conditions. Moreover, it appeared that the redundancy of structures can be maximized ([18], [20]).

2 THEORETICAL BACKGROUNDS

Every engineering object can be viewed as a system S, of events E_i with probabilities $p(E_i) = 1,2,\ldots,N$, where N is the total number of events. By operational modes and effects analysis, all, or at least all observable and important events in a lifetime service of a system, can be determined. Their probabilities can be calculated by quantitative methods. Beside events, a description of every system includes functional levels, functional states and functional modes. Since events, modes, states and levels can be of a different functional status, the EOSA requires somewhat complex notation to model the behavior of a redundant system [18]. The following designations for a functional status 's' are used: i-intact, c-collapse, t-transitive, n-non-transitive, o-operational, f-failure, d-damage, s-serviceability and combinations. A functional level is a system of events comprising of all functional states of an object. Redundant structures are modeled by systems that have 2 or more functional levels. Initial intact structure is described on the first functional level. After failure of one or more structural components, the system transits from the first level to the second level. Further failure causes the system to transit on the third level, and so on. Furthermore, on every level system can have one or more functional states, which are systems composed of modes and represent distinguished independent ways the object performs its functions with full or with reduced operational capacity. Finally, functional modes are subsystems of events with a common status 's'.

${}^{l}_{j}E^{s}_{i}$ is an event of status 's', where $l = 1,2,\ldots, n$ is a functional level and $j = 1,2,\ldots,{}^{l}n$ is a functional state of a level, $i = 1,2,\ldots,{}^{l}_{j}N^{s}$, and ${}^{l}_{j}N^{s}$ is the number of events within a functional level, state or mode. Systems and subsystems of events are usually presented as finite schemes [21]:

$$ {}^{l}_{j}S^{s} = \begin{pmatrix} {}^{l}_{j}E^{s}_{1} & \cdots & {}^{l}_{j}E^{s}_{i} & \cdots & {}^{l}_{j}E^{s}_{{}^{l}_{j}N^{s}} \\ p({}^{l}_{j}E^{s}_{1}) & \cdots & p({}^{l}_{j}E^{s}_{i}) & \cdots & p({}^{l}_{j}E^{s}_{{}^{l}_{j}N^{s}}) \end{pmatrix} $$

where s is a functional mode of a status 's'.

Events can be appropriately grouped according to their functional status and then adequate subsystems of events can be formed. Thus the system of events can be presented as a sum of subsystems. For redundancy calculation the subsystems of interest are intact, transitive and collapse:

$$ {}^{1}S = \left({}^{1}_{1}S \right) = \left({}^{1}_{1}S^{i} + {}^{1}_{1}S^{t} + {}^{1}_{1}S^{c} \right) $$

where $p\left({}^{l}_{j}S^{s} \right) = \sum\limits_{allE \in {}^{l}_{j}E^{s}} p\left({}^{l}_{j}E^{s} \right)$.

Systems are either complete when $\Sigma p(E_i)=1$ or incomplete when $\Sigma p(E_i)<1$.

2.1 General relations among the probabilities, uncertainties and entropy

EOSA applies the entropy concept to assess the effects of the number of events and dispersion of their probabilities, as well as the possible redistribution of loads after failures. The concept of entropy is known in the information theory ([16], [17], [21], [22]) entropy is a simple logarithmic function that measures uncertainty related to the occurrence of some event, i.e. a function that measures the information yielded by the event. The entropy of a single stochastic event E is thus defined as $H(p) = -\log p(E)$ and corresponding uncertainty of a system of events denoted as the Shannon's entropy is [16]:

$$ H(S) = -\sum_{i=1}^{n} p_i \log p_i \qquad (1) $$

For incomplete systems of events more appropriate is the Renyi's entropy [17].

The most important properties of the entropy are:

– increases with the increasing number of events in the system,
– maximum value is attained when probabilities of all events are equal,
– entropy is zero if there is no uncertainty, i.e. one event has a probability of occurrence 1,
– it does not depend on the sequence of the events and represents the only rational measure of uncertainty (uniqueness theorem)

Since events of a system can be grouped into adequate subsystems according to their operational or failure statuses, the system S can also be presented as a summa of operational and failure subsystem [4] as shown:

$$ S = \begin{pmatrix} E_1 & \ldots & E_N \\ p(E_1) & \ldots & p(E_N) \end{pmatrix} = \left(S^{o} + S^{f} \right) $$

where S^o and S^f are operational and failure subsystems:

$$ S^{o} = \begin{pmatrix} E^{o}_{1} & \ldots & E^{o}_{N} \\ p\left(E^{o}_{1}\right) & \ldots & p\left(E^{o}_{N_o}\right) \end{pmatrix}, S^{f} = \begin{pmatrix} E^{f}_{N_o+1} & \ldots & E^{f}_{N_o+N_f} \\ p\left(E^{f}_{N_o+1}\right) & \ldots & p\left(E^{f}_{N_o+N_f}\right) \end{pmatrix} $$

$N = N_o+N_f$ is the total number of events in the system $\boldsymbol{S}$.

The reliability of a system is equal to the probability of occurrence of the subsystem of operational events:

$$ R(S) = p\left(S^{o}\right) = \sum_{i=1}^{N_o} p\left(E^{o}_{i}\right) \qquad (2) $$

The probability of failure of a system is equal to the probability of a failure subsystem:

$$p_f\left(S\right)=p\left(S^f\right)=\sum_{i=N_o+1}^{N_o+N_f} p\left(E_i^f\right) \tag{3}$$

With presented schemes is possible to model any engineering object, including components of ship structures. EOSA can be applied to any relations among subsystems, inclusive or exclusive and with dependent or independent events under the condition of adequate partitioning of a system of events.

2.2 Redundancy definition

A system of events can be viewed conditionally, i.e. under a condition that it is operational:

$$S/S^o=\begin{pmatrix} E_1^o/S^o & \dots & E_N^o/S^o \\ \dfrac{p\left(E_1^o\right)}{p\left(S^o\right)} & \dots & \dfrac{p\left(E_{N_o}^o\right)}{p\left(S^o\right)} \end{pmatrix} \quad \text{where } S^o=\left(S^i+S^t\right).$$

The conditional entropy of a system is defined by using (1) as:

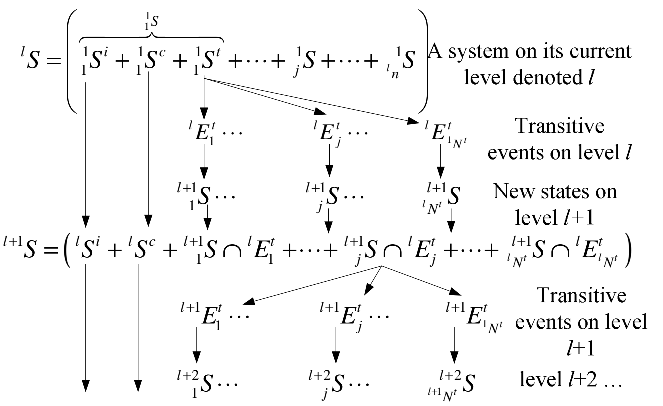
$$H_{N_o}\left(S/S^o\right)=-\sum_{i=1}^{N_o}\frac{p\left(E_i^o\right)}{p\left(S^o\right)}\log\frac{p\left(E_i^o\right)}{p\left(S^o\right)} \tag{4}$$

One can notice that entropy of an operational subsystem does not depend on the probability of a system $p(S)$ and also it does not depend whether the system is complete or incomplete. In EOSA the efficient redundancy of a system of events is comprehended as capability of a system to continue operations by performing different random operational modes of given probabilities in case of random failure of components. The redundancy is related only to the uncertainty of the operational subsystem [4] and it is defined (measured) as conditional entropy of a subsystem of operational events:

$$RED\left(S/S^o\right)=RED\left(S\right)=H_{N_o}\left(S/S^o\right) \tag{5}$$

2.3. Modeling redundant systems

A general procedure below, analogous with multi-level systems of events ([18], [19], [20], [23]), presents how to perform EOSA on redundant objects:



where ${}_j^lS\cap{}^{l-1}E_i^t$ is a compound functional state, i.e. a subsystem of those events on a considered level, which represent an emerged fully functional and independent state in a case that a specific transitive event occurred on the previous functional level. Transitive events on level l will cause emergence of new states on the level l+1 to emerge, transitive events on level l+1 will cause emergence of new states on the level l+2 to, and so on. On every level a system is analyzed: all the events are enumerated and sorted according to their functional modes into adequate subsystems.

3 EXAMPLE

The paper considers a part of longitudinally stiffened structure located at deck amidships of a double-hull tanker, (Fig.1), with following general characteristics: length between perpendiculars 174,8m; beam 31,4m; draught at full load 12,2m; C_b = 0,82; depth 17,5m; height of neutral axis from bottom 7,552m; displacement (full load draught) 47400tons [19].

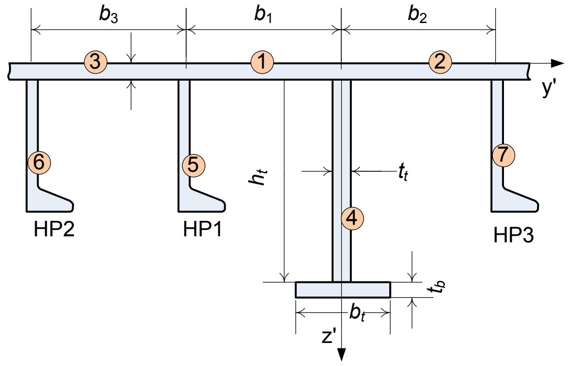


Figure 1. Part of the deck structure with marked structural elements (1 – 7), (Level 1 structural configuration)

The part of the deck consists of 3×bulb longitudinal (HP), one T-longitudinal and 3×plates between longitudinals. Generally, the finite element method (FEM) can be applied to predict the behavior and determine the strength of stiffened panels of ship structures. Although FEM will yield fairly accurate results, the analysis is time consuming and hence has not been adopted in this study. The example employs the DNV Rules for Ships for load and strength analysis ([24], [25]). Structural analysis approved that the considered structural configuration remained operational even when some components have failed, i.e. the remaining components were able to sustain redistributed load. Each level of functioning as well as individual operational states on each level was modeled by systems of events and redundancy calculation was carried out. All longitudinals and plating between longitudinals were involved in redundancy calculation as load carrying elements giving the total number of seven structural elements on the first functional level, six elements on the second functional level (Fig. 2) and five on the third functional level (Fig. 3).

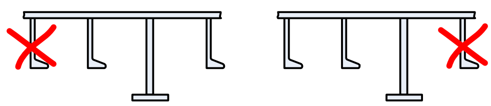


Figure 2. Level 2 configurations

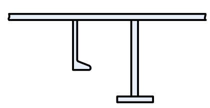


Figure 3. Level 3 configuration (non-redundant structure)

There is 11 failure modes associated with the seven structural elements considered, thus the first functional level has n=11 basic events designated ${}_j^l A_i$, i = 1, 2,…11. Reliability indexes are calculated by [26]:

$$ {}_j^l\beta_i = \frac{\mu_{{}_j^l C_i} - \mu_{{}_j^l D_i}}{\sqrt{\sigma^2_{{}_j^l C_i} + \sigma^2_{{}_j^l D_i}}} \tag{6} $$

where $\mu_{{}_j^l C_i}$ represent the mean values of stress random variables. In the cases of yield failure mode of longitudinals the mean values were taken as 60% of the material yield stress. In the cases of a buckling failure mode the mean values were taken as calculated from the DNV formulas for critical buckling stress ([24], [25]) for the corresponding structural element. $\mu_{{}_j^l D_i}$ represent the mean values of load stresses determined in structural analysis. $\sigma_{{}_j^l C_i}$ and $\sigma_{{}_j^l D_i}$ represent corresponding standard deviations of the variables. We assume here that all the stress random variables are independent and uncorrelated with linear safety margins for all functional levels and states. Also all stress variable are log-normally distributed with coefficient of variation (COV) of 0,7 ([27], [28]). Reliabilities and collapse probabilities are calculated respectively:

$$ R = p\left({}_j^1 A_i^i \right) = \Phi\left(-{}_j^1\beta_i \right) \quad \text{and} \quad p_f = p\left({}_j^1 A_i^c \right) = 1 - p\left({}_j^1 A_i^i \right) $$

where Φ is the standard normal density function. For levels with only one state j index can be omitted for simplicity. The results for the first functional level are given in Table 1.

The buckling of plates between longitudinals, the torsional buckling of longitudinals and the yielding of longitudinals due to deck design load may lead in total to 11 failure modes at the first functional level.

The mean values of the wave induced bending moments and design pressure on deck were taken as calculated according to the DNV Rules. The mean values of the still-water bending moments were taken as given in the trim and stability book for the full load state. Statistical properties of random variables were chosen from literature, tables 2-6, ([27], [29], [30]).

Table 2. Material characteristics (mild shipbuilding steel)

	Mean value	Distribution	COV
Yield stress	235,0 N/mm^2	Log-normal	0,06
Modulus of elasticity	206000N/mm^2	Normal	0,01

Table 3. Loads

	Mean value	Distr.	COV
Stillwater B.M.(sag.)	296252 kNm	Normal	0,4
Stillwater B.M.(hog.)	244690 kNm	Normal	0,4
Wave Induced B.M.(sag.)	1533336 kNm	Gumbel	0,09
Wave Induced B.M.(hog.)	1428791 kNm	Gumbel	0,09
Design pressure(deck)	13,6 kN/m^2	Normal	0,09

Table 4. Geometry (random variables)

	Mean value	Distribution	COV
Width of effective plate flange	800,0mm	Normal	0,01
Section modulus (longitudinals and plate flange b_i)	326,3cm^3	Log-normal	0,04
Midship section modulus at deck	16,14 m^3	Log-normal	0,04

Table 5. Properties of the deck structure (all levels)

	Level l=1	Level l=2	Level l=3
Panel cross sectional area in cm^2	392	360	327
Panel neutral axis in cm	9,3	8,9	8,4
Panel moment of inertia in cm^4	74021	71021	67785

Table 6. Geometry (deterministic variables)

Thickness of plating, t_p	14,0 mm
Bulb longitudinals	HP 220 ×11,5
Span of longitudinals, l	5,08 m
Spacing of longitudinals, s	0,8 m
Web height (T-profile), h_t	450 mm
Web thickness (T-profile), t_t	14 mm
Flange width (T-profile), b_t	100 mm
Flange thickness (T-profile), t_b	14 mm

The number of compound events ${}_j^l E_{{}^1N}$ on the first functional level is: ${}^1N = 2^n = 2^{11} = 2048$. There is only one intact functional state represented by the event ${}^1E_1^i$. Collapse of either one of the longitudinals HP2 or HP3 causes transition from the first functional level (Fig.1) to the second functional level (Fig.2). There is ${}^1N^t$ = 15 transitive events on the first functional level, denoted ${}_j^l E^t$. The remaining ${}^1N^c$=2032 events on the first level represent collapse of the structure.

Table 1. EOSA results for the first functional level l = 1, j = 1, intact mode

Basic event ${}_j^l A_i^i$	i	μ_C N/mm^2	μ_D N/mm^2	σ_C N/mm^2	σ_D N/mm^2	β	R	p_f
Buckling of plating 1	1	181,7	127,9	12,7	12,7	2,98265	0,99859	0,00141
Buckling of plating 2	2	181,7	127,9	12,7	12,7	2,98265	0,99859	0,00141
Buckling of plating 3	3	181,7	127,9	12,7	12,7	2,98265	0,99859	0,00141
Yielding of T longitudinal	4	141,0	63,7	9,87	12,7	4,79648	0,99999	0,00001
Buckling of T longitudinal	5	192,1	127,9	13,4	12,7	3,45939	0,99972	0,00027
Yielding of HP1 longitudinal	6	141,0	74,8	9,87	14,9	3,69367	0,99988	0,00011
Yielding of HP2 longitudinal	7	141,0	74,8	9,87	14,9	3,69367	0,99988	0,00011
Yielding of HP3 longitudinal	8	141,0	74,8	9,87	14,9	3,69367	0,99988	0,00011
Buckling of HP1 longitudinal	9	179,4	127,9	12,5	12,7	2,87316	0,99797	0,00203
Buckling of HP2 longitudinal	10	179,4	127,9	12,5	12,7	2,87316	0,99797	0,00203
Buckling of HP3 longitudinal	11	179,4	127,9	12,5	12,7	2,87316	0,99797	0,00203

Events are appropriately grouped according to their functional mode and the structure is modeled by the system of events with three subsystems: intact, transitive and collapse.

The system can be written as ${}^1S = {}^1_1S = \left({}^1_1S^i + {}^1_1S^t + {}^1_1S^c\right)$.

The probability of occurrence of intact state on the first functional level is calculated as:

$$p\left({}^1_1E^i_1\right) = p\left({}^1_1S^i\right) = \prod_{i=1}^{11} p\left({}^1_1A^i_i\right) = 0{,}9752.$$

The probabilities of transitive can be calculated likewise:

$$p\left({}^1E^t_1\right) = \prod_{z=1}^{6} p\left({}^1A^i_z\right)\cdot p\left({}^1A^c_7\right)\cdot \prod_{z=8}^{11} p\left({}^1A^i_z\right) = 0{,}1853\times10^{-3}$$

$$p\left({}^1E^t_2\right) = \prod_{z=1}^{7} p\left({}^1A^i_z\right)\cdot p\left({}^1A^c_8\right)\cdot \prod_{z=9}^{11} p\left({}^1A^i_z\right) = 0{,}1853\times10^{-3}$$

$$p\left({}^1E^t_3\right) = \prod_{z=1}^{9} p\left({}^1A^i_z\right)\cdot p\left({}^1A^c_{10}\right)\cdot p\left({}^1A^i_{11}\right) = 0{,}5985\cdot10^{-2}$$

The results for the remaining events are given:

$p\left({}^1E^t_4\right) = 0{,}5985\cdot10^{-2}$, $p\left({}^1E^t_5\right) = 0{,}3522\cdot10^{-7}$,

$p\left({}^1E^t_6\right) = 0{,}1137\cdot10^{-5}$, $p\left({}^1E^t_7\right) = 0{,}1137\cdot10^{-5}$,

$p\left({}^1E^t_8\right) = 0{,}1137\cdot10^{-5}$, $p\left({}^1E^t_9\right) = 0{,}1137\cdot10^{-5}$,

$p\left({}^1E^t_{10}\right) = 0{,}3673\cdot10^{-4}$, $p\left({}^1E^t_{11}\right) = 0{,}2162\cdot10^{-9}$,

$p\left({}^1E^t_{12}\right) = 0{,}2162\cdot10^{-9}$, $p\left({}^1E^t_{13}\right) = 0{,}6981\cdot10^{-8}$,

$p\left({}^1E^t_{14}\right) = 0{,}6981\cdot10^{-8}$, $p\left({}^1E^t_{15}\right) = 0{,}1327\cdot10^{-11}$

Probability of the subsystem of transitive events is:

$$p\left({}^1_1S^t\right) = \sum_{j=1}^{15} p\left({}^1E^t_j\right) = 0{,}0124.$$

The subsystem of transitive events can be considered as the residual strength of a redundant structure in case of failure of some components. The probabilities of individual collapse events are not listed here due to large number of events. Probability of the subsystem of collapse events is:

$$p\left({}^1_1S^c\right) = \sum_{j=1}^{2032} p\left({}^1E^c_j\right) = 0{,}0125.$$

The second functional level occurs when the one of longitudinal, HP2 or HP3, collapses. The remaining structure is still operational but with reduced carrying capacity. It is assumed that collapsed longitudinal has no carrying capacity at all. The load is then redistributed to the remaining elements of the deck and the new values of reliability and probability of failure must be calculated. When one longitudinal fails the structural configuration reduces to 6 carrying elements: 3×plating, T longitudinal and remaining HP longitudinals. Hence, the system of events on the functional level l=2 has 9 basic events with probabilities calculated in a same way as for the first functional level:

$p\left({}^2_1A^i_1\right) = 0{,}9955$, $p\left({}^2_1A^i_2\right) = 0{,}9955$, $p\left({}^2_1A^i_3\right) = 0{,}9955$,

$p\left({}^2_1A^i_4\right) = 0{,}9999$, $p\left({}^2_1A^i_5\right) = 0{,}9960$, $p\left({}^2_1A^i_6\right) = 0{,}9999$,

$p\left({}^2_1A^i_7\right) = 0{,}9999$, $p\left({}^2_1A^i_8\right) = 0{,}9855$, $p\left({}^2_1A^i_9\right) = 0{,}9855$.

Probabilities of occurrence of compound events ${}^2_jE_i$ can easily calculated as for the first level. For example, the probability of intact event is: $p\left({}^2_1E^i_1\right) = \prod_{i=1}^{9} p\left({}^2_1A^i_i\right) = 0{,}9541.$

The second level also includes three subsystems: intact, collapse and transitive, since there are events on this level that can cause emergence of the third functional level (failure of HP2 or HP3 longitudinal). The second level can be modeled by following system of events:

$${}^2S = \left({}^1S^i, {}^2_1S\cap{}^1E^t_1,\ldots, {}^2_jS\cap{}^1E^t_j,\ldots, {}^{\;2}_{{}^1N^t}S\cap{}^1E^t_{{}^1N^t}, {}^1S^c\right).$$

This level consist of non-transitive events from the first level (intact, collapse) and new states that emerge due to occurrence of some transitive event on the previous level (compound events ${}^2_1S\cap{}^1E^t_1,\ldots$). From 15 transitive event on the first level the 15 compound states j=1,2,…,15 will emerge on the second level, but only six of those are transitive. Every transitive compound state has 3 transitive events causing emergence of 18 new states on the third level (Fig.4). The probabilities of occurrence of transitive state j=1 on the second level are calculated as follows:

$$p\left({}^2_1E^t_1\right) = \prod_{i=1}^{6} p\left({}^2_1A^i_i\right)\cdot p\left({}^2_1A^c_7\right) p\left({}^2_1A^i_8\right) p\left({}^2_1A^i_9\right) = 0{,}9555\cdot10^{-5}$$

$$p\left({}^2_1E^t_2\right) = \prod_{i=1}^{8} p\left({}^2_1A^i_i\right) p\cdot\left({}^2_1A^c_9\right) = 0{,}1407\cdot10^{-1}$$

$$p\left({}^2_1E^t_3\right) = \prod_{i=1}^{6} p\left({}^2_1A^i_i\right)\cdot p\left({}^2_1A^c_7\right) p\left({}^2_1A^i_8\right) p\left({}^2_1A^c_9\right) = 0{,}1408\cdot10^{-6}$$

Probabilities of the remaining 15 transitive events and compound collapse events can be calculated in a same way Since the total number of compound events on the second level is ${}^2N$ = 9713 the probabilities of remaining events are not listed here. The reliability $p\left({}^2S^i\right)$ and probability of failure $p\left({}^2S^c\right)$ for the second functional state are:

$$p\left({}^2S^i\right) = \sum_{j=1}^{{}^1N^t} p\left({}^1E^t_j\right) \sum_{i=1}^{{}^2_jN^i} p\left({}^2_jE^i_i\right) = \sum_{j=1}^{{}^1N^t} p\left({}^1E^t_j\right) p\left({}^2_jS^i\right) = 0{,}0118$$

$$p\left({}^2S^c\right) = \sum_{j=1}^{{}^1N^t} p\left({}^1E^t_j\right) \sum_{i=1}^{{}^2_jN^c} p\left({}^2_jE^c_i\right) = \sum_{j=1}^{{}^1N^t} p\left({}^1E^t_j\right) p\left({}^2_jS^c\right) = 5{,}6\cdot10^{-4}$$

The cumulative (overall) reliability of the system includes all the probabilities of intact states on the first level as well as all the probabilities of compound intact states on the second level and transitive states on the first level:

$$p\left({}^2S^o\right) = p\left({}^1S^i\right) + p\left({}^2S^i\right) = 0{,}9869.$$

Adequately, the total probability of failure is then:

$$p\left({}^2S^f\right) = p\left({}^1S^c\right) + p\left({}^2S^c\right) = 0{,}0125 + 5{,}689\cdot10^{-4} = 0{,}0131.$$

The third functional level l = 3 arises when failure of both longitudinals, HP2 and HP3, occurs and not necessarily simultaneously. On the third level there is 5 structural elements remained to carry the load (Fig. 3): 3 × plating, T longitudinal and HP1 longitudinal. That structural configuration represents non-redundant structure, since further damage of any element will cause the entire structure to collapse. The number of basic events on the third level is connected to the number of possible types of failure of the elements hence giving 7 basic events ${}_{j}^{3}n = 7$.

There are 18 operational states at this level emerging from j=1,2,…,18 compound transitive events on the second functional level. Each of the 18 operational states on the third level has one intact state. The remaining events are all collapse events, i.e. there are no transitive events on the third level.

The total number of events on the third level is ${}^{3}N$=12017. The probabilities of the events for the j = 1 are:

$p\left({}_{1}^{3}A_{1}^{i}\right) =$ 0,9874, $p\left({}_{1}^{3}A_{2}^{i}\right) = 0{,}9874$, $p\left({}_{1}^{3}A_{3}^{i}\right) = 0{,}9874$,

$p\left({}_{1}^{3}A_{4}^{i}\right) = 0{,}9999$, $p\left({}_{1}^{3}A_{5}^{i}\right) = 0{,}9887$, $p\left({}_{1}^{3}A_{6}^{i}\right) = 0{,}9999$,

$p\left({}_{1}^{3}A_{7}^{i}\right) = 0{,}9660$

Probabilities of intact and collapse states are:

$$p\left({}_{j}^{3}S^{i}\right) = p\left({}_{j}^{3}E_{1}^{i}\right) = \prod_{i=1}^{7} p\left({}_{j}^{3}A_{i}^{i}\right) 0{,}919547, \quad j = 1,2,...,18$$

$$p\left({}_{j}^{3}S^{c}\right) = 0{,}0804, j = 1,2,...,18.$$

The third level is modeled as the system of events consisting of the non-transitive events on the second level together with the new states on the third level (compound events):

$$
{}^{3}S = \left(\begin{array}{l}
{}^{1}S^{i} + {}^{1}S^{c} + {}_{1}^{2}S \cap {}^{1}E_{1}^{t} + {}_{2}^{2}S \cap {}^{1}E_{2}^{t} + \\
+\ldots + {}_{15}^{2}S \cap {}^{1}E_{15}^{t} + \\
+\left({}_{1}^{3}S \cap {}^{2}E_{1}^{t}\right) \cap {}^{1}E_{1}^{t} + \left({}_{2}^{3}S \cap {}^{2}E_{2}^{t}\right) \cap {}^{1}E_{1}^{t} + \\
+\left({}_{3}^{3}S \cap {}^{2}E_{3}^{t}\right) \cap {}^{1}E_{1}^{t} + \left({}_{4}^{3}S \cap {}^{2}E_{4}^{t}\right) \cap {}^{1}E_{2}^{t} + \\
+\left({}_{5}^{3}S \cap {}^{2}E_{5}^{t}\right) \cap {}^{1}E_{2}^{t} + \left({}_{6}^{3}S \cap {}^{2}E_{6}^{t}\right) \cap {}^{1}E_{2}^{t} + \\
+\ldots + \\
+\left({}_{18}^{3}S \cap {}^{2}E_{18}^{t}\right) \cap {}^{1}E_{9}^{t} + \left({}_{18}^{3}S \cap {}^{2}E_{18}^{t}\right) \cap {}^{1}E_{9}^{t} + \\
+\left({}_{18}^{3}S \cap {}^{2}E_{18}^{t}\right) \cap {}^{1}E_{9}^{t}
\end{array}\right)
$$

Compound probabilities on the third level are equal to the probabilities of the transitive events on the previous levels:

$$p\left[\left({}_{j}^{3}S \cap {}^{2}E_{j}^{t}\right) \cap {}^{1}E_{k}^{t}\right] = p\left({}_{j}^{3}S\right) p\left({}^{2}E_{j}^{t}\right) p\left({}^{1}E_{k}^{t}\right).$$

Reliabilities $p\left({}^{3}S^{i}\right)$ and probabilities of failure $p\left({}^{3}S^{c}\right)$ are:

$$
\begin{aligned}
p\left({}^{3}S^{i}\right) = & \sum_{k=1}^{4}\sum_{j=1}^{3} p\left({}^{1}E_{k}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E^{i}\right) + \\
& + \sum_{j=1}^{3} p\left({}^{1}E_{6}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E^{i}\right) + \\
& + \sum_{j=1}^{3} p\left({}^{1}E_{9}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E^{i}\right) = 1{,}5977 \cdot 10^{-4}
\end{aligned}
$$

$$
\begin{aligned}
p\left({}^{3}S^{c}\right) = & \sum_{k=1}^{4}\sum_{j=1}^{3} p\left({}^{1}E_{k}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E^{c}\right) + \\
& + \sum_{j=1}^{3} p\left({}^{1}E_{6}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E^{c}\right) + \\
& + \sum_{j=1}^{3} p\left({}^{1}E_{9}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E^{c}\right) = 1{,}3979 \cdot 10^{-5}
\end{aligned}
$$

The third functional level can be also viewed as a system compound of subsystems ${}^{3}S = \left({}^{3}S^{o} + {}^{3}S^{f}\right)$, where the subsystem of operational states is ${}^{3}S^{o} = \left({}^{1}S^{i} + {}^{2}S^{i} + {}^{3}S^{i}\right)$ and subsystem of failure states is ${}^{3}S^{f} = \left({}^{1}S^{c} + {}^{2}S^{c} + {}^{3}S^{c}\right)$.

The overall reliability includes all probabilities of intact states of the first level as well as compound probabilities of transitive and intact states on the second and the third level is:

$$p\left({}^{3}S^{o}\right) = p\left({}^{1}S^{i}\right) + p\left({}^{2}S^{i}\right) + p\left({}^{3}S^{i}\right) = 0{,}9870.$$

The total probability of collapse includes all probabilities of collapse states on the first level as well as compound probabilities of transitive and collapse states on the second and third level:

$$p\left({}^{3}S^{f}\right) = p\left({}^{1}S^{c}\right) + p\left({}^{2}S^{c}\right) + p\left({}^{3}S^{c}\right) = 0{,}0130.$$

One can see that $p\left({}^{3}S^{o}\right) + p\left({}^{3}S^{f}\right) = 0{,}9870 + 0{,}0130 = 1$, i.e. the system of events that models the structure is complete.

3.1 Redundancy calculation

The maximum entropy of the system of events on the first functional level is log (${}^{1}N$)=log (2048) = 11,0 [19]. The conditional entropy of the first functional level with respect to the operational modes denoted as redundancy with respect to the operational mode is according to (4) and (5):

$$
\begin{aligned}
& H\left({}^{1}\mathrm{S} / {}^{1}\mathrm{S}^{o}\right) = RED\left({}^{1}\mathrm{S}^{o}\right) = \\
& = -\frac{p\left({}^{1}E_{1}^{i}\right)}{p\left({}^{1}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{1}^{i}\right)}{p\left({}^{1}\mathrm{S}^{i}\right)} - \sum_{j=1}^{15} \frac{p\left({}^{1}E_{j}^{t}\right)}{p\left({}^{1}\mathrm{S}^{t}\right)} \log \frac{p\left({}^{1}E_{j}^{t}\right)}{p\left({}^{1}\mathrm{S}^{t}\right)} = 0{,}1125
\end{aligned}
$$

The conditional entropy of the system of events for the second level is then:

$$
\begin{aligned}
& H\left({}^{2}\mathrm{S} / {}^{2}\mathrm{S}^{i}\right) = RED\left({}^{2}\mathrm{S}^{i}\right) = \\
& = -\sum_{j=1}^{{}^{1}N^{t}=15} \frac{p\left({}^{1}E_{j}^{t}\right) p\left({}_{j}^{2}E_{1}^{i}\right)}{p\left({}^{2}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{j}^{t}\right) p\left({}_{j}^{2}E_{1}^{i}\right)}{p\left({}^{2}\mathrm{S}^{i}\right)} = 1{,}0245
\end{aligned}
$$

Redundancy of the system of events after inclusion of the third level in the model is calculated by:

$$
\begin{aligned}
& H\left({}^{3}\mathrm{S} / {}^{3}\mathrm{S}^{i}\right) = RED\left({}^{3}\mathrm{S}^{i}\right) = \\
& = -\sum_{j=1}^{3} \frac{p\left({}^{1}E_{1}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{1}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} - \\
& -\sum_{j=4}^{6} \frac{p\left({}^{1}E_{2}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{2}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} - \\
& -\sum_{j=7}^{9} \frac{p\left({}^{1}E_{3}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{3}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} - \\
& -\sum_{j=10}^{12} \frac{p\left({}^{1}E_{4}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{4}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} - \\
& -\sum_{j=13}^{13} \frac{p\left({}^{1}E_{6}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{6}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} - \\
& -\sum_{j=16}^{18} \frac{p\left({}^{1}E_{9}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)} \log \frac{p\left({}^{1}E_{9}^{t}\right) p\left({}^{2}E_{j}^{t}\right) p\left({}_{j}^{3}E_{1}^{i}\right)}{p\left({}^{3}\mathrm{S}^{i}\right)}
\end{aligned}
$$

$RED\left({}^{3}\mathrm{S}^{i}\right) =$ 1,1971 and $RED\left({}^{3}\mathrm{S}^{i}\right)_{\max} = 4{,}1699$.

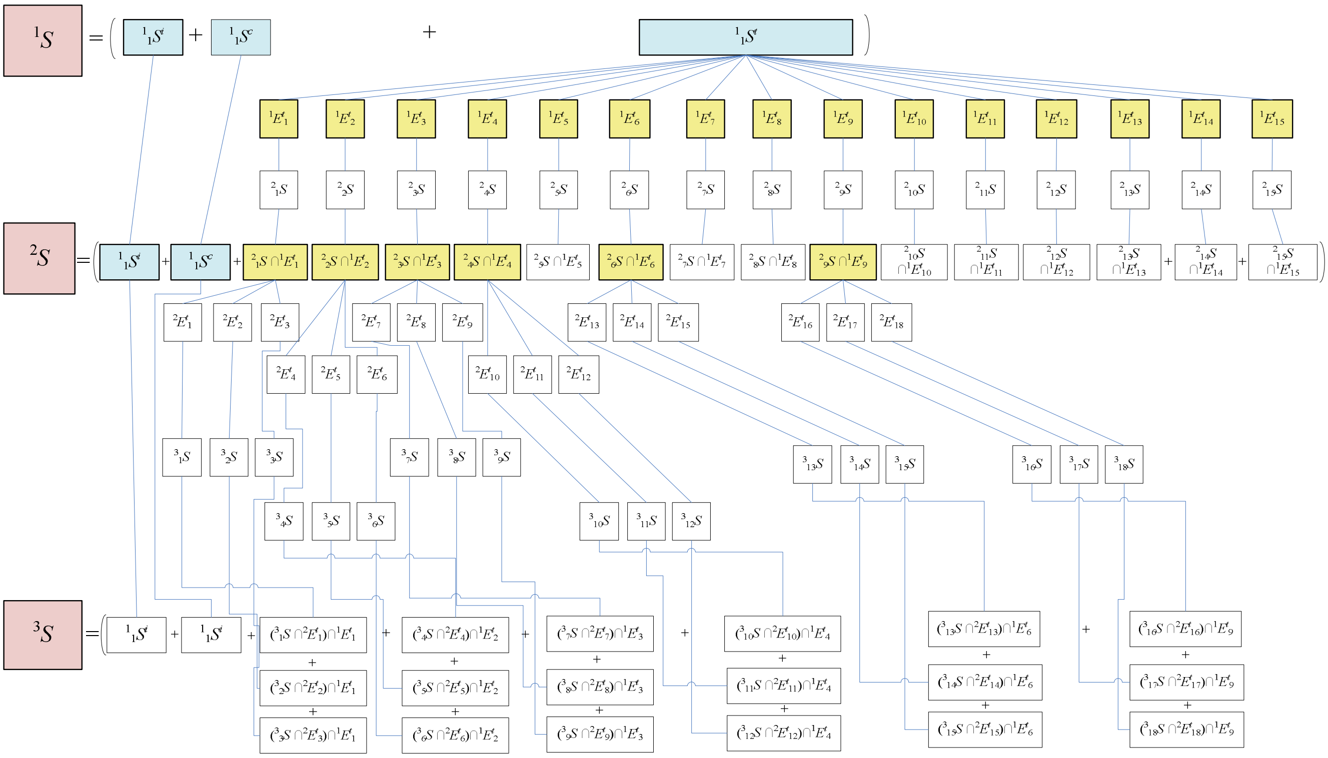


Figure 4. EOSA model of multi-level system of events that represents part of the deck structure

3.2 Redundancy analysis

The starting configuration, Fig. 1, is modified by changing spacing between longitudinals. The structural redundancy is investigated for spacing b_1 and b_2 ranging between 63cm and 97cm (initial spacing is 80cm). For a number of obtained systems of events the redundancies as well as the reliabilities and probabilities of failure of the systems are calculated. The following requirements have to be satisfied in the analysis:

a) The reliability of a modified structure must be equal or larger than the reliability of the initial structure.

b) The weight of a modified structure has to be equal to that of initial structure, i.e. the increase of the system's redundancy is not gained by increasing the strength of the existing structural elements or by introducing additional elements to the structure.

The results of the analysis are presented in Fig.5 and Table 7.

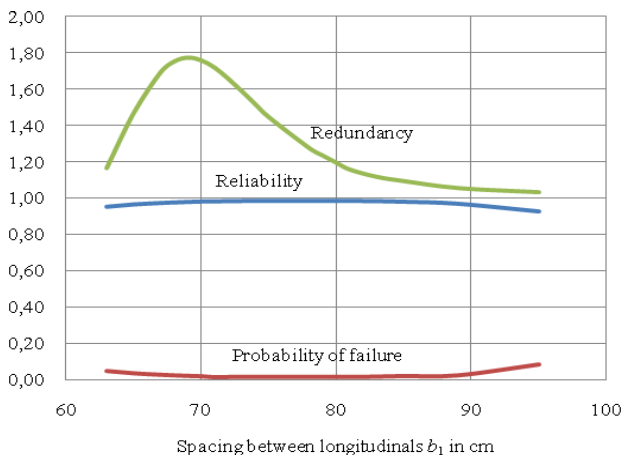


Figure 5. Redundancy based design - results

Table 7. Results of the redundancy analysis

Spacing between longitudinals in cm	Reliability $p\left({}^3S^o\right)$	Prob. of failure $p\left({}^3S^c\right)$	$RED\left({}^3S^i\right)$
b_1=63; b_3=97	0,954	0,045	1,1663
b_1=65; b_3=95	0,967	0,033	1,4705
b_1=67; b_3=93	0,975	0,025	1,6959
b_1=68; b_3=92	0,978	0,022	1,7553
b_1=69; b_3=91	0,981	0,019	1,7769
b_1=70; b_3=90	0,983	0,017	1,7621
b_1=71; b_3=89	0,984	0,120	1,7214
b_1=72; b_3=88	0,985	0,013	1,6607
b_1=73; b_3=87	0,986	0,013	1,5924
b_1=74; b_3=86	0,987	0,013	1,5199
b_1=75; b_3=85	0,987	0,013	1,4486
b_1=78; b_3=82	0,987	0,013	1,2774
b_1=79; b_3=81	0,987	0,013	1,2362
$b_1 = b_3 = 80$	0,987	0,013	1,1971
b_1=81; b_3=79	0,987	0,013	1,1502
b_1=85; b_3=75	0,982	0,018	1,0957
b_1=89; b_3=71	0,971	0,022	1,0573
b_1=95; b_3=65	0,928	0,082	1,0342

The following points are outlined from this analysis:

- The study presented herein allows design selection based on maximal redundancy, for different levels of primary, secondary and overall reliability.
- For a number of considered structural configurations reliability changes only slightly while the redundancy has clearly expressed maximum.
- The maximal redundancy, is attained for b_1=69cm and b_3=91cm, indicating the most uniformly distributed compound probabilities of intact modes

for each structural element. Maximum redundancy is achieved without incrasing the weight of the structure.

- For b_1=74cm and b_3=86cm, the redundancy of reconfigured deck structure is larger than that of the intial structural configuration while at a same time the reliability remains unchanged. This shows that EOSA can be used to indicate a different design solutions without affecting existing criteria.

CONCLUSION

This paper indicates that the event-oriented analysis of redundant ship structural components exposed to successive component failures, which change the system configuration and provoke a redistribution of loads and capabilities, is a complicated but feasible task. The event-oriented analysis is performed entirely in the event space where different types of events can be identified and involved in the analysis. The relation between the event space and the physical world can be defined empirically by statistical methods, theoretically by employing random variable models or by their combinations. Ship structural components acquire new functional states after reconfiguration due to element failures and a redistribution of loads. Minimal probabilistic safety requirements are considered by employing reliability methods. The applied redundancy measure accounts for a number of events as well as for probability distributions and is expressed by the conditional entropy of transitive and operational functional modes, independent of the system reliability and residual strength. EOSA provides probabilities of successive operational levels and functional states, regardless of the ordering and succession of events. High redundancy indicates a uniform distribution of probabilities, as well as more economical allocation of system capabilities with respect to the system performance.

An example of a part of longitudinally stiffened structure located at deck amidships of a double-hull tanker under random loads demonstrates application of EOSA in description of multi-level transitive behavior of redundant ship structural components. The part of the deck is modeled by systems of events on three functional levels. Typical types of failure of considered structural elements are included in the analysis. Intact, transitive and collapse event sequences, independent of a time, are appropriately grouped into subsystems of different functional modes. The proper partitioning of systems allows measuring redundancy as conditional entropy of operational subsystem of events. The example indicates that for the complete insight into redundant system behavior all functional levels should be considered. The check of reliability of operational states on the second and third level, which emerge after failure of structural element occurs, are important for assessing the overall system redundancy. The redundancy analysis showed how design variables influence reliability and redundancy. A detailed numerical investigation illustrates the results of redundancy analysis in order to demonstrate benefits of the EOSA. It is demonstrated that it is possible to maximize redundancy without increasing the weight of a structure. Moreover, it is possible to detect a structural configuration with higher redundancy without affecting existing design criteria like reliability of the structure. The example confirmed that components of ship structures can be modeled by systems of events and that redundancy of ship structures can be quantified in a different and useful manner.

Possible problems in application of EOSA to ship structures should also be pointed. EOSA of complex parts of ship structures involves grueling counting and grouping of a large number of events. Also, highly-redundant parts of ship structures are multi-level operational with possible complex interactions between functional levels provoking considerable increase of computational efforts. These problems can be partially reduced by careful partitioning of systems of events and separate calculations for subsystems on different functional levels due to additivity property of entropy. The numerical difficulties can be overcomed by powerful computational means.

But regardless of some difficulties this paper points out that EOSA approach may contribute to design improvement and assure more appropriate lifetime service of ship structural components under uncertain circumstances.

REFERENCES

1. Casella, G., Dogliani, M., Guedes Soares, C., 1996, "Reliability Based Design of the Primary Structure of Oil Tankers", *Proceedings of the 15th International conference on Offshore Mechanics and Arctic Engineering (OMAE'96),* C. Guedes Soares, et al (Eds.), ASME, New York, Vol. II, pp. 217-224.
2. Guedes Soares, C., Teixeira, A.P., 2000, "Structural reliability of two bulk carrier designs", Marine Structures **13**, pp. 107–128.
3. Parunov, J., Guedes Soares, C., 2007, "Effects of Common Structural Rules on hull-girder reliability of an Aframax oil tanker", Reliability Engineering & System Safety, 2007, doi:10.1016/j.ress.2007.07.011.
4. Žiha, K., 2000, "Event Oriented System Analysis", Probabilistic Engineering Mechanics, Vol. **15**(3), pp. 261-275.
5. Žiha, K., 2000, "Redundancy and Robustness of Systems of Events", Probabilistic Eng. Mech., Vol. **15**(4), pp. 347-357.
6. Nikolaidis, E., Kapania, R.K., 1990, "System reliability and redundancy of marine structures: a review of the state of the art", Journal of Ship Research; Vol. **34**(1).
7. Žiha, K., 2001, "Event –oriented Analysis of Series Structural Systems", Structural Safety, Vol. **23**, pp. 1-29
8. Hendawi, S., Frangopol, D. M., 1994, "System Reliability and Redundancy in Structural Design and Evaluation", Structural Safety, Vol. **16**, pp. 47-71.
9. Stiansen, S.G., 1984, "Interrelation between Design, Inspection and Redundancy in Marine Structures", *The Role of Design, Inspection and Redundancy in Marine Structural Reliability, Proceeding,.* National Academy Press Washington D.C.
10. Rao, S. S., 1992, *Reliability Based Design*, McGraw-Hill, NY.
11. Gnedenko, B, Ushakov, I., 1995, *Probabilistic Reliability Engineering*, Wiley, New York.

12. De Oliveira, J.G., Zimmer, R.A., 1984, "Redundancy Consideration in the Structural Design of Floating Offshore Platforms", *The Role of Design, Inspection and Redundancy in Marine Structural Reliability, Proceedings,* National Academy Press, Washington.
13. Chen, K., Zhang, S., 1996, "Semi-probabilistic method for evaluating systems redundancy of existing offshore structures", Ocean Engineering Vol. **23**(6), pp. 455-464.
14. Palion, C., Shinozuka, M., Chen, Y. N., 1987, "Reliability Analysis of Offshore Structures", *Marine Structural Reliability Symposium, SNAME*, New York.
15. De, R., S., Karamchandani, A., Cornell, C., A., 1990, "Study of redundancy in near-ideal parallel structural systems", *Structural Safety and Reliability*, *ASCE*, Vol. **II**, pp. 975-982.
16. Shannon, C.E., Weaver, W., 1949, *The mathematical theory of communication*, Urbana University of Illinois Press.
17. Renyi, A., 1970, *Probability theory*, Amsterdam North-Holland.
18. Žiha, K., 2003, "Redundancy based design by event oriented analysis", Transactions of FAMENA, Vol. **27** (2), Univesity of Zagreb, pp. 1-12.
19. Žiha, K., 2003, "Event-oriented analysis of fail-safe objects", Transactions of FAMENA, Vol. **27** (1), Univesity of Zagreb, pp. 11-22.
20. Blagojević, B., 2005, *Modeling of ship structural systems by events*, PhD Thesis, Zagreb, Croatia (in Croatian).
21. Khinchin, A.I., 1957, *Mathematical Foundations of Information Theory*, Dover Publications, NY.
22. Aczel, J., Daroczy, Z., 1975, *On measures of information and their characterization*, Academic Press, New York.
23. Žiha, K., 2001, "Uncertainty of multi-level systems of events", *Proceedings ITF2001 Conference*, Pula, Croatia, pp.
24. Det Norske Veritas, 1995, *Ship's Load and Strength Manual*, Oslo.
25. Det Norske Veritas, 2003, *Rules for Classification of Ships*, Oslo.
26. Ditlevsen, O., Madsen, H. O., 1996, *Structural Reliability methods*, Wiley, NY.
27. Assakkaf, I.A., 1998, *Reliability-Design of Panels and Fatigue Details of Ship Structures*, PhD Thesis, University of Maryland.
28. Atua, K., Assakkaf, I., Ayyub, M., 1996, "Statistical Characteristics of Strength and Load Random Variable of Ship Structures", *Proc. 7th Specialty Conference on 'Probabilistic Mechanisms and Structural Reliability', ASCE*, Massachusetts.
29. Guedes Soares, C., Dogliani, P M., 2000, "Probabilistic modeling of time-varying still-water load effects in tankers, Mar Struct 13, pp. 129–143.
30. Guedes Soares, C., Moan T., 1988, "Statistical analysis of still-water load effects on ship structures", Transactions of SNAME, New York, Vol **96** (4), pp. 129–56.