Tuning Elliptic Filters with a 'Tuning Biquad'

Dražen Jurišić and Neven Mijat
Electronic Systems and Information Processing Dept.
University of Zagreb
Unska 3, HR-10 000 Zagreb, Croatia
drazen.jurisic@fer.hr, neven.mijat@fer.hr

George S. Moschytz
School of Engineering
Bar-Ilan University
IL-52 900 Ramat-Gan, Israel
moschytz@isi.ee.ethz.ch

***Abstract*— This paper presents an optimum tuning procedure for high-order low-pass (LP) elliptic filters. Since elliptic filters are often used to satisfy very tight specifications, they often need to be tuned accurately. In this paper, we describe the tuning of one biquad, the 'tuning biquad', in a cascade of biquads. It is shown by Matlab simulations that the best choice for the tuning biquad consists of the pole pair with the highest pole Q ('maximum-Q poles') combined with the zero pair with the lowest frequency ('minimum-frequency zeros'). We also show how standard tuning procedures, such as those for the Tow-Thomas biquad, lead to excellent results. As an example, the tuning procedure is performed on a normalized seventh-order elliptic LP filter.**

I. Introduction

The realization of selective, elliptic, high-order, low-pass filters often presents problems with the accuracy of the cut-off frequency of the amplitude response. Contrary to previous studies that have examined optimal tuning methods for individual biquads, we investigate an approach that attempts to tune only one biquad in a filter, by selecting the pole-zero pair for the so-called 'tuning biquad' that will most effectively tune the critical characteristics of the overall higher-order elliptic lowpass filter. Having selected the pole-zero pair for the tuning biquad, the rest of the filter can be realized either by a biquad cascade, or by any other filter structure such as a ladder filter of reduced degree. Since the derivation of a reduced-order ladder filter cascaded with a tuning biquad is a separate - and non-trivial - problem (which we are presently investigating), we here present the intermediate solution to the problem of selecting an optimal tuning biquad from a higher-order biquad cascade.

II. Filter Design by Partitioning of the Filters Transfer Function

Consider a seventh-order elliptic filter which, according to [1], is referred to as CC07 25 50. Its transfer function magnitude $\alpha(\omega)$[dB] is shown in Fig. 2. In [1] it is given by the filter realization shown in Fig. 1, and by the following normalized real and imaginary values for the poles and zeros of the transfer function $T(s)$:

$$p_0 = -\sigma_0 = -0.3764$$

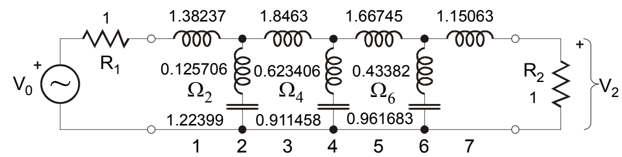

Figure 1. Doubly terminated LC-ladder filter CC 07 25 50 realization.

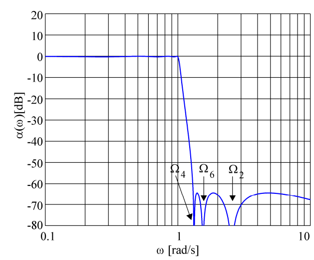

Figure 2. Magnitude of CC 07 25 50 filter.

$$\begin{aligned} p_1, p_1^* &= \sigma_1 \pm j\Omega_1 = -0.0408 \pm j\,1.0127 \\ p_3, p_3^* &= \sigma_3 \pm j\Omega_3 = -0.1459 \pm j\,0.8853 \\ p_5, p_5^* &= \sigma_5 \pm j\Omega_5 = -0.2911 \pm j\,0.5573 \\ z_2, z_2^* &= \pm j\Omega_2 = \pm j\,2.5494 \\ z_4, z_4^* &= \pm j\Omega_4 = \pm j\,1.3266 \\ z_6, z_6^* &= \pm j\Omega_6 = \pm j\,1.5482 \end{aligned} \tag{1}$$

The corresponding transfer function is given by:

$$T(s) = \frac{0.0044883 \cdot (s^2 + 1.76)}{(s + 0.3764)(s^2 + 0.5822s + 0.3953)} \times \\ \times \frac{(s^2 + 2.397)(s^2 + 6.499)}{(s^2 + 0.2918s + 0.8501)(s^2 + 0.08155s + 1.027)} \tag{2}$$

The Orchard's theorem [2] proves that ladder *LC*-filters terminated in both ends with resistors have minimum sensitivity to passive component tolerances in the pass-band region. This fact started the idea to simulate the ladder *LC*-filter using active-*RC* filter realizations, that is, realizations without inductances. Therefore, for the purpose of our research, the filter in Fig. 1 is realized by an active-*RC* ladder simulation (e.g. simulated by signal-flow-graph technique), and have the low sensitivity, as well. PSpice Monte Carlo runs in the vicinity of the cut-off frequency with 1% Gaussian distribution, zero-mean resistors and capacitors were carried out for the resulting active-*RC* filter and presented

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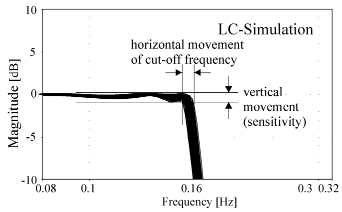

Figure 3. MC runs of the filter in Fig 1 (realized by *LC*-simulation) in the vicinity of the cut-off requency.

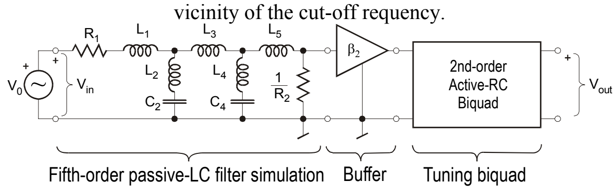

Figure 4. Seventh-order elliptic filter realized by filter partitioning.

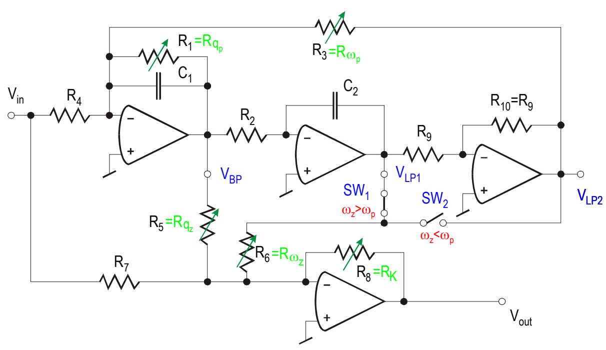

Figure 5. Tuning biquad filter circuit.

in Fig. 3. The nature of simulated ladder-*RLC* filters is very similar to that of conventional ladder-*RLC* filter, not only with respect to low sensitivity but also to other less desirable characteristics. Note that the vertical spread of the characteristics is very low because of low sensitivity. On the contrary, the horizontal movement of the magnitude's cut-off frequency is significant. From the latter we conclude that in the practical precision analog filters design apparently there exists need for tuning. But, the interdependence of the elements in ladder-*RLC* filter in Fig. 1 is so strong that any change in one coil will require a corresponding change to be made on all other filter coils, which makes the tuning of the high-order filters to tight initial design tolerances impossible, not to speak about expensive and time consuming production. The lack of widespread usage of simulated ladder-*RLC* filters is caused by this tuning impossibility.

The way in which the tuning problem could be solved is called the 'filter partitioning' design, or factoring, of the original n^{th}-order transfer function into the product of an (n–2)-order transfer function and a biquadratic function, and the appropriate realization of the two functions by a simulated active-*RC* ladder network and a biquad, respectively. The 'partitioning' a given filter into the cascade of an LCR ladder filter and a second-order filter building block or 'biquad' is shown in Fig. 4. The former has low sensitivity to component tolerances, the latter, which is selected to contain the pole-zero pair determining the filter band edges, is readily tunable. The two filters are separated by a buffer amplifier that also provides the additional gain necessary to overcome the insertion loss of the overall filter. Partitioning the filter into the cascade of an (n–2)-order ladder filter and a biquad will deteriorate the sensitivity characteristics of the original ladder filter to some degree. However, the advantages of combining a readily tunable biquad (preferably a so-called 'multi-amplifier' biquad) with the still-low sensitivity of the remaining doubly-terminated ladder filter (simulated by an inductorless, active-*RC* circuit), outweighs this slight increase in filter sensitivity.

The problem of tuning and selecting the pole-zero pair in the 'tuning biquad' is investigated in this paper. We examine the selection of pole-zero pair for the tuning biquad, such that the overall transfer function can most effectively be tuned. This implies finding the answers to the questions (*i*) 'Which pole and zero pair should most appropriately be combined to realize the tuning biquad?' and (*ii*) 'How is the tuning procedure to be accomplished?' The simulations to find the answers to these questions have been carried out using Matlab. The whole (and non-trivial) problem of 'filter partitioning' together with the dynamic range and noise will be considered in the future publications.

III. The Tuning Biquad Circuit

An opamp biquad circuit that has proved to be advantageous for various reasons including its good dynamic- range properties (because it requires only inverting opamps with virtual ground rather than common-mode inputs), and its excellent tuning properties (see below), is that shown in Fig 5. This circuit is sometimes referred to as the Tow-Thomas biquad [3] or, more often, the multi-amplifier biquad [4][5]. The transfer function is given by:

$$T_{Biq}(s)=\frac{V_{out}}{V_{in}}=-k_{Biq}\frac{s^2+(\omega_z/q_z)s+\omega_z^2}{s^2+(\omega_p/q_p)s+\omega_p^2}\,, \qquad (3)$$

with: $\omega_p=1/\sqrt{R_2R_3C_1C_2}$, $q_p=R_1\sqrt{C_1/(R_2R_3C_2)}$,

$$\omega_z=\omega_p\sqrt{1\pm R_3R_7/(R_4R_6)}\,,\ k_{Biq}=R_8/R_7\,, \qquad (4)$$

where the plus sign is for switch SW_1 closed, SW_2 open, and $\omega_z>\omega_p$; the minus sign is for SW_2 closed, SW_1 open, and $\omega_z<\omega_p$. When both switches are open then $\omega_z=\omega_p$. For elliptic lowpass filters we generally have $\omega_z>\omega_p$ and the amplitude response as in Fig. 6(c). The middle term in the numerator of (3) is equal to:

$$\omega_z/q_z=(\omega_p/q_p)[1-R_1R_7/(R_4R_5)]\,. \qquad (5)$$

For a notch filter, this term must be zero, that is, q_z=∞; this is obtained with the appropriate value of R_5.

From (4) it is possible to formulate the non-iterative ('orthogonal') tuning procedure for the circuit in Fig. 5 [3][6]. It follows that changing the value of R_3 can tune ω_p, while keeping ω_p/ω_z ratio constant. If the pole Q, q_p, is to be kept constant and the zero Q, q_z=∞, then R_1 and R_5 must be adjusted as well. This complicates the tuning procedure.

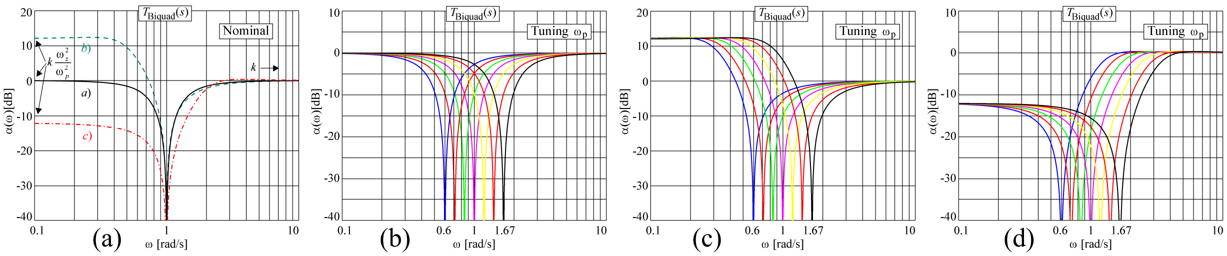

Figure 6. (a) Second-order notch transfer function. Notch frequency ω_p tuning: (b) when $\omega_z=\omega_p$; (c) $\omega_z>\omega_p$; (d) $\omega_z<\omega_p$.

However, if the change in R_3 is small, then ω_p tuning can often be considered ideal. In what follows, this is referred to as "ω_p-tuning". Similarly, ω_z can be tuned, independently of ω_p and q_p, by adjusting R_6. If we tune only ω_z, while keeping the value of ω_p fixed, we shall refer to this as "ω_z-tuning". Those two frequency tuning procedures are "standard" for the biquad in Fig. 5. Their influence on tuning second-order notch filter (that can be realized by the circuit in Fig. 5 solely), as well as, higher-than-second-order elliptic filters will now be discussed. Finally, changing the value of R_8 can correct the pass-band gain k_{Biq}.

IV. Tuning A Second-Order Notch Filter

Consider the second-order transfer function with finite zeros, which has a form (3) with the middle term equal to zero. In Fig. 6(a) we present three examples of normalized transfer function magnitudes with k=1, q_p=1, and *a*) $\omega_p=\omega_z=1$; *b*) $\omega_p=0.5$, $\omega_z=1$ ($\omega_z>\omega_p$); *c*) $\omega_p=2$, $\omega_z=1$ ($\omega_z<\omega_p$). Note that the d.c. gain is $k\omega_z^2/\omega_p^2$, and at infinite frequency the gain is k. We proceed with ω_p-tuning; we maintain the ratio between ω_p and ω_z, and tune ω_p in the range from $\omega_L=0.6\omega_p$ to $\omega_H=1.67\omega_p$ in 7 steps ($\omega_p^2=\omega_L\omega_H$), we influence the horizontal shift of the curves in all three notch filter types as shown in Fig. 6(b)-(d) (the nominal curve is in the middle). Note that the pole Q factor, q_p, is kept constant. In this way the biquadratic transfer function of the filter can be tuned to the exact zero (i.e. notch) frequency, while keeping the normalized stop bandwidth B/ω_p constant. At the same time, the magnitude of the frequency response has no vertical shift and does not change its shape.

V. Tuning A Seventh-Order Elliptic Filter

Poles and zeros of a seventh-order elliptic filter CC 07 25 50 are given by (1). The pole-zero plot is shown in Fig. 7. The filter has three pole Q factors: q_{p1}=12.4291 (max Q), q_{p3}=3.07528 (mid Q) and q_{p5}=1.07996 (min Q), and three zeros: Ω_4=1.3266 (min Z), Ω_6=1.5482 (mid Z) and Ω_2=2.5494 (max Z). To realize 0-dB at ω=0, we obtain k_{PB}=0.0044883 (see [6]).

Tuning the tuning-biquad should be effective in tuning the overall transfer function $T(s)$. With this in mind, a critical problem is to decide which pole-zero pair to select for the tuning biquad. We can pair each zero with poles having high, medium, or low pole Qs. There are altogether 9 ways in which we can pair the poles and zeros, i.e. we have the following set of pairs: {{min Z, min Q}, {min Z, mid Q}, {min Z, max Q}, {mid Z, min Q}, {mid Z, mid Q}, {mid Z, max Q}, {max Z, min Q}, {max Z, mid Q}, {max Z, max Q}}. Two examples: {max Z, min Q} and {mid Z, min Q} are shown with solid and dashed lines, respectively, in Fig. 7.

In what follows we examine the optimum design, and the tuning procedure for the tuning biquad. We start with "ω_p-tuning". If we hold the ratio ω_p/ω_z constant and then tune ω_p in the range from $\omega_L=0.95\omega_p$ to $\omega_H=1.052\omega_p$ (≈±5%) in 7 steps we influence the shift of the curves as shown in the left upper side in Fig. 8 (with the pass-band region magnified). Note that in the left upper side of Fig. 8 the tuning curves in the pass-band are tilted. The resulting filter response can be expected to exceed the specifications. Although all 9 possible tuning biquads with different pole-zero combinations were examined with regard to their tuning capabilities for the overall filter, we have presented only examples with extreme values in Fig 8.

Continuing with "ω_z-tuning" (i.e. tuning only ω_z while keeping the value of ω_p fixed), we obtain the curves shown in the upper right side in Fig. 8. Finally, in the lower part of Fig. 8 we have examples showing the stop-band responses for various Z values. Both tuning types, ω_p and ω_z combined with min Q, mid Q, and max Q values, produce very similar curves in the stop band. Thus, only the curves with various Z values are presented.

From the characteristics in Fig. 8 we note that with ω_p -tuning the frequency response is distorted (i.e. a tilt in the pass band) because at ω=0 the gain remains constant. By contrast, with ω_z -tuning the slope of the frequency response in the pass band remains relatively constant, while only the cut-off frequency and slope are shifted. The vertical movement of the curves represents a change in gain, but this can readily be corrected for within the filter. Thus, as a general rule, ω_z -tuning is preferable to ω_p - tuning.

Furthermore, it is apparent that the tuning process is more effective with the small-Z large-Q combinations. In other words, for the tuning biquad, *the pole pair with maximum pole Q should be combined with the zero pair with lowest frequency* i.e. the 'minimum-frequency zero pair'.

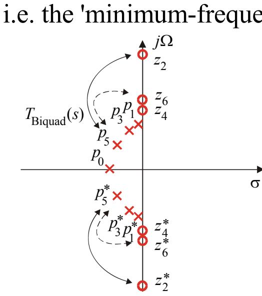

Figure 7. Pole-zero plot for CC 07 25 50 filter

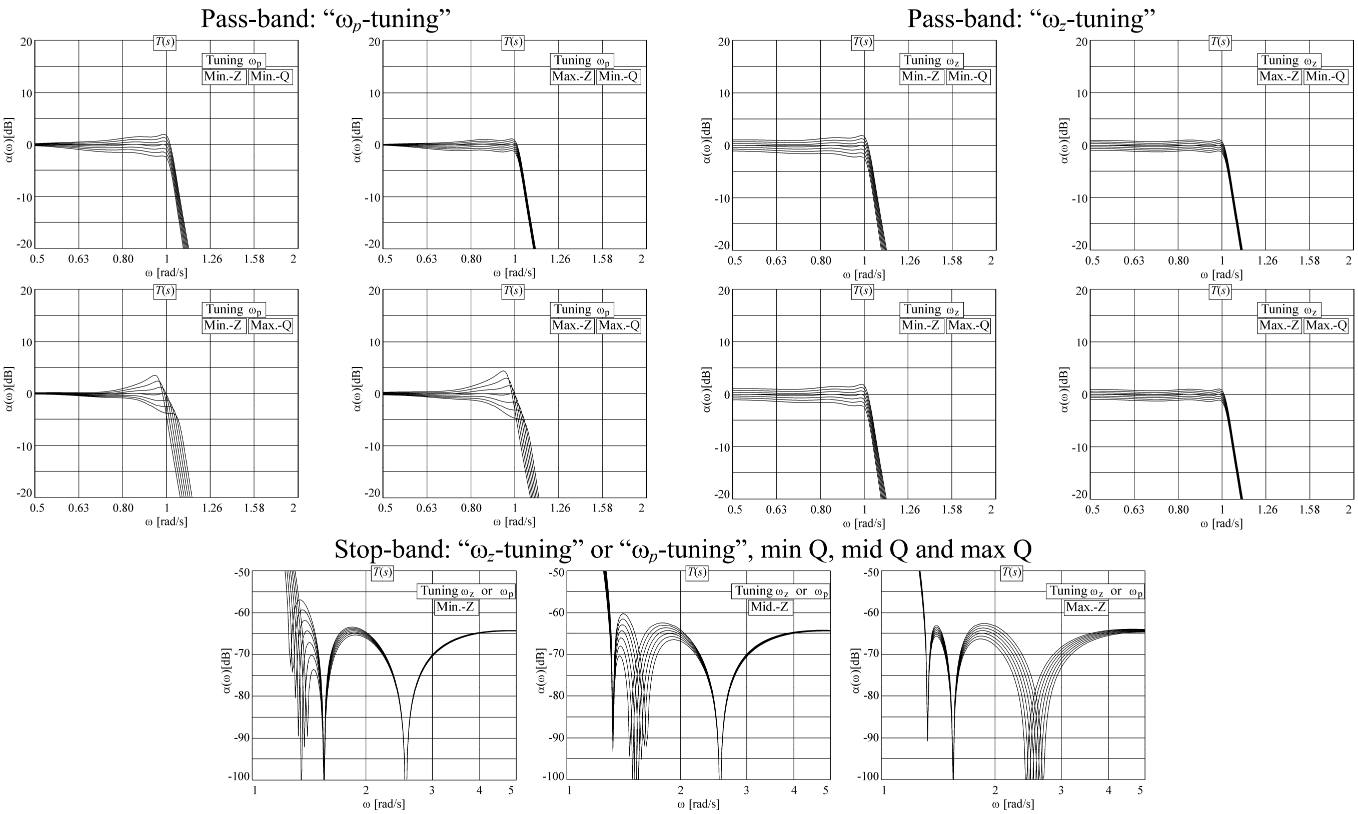

Figure 8. Tuning curves of the CC 07 25 50 filter in the pass-band and in the stop-band.

This is a rather serendipitous result, since it has been established elsewhere (see, for example [4] and [6]) that for *a biquad with minimum sensitivity to component changes and maximum dynamic range the pole pair with the highest pole Q should be combined with the closest zero pair*. Since the pole- pair with the highest pole Q is generally at the upper filter band edge, this is equivalent to combining the pole- pair with the highest pole Q with the 'minimum-frequency zero pair' *which is precisely what we are recommending for our tuning biquad.*

This result may at first seem contradictory, since the tuning biquad needs to have a *high* sensitivity of some crucial part of its frequency response to its particular tuning component. However, in the case of the multi-amplifier biquad (see Fig. 5) for which the zero frequency can be tuned for *orthogonally* to the pole Q, the zero, or notch frequency of the tuning biquad can be tuned for easily with one resistor (R_6 in Fig. 5). In doing so, the cut-off frequency and slope of the overall higher-order filter will be shifted effectively, yet with minimum effect on the rest of the frequency response. Furthermore, the biquad with which the notch is being tuned (the 'tuning biquad') has a minimum sensitivity to tolerances or unintentional changes of its components.

VI. CONCLUSIONS

In this paper, we have presented a procedure by which the cutoff frequency and slope of the overall frequency response of a high-order elliptic filter can be tuned with a so-called tuning biquad by adjusting the zero, or notch frequency of the tuning biquad. The rule of thumb in the design of the tuning biquad is to combine the pole pair with the highest pole Q, with the zero pair at the lowest frequency. The remaining transfer function can be realized by an arbitrary filter structure. Preferably, this can be either an active-RC ladder circuit with its inherent advantage of low-sensitivity to component tolerances, or a cascade of biquads. Then, by tuning the notch frequency ω_z of *only* the tuning biquad, while keeping its ω_p value constant, the cut-off frequency and slope of the overall filter can be adjusted. Changes in the pass-band gain incurred by this tuning process can generally be readily corrected for within the overall filter or its embedding circuitry.

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