

# Fatigue Yield

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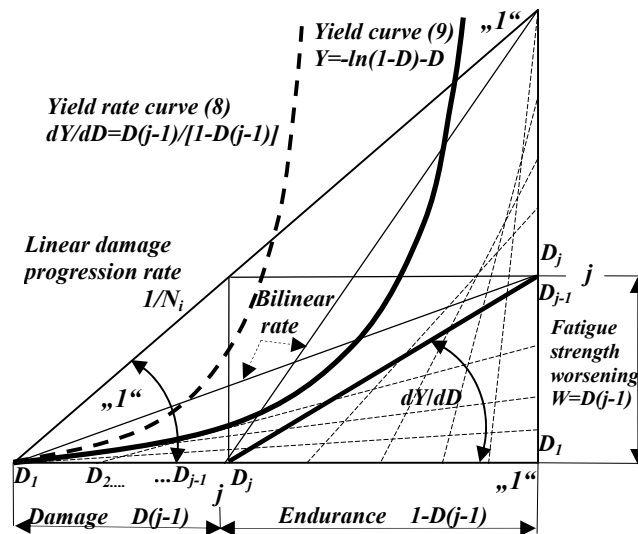
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**Abstract:** The paper starts with a brief review of fatigue damage progression under block loadings and then investigates in more details a fatigue yield model. The study identifies the fatigue yield as a cause-effect interaction of damage and endurance rather than as a physical process in the traditional fatigue engineering. The experimental data under variable and constant stress amplitude loadings reported up to date were used for the evaluation of the fatigue yield approach. Analytical, numerical and recursive procedures for fatigue yield assessments were carried out and illustrated by examples.

**Keywords:** block loading, life prediction, variable amplitude fatigue, damage progression, fatigue yield.

## 1. Introduction

The damage progression theories have been dominating fatigue research in engineering since commencement of the linear damage rule, e.g. review by Fatemi and Yang[9]. Therefore, the fatigue yield approach in this study primarily adapts to the practical engineering methods such as the basic linear damage rule (LDR) by Palmgren [1]-Miner [2] and to the nonlinear Marco-Starkey model [3]. The study also follows the damage theories based on endurance limit reduction Kommers[4] and Bennett [5], theories of load interaction effects Corten-Dolon [6] and Freudenthal-Heller [7] and to the double linear rule (DLDR) or double damage curve (DDCA) approaches Manson, Halford [8]. The paper reveals the fatigue yield as a fatigue life shortening only due to the damage-endurance interaction or fatigue strength worsening due to formerly accumulated damages under block loadings. Consequently the fatigue yielding is a nonlinear ageing process supplementary to the linear damage progression that for itself accounts for material properties and types of fracture. The reported results of laboratory fatigue experiment under variable amplitude block loads Socha [10, 11] were borrowed for the fatigue yield concept evaluation as well as the approved standard S-N curves accompanied with fatigue strength reference values obtained by the constant stress amplitude tests, IIW [12], also (ECCS) Fatigue Recommendations [13] and Eurocode 3 [14]. However, variable amplitude load fatigue tests are rare and involve additional uncertainties Johannesson and Svensson [15, 16]. Finally, the paper puts forward practical amendments of yield effects on linear fatigue damage rules.



**Fig. 1.** Models of damage progression and fatigue yield

## 2. Damage progression

The damage progression rate (DPR) under  $j^{\text{th}}$  loading block of  $n_j$  load cycles for  $i^{\text{th}}$  stress amplitude  $\Delta\sigma_i$  also relating to the fatigue lifetime  $N_i$  Socha [10, 11], Fig. 2a, is of the general form:

$$\frac{dD_{j/i}}{dn_j} = f[\Delta\sigma_i(N_i)] \quad (1)$$

The fatigue damage fraction is approximated for practical purposes by piece-wise linearization of the damage progression rate (1) over blocks of  $n_j$  cycles of  $\Delta\sigma_i$  stress amplitude, as follows:

$$D_{j/i} = \int_n \frac{dD}{dn} dn = n_j \cdot \frac{dD_{j/i}}{dn_j} \quad (2)$$

The commonly adopted linear rule takes for granted that the damage progression rate DPR (1) is linear and constant in amount of  $dD_{j/i}/dn_j = 1/N_i$  (LDPR), Fig. 2a, at any stress amplitude  $\Delta\sigma_i$ .

The Palmgren-Miner [1,2] linear damage rule estimates the cumulative fatigue damage  $D(k)$  up to  $k$  block loadings by summing up all contributing damage fractions  $D_{j/i}$  irrespective of their sequence that are simply taken as the cyclic ratio  $r_{j/i} = n_j/N_i$  for all  $i$ , as it is put down next:

$$D(k) = \sum_{j=1}^k r_{j/i} \quad (3)$$

The Marco-Starkey model [3] introduces the exponential relation on successively accumulating damage fractions into (3) in the following form:

$$D(k) = \sum_{j=1}^k r_{j/i}^\alpha \quad (4)$$

Bilinear models (double linear damage rule DLDR or the double damage curve approach DDCA) by Manson, Halford [8] combines the slower starting fatigue escalation and the accelerated damage progression to fatigue failure, Fig. 1. The relative Miner rule by Schütz and others, e.g. Johannesson and Svensson [15], employs S-N data from constant amplitude tests, and then estimates the parameters from spectrum reference tests. Proposal of P. Marti in PhD thesis of L. Eskola [17], considers the residual fatigue damage on concrete structures by extending the Miners rule with the  $n_s$  value dependent on variable amplitude loading  $M = \sum 1/(N - n_s)$ .

The study starts by introducing the fatigue yield rate also appropriate as a yield factor  $y_i$  that relates the general fatigue damage progression rate DPR (1) to the linear damage progression rate  $1/N_i$  LDPR for each block load, Fig. 2a and b, in the form:

$$\frac{dY_{j/i}}{dD_j} = y_i = \left( \frac{dD_{j/i}}{dn_j} \right) / \left( \frac{1}{N_i} \right) \quad (5)$$

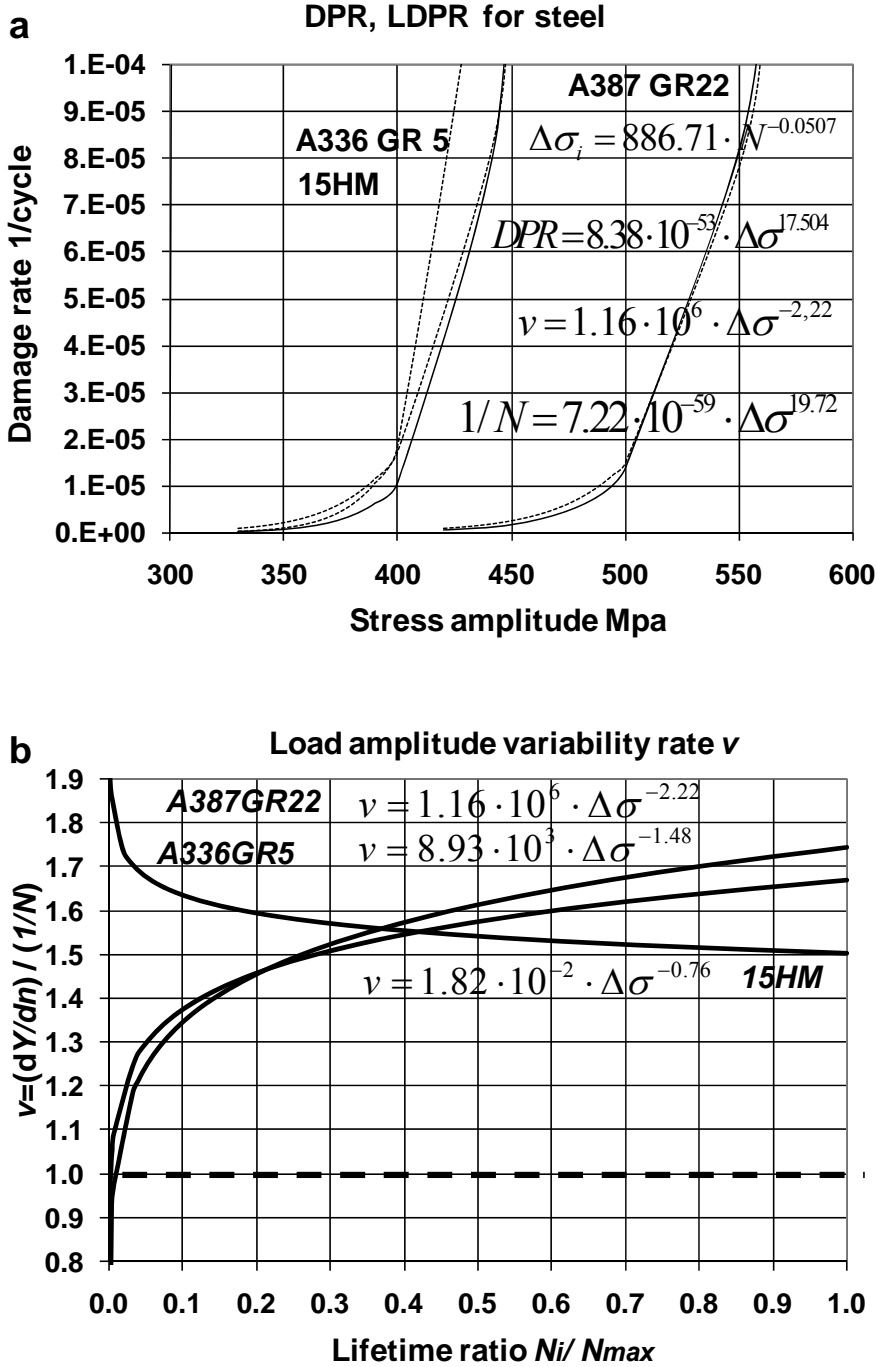
The yield factor  $y_i$  (5) corrects the LDR (3) for the effect of yielding as shown:

$$D'(k) = \sum_{j=1}^k y_i \cdot r_{j/i} \quad (6)$$

In the rough and ready application of (6), an overall correction factor  $Y$  for formerly accumulated linear damage  $D(k)$  can substitute yield factor  $y_i$  for each  $\Delta\sigma_i$  as follows:

$$D'(k) = [Y \cdot D(k)]^\alpha \approx Y \cdot D(k) \quad (7)$$

The correction factor  $Y$  is attainable either by linear regression analysis on (6) or employing the crossing with  $D(r)=1$  of the  $D$ - $r$  curve (6), Figs. 3-5.



**Fig. 2.** Experimental DPR (dashed lines) and LDPR  $1/N$  (a), load variability factor  $\nu$  (b) [10, 11]

### 3. Fatigue yield

The theoretical fatigue yield model in this study takes the formerly accumulated successive fatigue damage fractions  $D_{j/i}$  for the measure of fatigue strength worsening  $W_j = \sum_{k=1}^{j-1} D_{k/i} = D_i(j-1)$  for all  $i$ .

The underlying hypothesis of the paper is that the fatigue life shortens due to strength diminution in proportion to the endurance reduction  $E_j = 1 - D_i(j-1)$ , Fig. 1. Consequently the fatigue yield rate relates the strength worsening  $W_j$  to the endurance  $E_j$  as follows:

$$\frac{dY_j}{dD} = y_j = \frac{W_j}{E_j} = \frac{(\varphi + \delta) - \delta \cdot D_i(j-1)}{1 - D_i(j-1)} \quad (8)$$

The fatigue yield rule (FYR) uses the preceding fatigue yield rate (8) to evaluate the yielding progression  $Y(D)$  in a recursive manner. In contrast to the other damage progression theories [1, 2, 3, 4, 5, 6, 7 and 8] the study reveals the fatigue yield approach in order to modify the LDR for the endurance effect on damage progression by applying the FYR. The relation (5) is the missing link between the FYR and LDR. Thus for infinitesimally small amounts of damage progressions  $D$ , the integral of (8) indicates the logarithmic and the linear components of the fatigue yield  $Y(D)$  as it is put down next:

$$Y(D) = \int_0^D \frac{dY}{dD} dD = -\varphi \cdot \ln(1-D) + \delta \cdot D \approx \sum_{j=1}^k y_j \cdot D_j \quad (9)$$

The fatigue yield intensity factor  $\delta$  in (8) influences the linear part of (9). The factor  $\varphi$  in (8) expresses the initial propensity to fatigue yield that affects the nonlinear component in (9), Figs. 3. Note that intensity affects the propensity (8). The integral (9) can also be numerically assessed up to the  $k^{th}$  block by using (8) in a recursive manner analogously to (6). The life time due to nonlinear yield  $D(Y=1)$  (9) reduces e.g. for  $\delta=-1$  and  $\varphi=+1$  at 84.14% of the life under linear progression assumption  $D$  (3), Figs. 3, 4 and 5. The adjusted correction factor in (7) is then  $Y = 1/0.8414 = 1.19$ .

#### 4. Fatigue yield evaluation on test data

The aim of the next study is to deduce information on fatigue yielding from  $\Delta\sigma - N$  statistically averaged fatigue test data commonly given by fatigue strength and life relation  $\Delta\sigma_i \cdot N_i^{1/m} = C$ .

Fatigue yielding in this study represents the fatigue strength worsening  $W$  due to earlier accumulated damages with respect to the fatigue strength  $\Delta\sigma_i$  of an intact specimen as normally used by LDR.

The reduction of the lifetime  $N_i$  in proportion to the damage progression  $D_{j/i} = n_j / N_i$  affects the fatigue strength worsening, Fig. 3, as it is put down next:

$$W = \Delta\sigma_i \cdot (1 - D_{j/i})^{-1/m} \quad (10)$$

The fatigue yield rate relates the fatigue worsening  $W$  to the fatigue strength  $\Delta\sigma$  from the appropriate S-N data as shown in a more general form:

$$\frac{dY}{dD} = \frac{W}{\Delta\sigma} = (1 - \alpha \cdot D_{j/i})^{-1/m} \quad (11)$$

Parameter  $\alpha$  in (11) represents the fatigue yield interaction intensity.

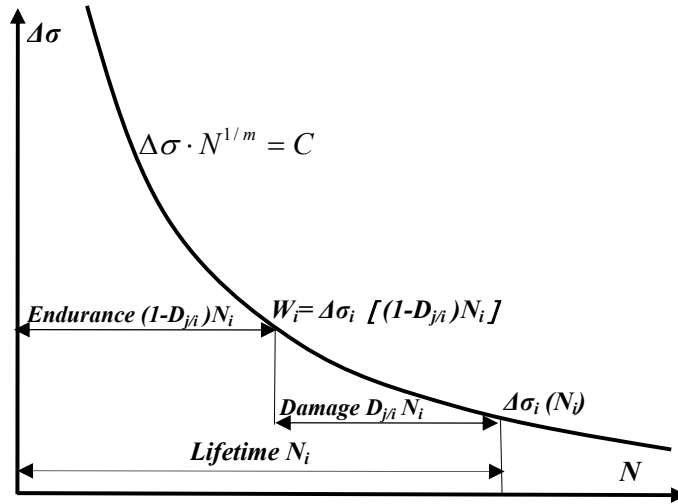
The fatigue yield is then the integral of (11), Fig. 3, in the following general form:

$$Y(D) = \int_0^D \frac{dY}{dD} dD = \alpha \cdot \frac{m}{m-1} \cdot \left[ 1 - (1 - \alpha \cdot D_{j/i})^{\frac{m-1}{m}} \right] \quad (12)$$

The lifetime under yielding follows from the condition  $Y(D_1) = 1$  (12) as shown:

$$D_1 = \frac{1}{\alpha} \left[ 1 - \left( \frac{\alpha m}{\alpha m - m + 1} \right)^{\frac{m}{m-1}} \right] \quad (13)$$

The fatigue lifetime (13) shortens by the inverse slope  $m$  of the S-N curves and indicates the resistance to fatigue yielding, Fig. 7. Moreover  $Y(D) = [1/D_1] \cdot D$  is the linearized fatigue yield approximation.



**Fig. 3.** Experimental S-N data and the fatigue yield model

The analytical fatigue yield (12) under constant stress amplitude loads for three types of steels (A336GR5:  $m=17.54$ , 15HM:  $m=20.79$ , A387GR22:  $m=19.72$ ) [10, 11], Fig. 4, are given by an average curve for all steels as follows:

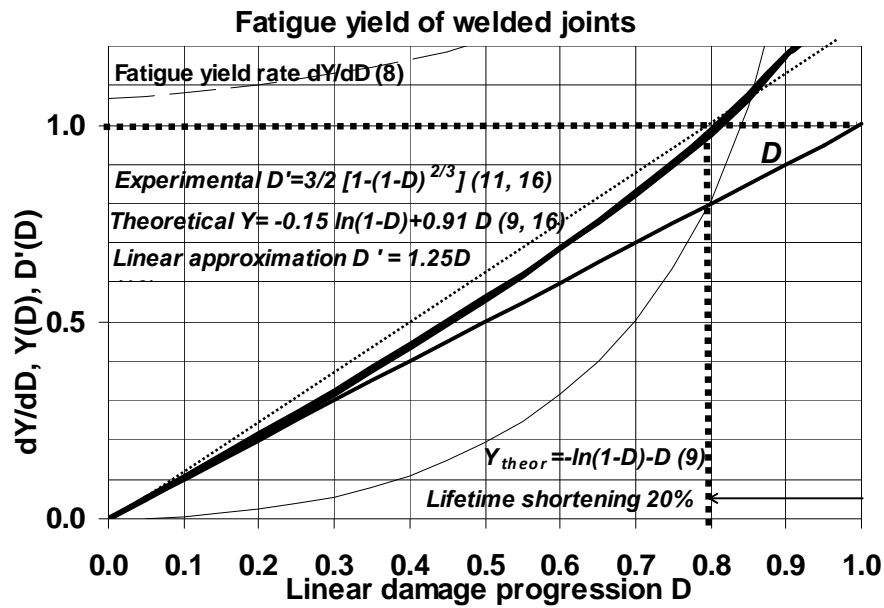
$$Y(D) = 1.06 \cdot [1 - (1 - D)^{0.94}] = -0.02 \cdot \ln(1 - D) + 1.00 \cdot D \approx 1.06 \cdot D \quad (14)$$

The yield parameters  $\varphi$  and  $\delta$  in (9) are defined by least square methods with respect to S-N data.

The fatigue yield under variable stress amplitude fatigue test data for steel Socha [10, 11], Fig. 4, is presented for an average load amplitude variability factor of  $\nu = 1.4$ , Fig. 2b, as follows:

$$Y(D) = 1.48 \cdot [1 - (1 - D)^{0.94}] = -0.02 \cdot \ln(1 - D) + 1.4 \cdot D \approx 1.48 \cdot D \quad (15)$$

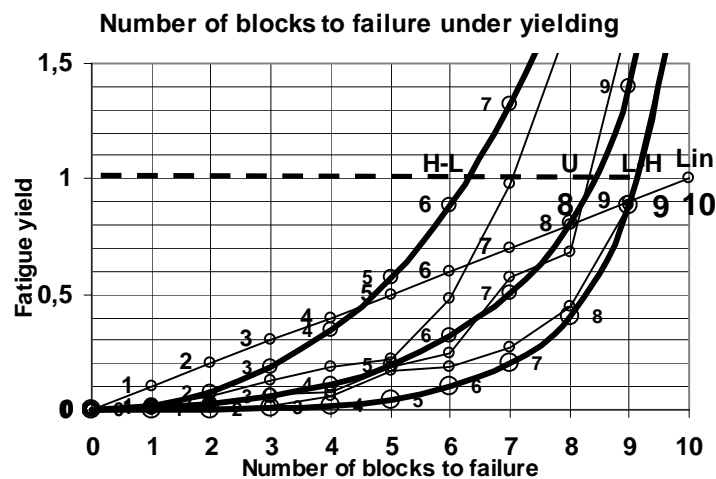




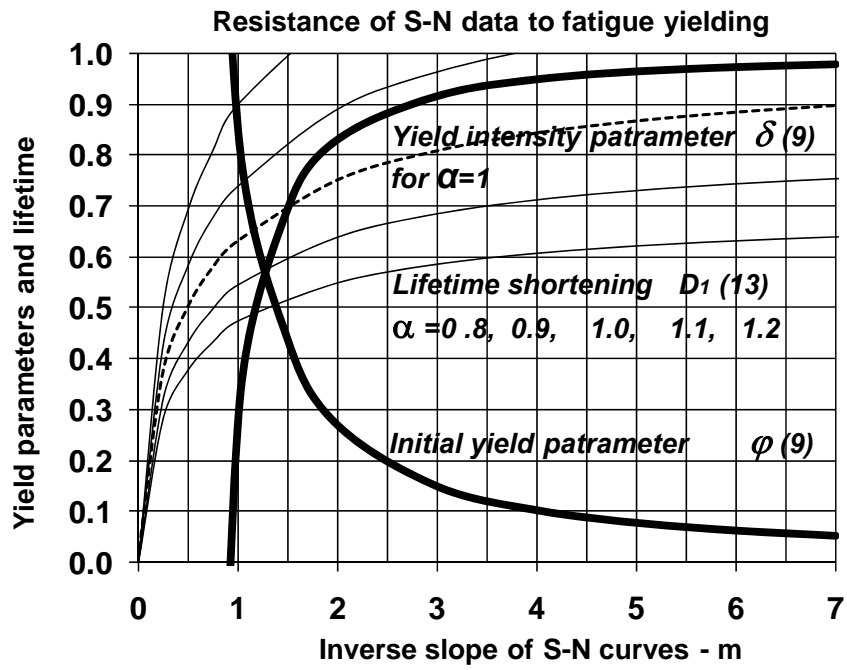
**Fig. 5.** Fatigue yield for welded joints [12, 13, 14]

The results of the study point to almost coinciding results of theoretical, experimental and numerical calculations. It is also worthwhile to note that the linear trend-lines of the fatigue yield curves are close, although higher of the linear damage progression rates, Figs. 4 and 5.

Finally the study investigates the effects of different loading sequences on a number of blocks to failure under fatigue yield. The highest value of fatigue yield  $Y[D(i)] \leq 1$  (9) defines the last  $i^{th}$  block in the yielding sequence prior to the fatigue failure, Figs. 4 and 5. The FYR elucidates that the numbers of blocks to failure depend on the damaging sequence such as Linear (Lin), Uniform (U), Low-High (L-H), High-Low (H-L), Fig. 6. High to low damaging sequences provide a fewer number of blocks to failure than the uniform and low to high sequences. The numbers of blocks for some other damage sequences (e.g. Random, L-H-L, H-L-H, L-H-L-H, H-L-H-L, Alternating, Irregular) are normally between L-H and H-L sequences, Fig. 6.



**Fig. 6.** Number of block loadings to failure under intermittent operation



**Fig. 7.** Resistance of S-N data and fatigue life under yielding



## Conclusions

An intuition that the fatigue yield information can be theoretically deduced and practically evaluated from the experimental data prompted this study. The fatigue yield model in this paper presumes the fatigue strength worsening and endurance reduction as a cause-effect interaction rather than a physical process. The theoretical fatigue yield rate  $D/(1-D)$  demonstrates how the effect-endurance reduction  $(1-D)$  influences the cause-damage progression  $D$ . The application of the fatigue yield rate with permanently changing slopes of the yield curve  $Y=-\ln(1-D)-D$  due to damage-endurance interaction implies a recursive approach to fatigue yield assessment that necessitate accounts of damaging sequences. Consequently the fatigue yield combines the nonlinear yield by logarithmic rule and the damage progression by linear rule. The theoretical fatigue yield curve can be adjusted to experimental damage progression results by setting the fatigue yield intensity factor  $\delta$  and the factor of initial propensity to yielding  $\phi$  at appropriate values.

The study indicates that the fatigue life due to yielding becomes shorter since the formerly accumulated damages reduce endurance. The presented yield model accounts for the accelerated strength worsening as a fatigue ageing process and makes possible to trace how the fatigue damage escalates slower at the beginning and then yields rapidly towards fatigue failure. The fatigue yield approach provides distinction between a numbers of variable damaging blocks to failure. High to low damaging sequences provide a fewer number of blocks to failure than the uniform and low to high loading sequences.

The observation that the linear trendlines of the nonlinear fatigue yield curves are close to the linear damage progression rates clarifies the lasting importance of the practical linear damage rule.

However, the fatigue yield procedure applied to experimental data for steel materials and welded joints provides an insight to the damage escalation and to the fatigue life shortening due to yielding. The combined procedure for fatigue yield assessment in its first step applies the traditional linear damage progression rule with the same set of experimental data. The procedure goes on by correction of the linearly accumulated fatigue damage for the yielding effect. Therefore the paper recommends the fatigue yield procedure for variable and constant amplitude intermittent block loading analysis in engineering to modify the results based on linear damage rule.

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