Fitting GPDs to DVCS data: at NLO and beyond

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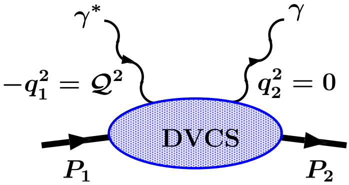
Outline

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]



$$P = P_1 + P_2\ , \qquad q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

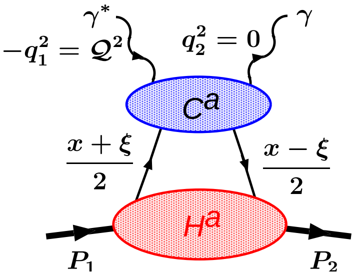
$$\vartheta = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \approx \frac{\eta}{\xi} \stackrel{\text{DVCS}}{=} 1$$

$$t = (P_2 - P_1)^2$$

- To leading twist-two accuracy cross-section can be expressed in terms of Compton form factors (CFFs):

$$\mathcal{H}(\xi, t, \mathcal{Q}^2), \mathcal{E}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2), \ldots$$

Factorization of DVCS $\longrightarrow$ GPDs



$$P = P_1 + P_2 \,, \qquad q = (q_1 + q_2)/2$$
$$t = (P_2 - P_1)^2$$

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$
$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

- Compton form factor is a convolution:

$$ {}^{a}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \; H^a(x, \eta = \xi, t, \mu^2)$$
$$a = \text{NS}, \text{S}(\Sigma, G)$$

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)

- C^a: LO, NLO (1st order in α_s)

  [Ji et al, Belitsky et al, Mankiewicz et al, '97]

  $\Rightarrow$ need NNLO to stabilize perturbation series and investigate convergence
- H^a:
  - Complete deconvolution is impossible, so to extract GPDs from the experiment we need to model their functional dependence.
  - Evolution known to NLO order and not trivial to implement.

$\Downarrow$

Conformal moment series representation of CFFs

$\Downarrow$

Mellin-Barnes representation of CFFs

- factorization formula for singlet DVCS CFFs:

$$ {}^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x\, \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}(x, \xi, t, \mu^2) $$

- ... in terms of conformal moments

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, t, \mu^2) $$

$$ H_j^q(\eta, \dots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \dots) $$

...

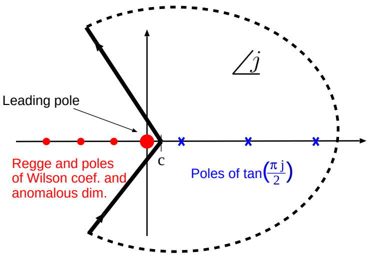
H_j^a even polynomials in η with maximal power η^{j+1}

- series summed using Mellin-Barnes integral over complex j:

$$ = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right) \right] \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, t, \mu^2) $$

Mellin-Barnes representation of CFFs

$$
{}^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right)\right] \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, t, \mu^2)
$$



Advantages of conformal moments and Mellin-Barnes representation

- **NNLO corrections** accessible by making use of conformal OPE and known NNLO DIS results
- enables simpler inclusion of **evolution** effects
- possible efficient and stable numerical treatment $\Rightarrow$ stable and fast **computer code** for evolution and fitting
- powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
- opens the door for interesting **modelling of GPDs**
- moments are equal to matrix elements of local operators and are thus directly accessible on the **lattice**

OPE

- DVCS belongs to a class of two-photon processes (DIS, DVCS, two-photon production of hadronic states ...) calculable by means of OPE

$$T_{\mu\nu}(q, P, \Delta) = \frac{i}{e^2} \int d^4x \, e^{ix\cdot q} \langle P_2, S_2 | T j_\mu(x/2) j_\nu(-x/2) | P_1, S_1 \rangle$$
$$\rightarrow C_j \, O_j$$
$$\downarrow$$

$$\left.\begin{array}{l}\text{generalized Bjorken kinematics}\\ \text{conformal symmetry}\end{array}\right\} \rightarrow \text{unified description}$$

Conformal OPE (COPE)

- COPE prediction for general kinematics reads

$$C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^* = \textit{fixed})$$

$$= \quad c_j(\alpha_s^*)\, {}_2F_1\!\left({(2+2j+\gamma_j(\alpha_s^*))/4,\,(4+2j+\gamma_j(\alpha_s^*))/4 \atop (5+2j+\gamma_j(\alpha_s^*))/2} \,\middle|\, \frac{\eta^2}{\xi^2} \right) \left(\frac{\mu^2}{Q^2}\right)^{\frac{\gamma_j(\alpha_s^*)}{2}}$$

- $\eta = 0$: DIS

$$\lim_{\eta\to 0} C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^*) = c_j^{DIS}(\alpha_s^*)|_{\beta=0} \left(\frac{\mu^2}{Q^2}\right)^{\frac{\gamma_j(\alpha_s^*)}{2}}$$

- $\eta = \xi$: DVCS
- $\eta = 1$: photon-to-pion transition form factor

Breaking of conformal symmetry

- massless QCD is conformally symmetric at the tree level
- conformal symmetry broken at the loop level (renormalization introduces mass scale)
  - running of the coupling constant $\Rightarrow \beta \neq 0$
  - renormalization of the composite operators $\Rightarrow$ non-diagonal anomalous dimensions $\gamma_{jk} = \delta_{jk}\gamma_j + \gamma_{jk}^{\mathrm{ND}}$

$$\mu \frac{d}{d\mu} O_j(\ldots, \mu^2) = -\sum_{k=0}^{j} \gamma_{jk}(\alpha_s(\mu))\, \eta^{j-k}\, O_k\,(\ldots, \mu^2)\,,$$

$$\mu \frac{d}{d\mu} C_j(\ldots, Q^2/\mu^2, \alpha_s(\mu))] = \sum_{k=j}^{\infty} C_k(\ldots, Q^2/\mu^2, \alpha_s(\mu))\gamma_{kj}(\alpha_s(\mu)) \left(\frac{\eta}{\xi}\right)^{k-j}$$

Conformal scheme

- non-diagonal terms of anomalous dimensions ($\overline{\text{MS}}$ scheme) can be removed by finite renormalization, i.e, specific choice of factorization scheme $\rightarrow$ conformal subtraction ($\overline{\text{CS}}$) scheme:

$$C^{\overline{\text{MS}}}\, O^{\overline{\text{MS}}} = C^{\overline{\text{MS}}}\, B\, B^{-1}\, O^{\overline{\text{MS}}} = C^{\overline{\text{CS}}} O^{\overline{\text{CS}}}$$

$$\gamma_{jk}^{\overline{\text{CS}}} = \delta_{jk}\gamma_k + \frac{\beta}{g}\Delta_{jk}$$

- moreover, there is ambiguity in $\overline{\text{MS}} \rightarrow$ CS rotation matrix:

$$B = B^{(\beta=0)} + \frac{\beta}{g}\delta B$$

and by judicious choice of δB one can "push" mixing to NNLO ($\overline{\text{CS}}$ scheme, [Melić et al. '02]) $\rightarrow \Delta_{jk}$ — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected

NNLO DVCS

- Finally

$$C_j^{\overline{\mathrm{CS}},\mathrm{DVCS}}(Q^2/\mu^2, \alpha_s(\mu)) = C_k(1, \alpha_s(Q))\, \mathcal{P}\exp\left\{\int_Q^{\mu} \frac{d\mu'}{\mu'}[\gamma_j(\alpha_s(\mu'))\delta_{kj}\right\}$$

with

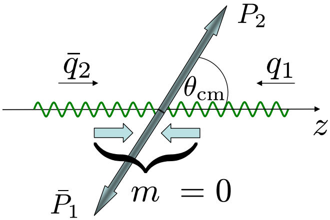
$$C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2}\Gamma\left(\frac{5}{2}+j+\gamma_j(\alpha_s)/2\right)}{\Gamma(3/2)\Gamma\left(3+j+\gamma_j(\alpha_s)/2\right)}\, c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$$

- we take
  $c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$ from [Zijlstra, v. Neerven '92,'94, v. Neerven and Vogt '00]
  γ_j from [Vogt, Moch and Vermaseren '04]

- We have used previously developed formalism to
  1. investigate size of NNLO corrections to non-singlet [D. Müller, Phys.Lett. **B634** (2006), hep-ph/0510109] and singlet Compton form factors in $\overline{\mathrm{CS}}$ scheme [K. Kumerički, D. Müller, K. P-K., A. Schäfer, Phys.Lett. **B648** (2007), hep-ph/0605237]
  2. compare the $\overline{\mathrm{CS}}$ NLO predictions to complete $\overline{\mathrm{MS}}$ NLO predictions (non-diagonal evolution included) and analyze the latter [K. Kumerički, D. Müller, K. P-K., Nucl. Phys. **B794** (2008), hep-ph/0703179]
  3. perform fits (in both schemes) to DVCS (and DIS) data and extract information about GPDs [K. Kumerički, D. Müller, K. P-K., Nucl. Phys. **B794** (2008), hep-ph/0703179]
  4. analyze the new HERA data to LO (investigating η-dependent GPD model) [K. Kumerički, D. Müller, K. P-K., in preparation]

Modelling conformal moments of GPDs

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^*\gamma \to pp$



When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

- ...and dependence on $\theta_{\rm cm}$ in t-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta,\ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta},\ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

GPD Ansatz (I)

$$\mathbf{H}_j(\eta,t) = \underbrace{\begin{pmatrix} N'_\Sigma\, F_\Sigma(t)\, \mathrm{B}\big(1+j-\alpha_\Sigma(0),8\big) \\ N'_G\, F_G(t)\, \mathrm{B}\big(1+j-\alpha_G(0),6\big) \end{pmatrix}}_{\text{Leading partial wave}} + \begin{pmatrix} s_\Sigma \\ s_G \end{pmatrix} \left(\text{subleading partial waves, } \eta\text{-dependence!}\right)$$

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j+1-\alpha(0)}{j+1-\alpha(t)} \left(1 - \frac{t}{M_0^{a\,2}}\right)^{-p_a}$$

- Leading wave: for $t=0$ corresponds to x-space PDFs of form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\,(1-x)^7\,; \qquad G(x) = N'_G\, x^{-\alpha_G(0)}\,(1-x)^5$$

$$\Downarrow$$

- analysis of radiative corrections (with generic parameters)
- fit of N_Σ, $\alpha_\Sigma(0)$, M_0^Σ, N_G, $\alpha_G(0)$, M_0^G

(for small ξ valence quarks less important: $\Sigma \approx$ sea)

GPD Ansatz (II)

- at NLO data can be fitted with leading wave only, but at LO we need η-dependence! ($\rightarrow$ included in the new LO analysis)

- $H_j(\eta \rightarrow 0, t)$ limit is non-trivial (branch point) $\Rightarrow$ resummation

GPDs and sum rules

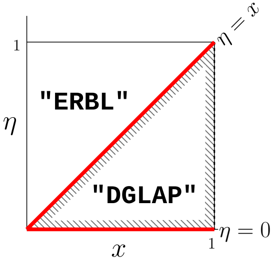
[K. Kumerički, D. Müller, K. P-K., arXiv:0805.0152 [hep-ph]]

- LO perturbative prediction

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \overset{\rm LO}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mu^2 = \mathcal{Q}^2)$$

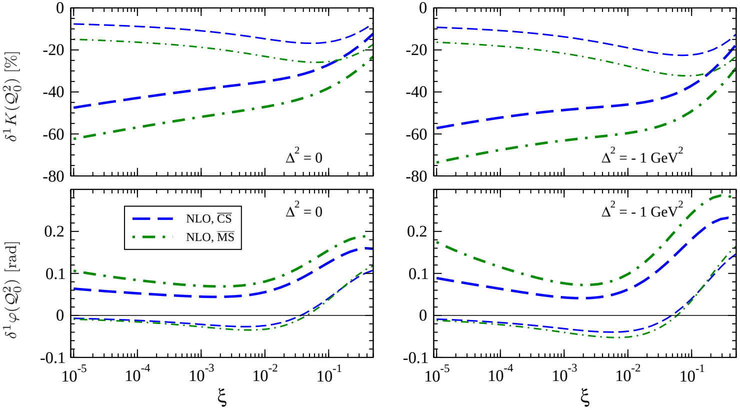
$$\Downarrow$$

$$H(x, \xi = x, t, \mathcal{Q}^2) - H(-x, \xi = x, t, \mathcal{Q}^2) \overset{\rm LO}{=} \frac{1}{\pi} \Im\mathrm{m} \mathcal{F}(\xi = x, t, \mathcal{Q}^2)$$



- dispersion relations ⇒ sum rules for GPDs

NLO corrections

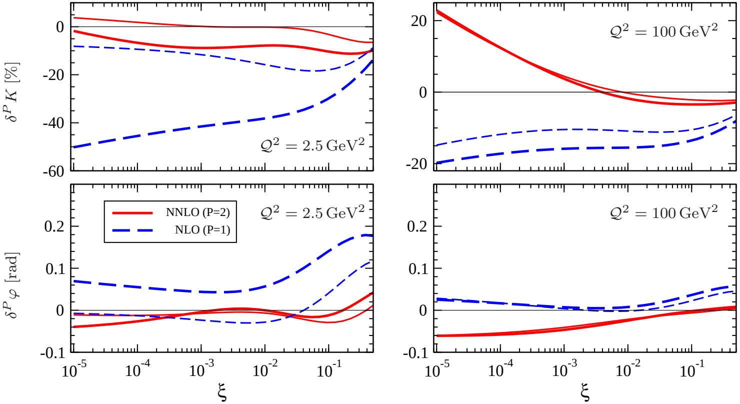


Thick lines: "hard" gluon $N_G = 0.4$ $\alpha_G(0)=\alpha_\Sigma(0) + 0.05$

Thin lines: "soft" gluon $N_G = 0.3$ $\alpha_G(0)=\alpha_\Sigma(0) - 0.02$

$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

NNLO corrections



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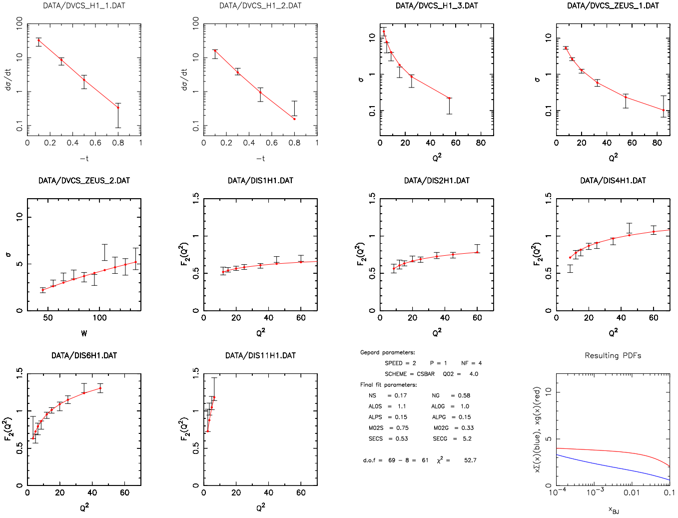
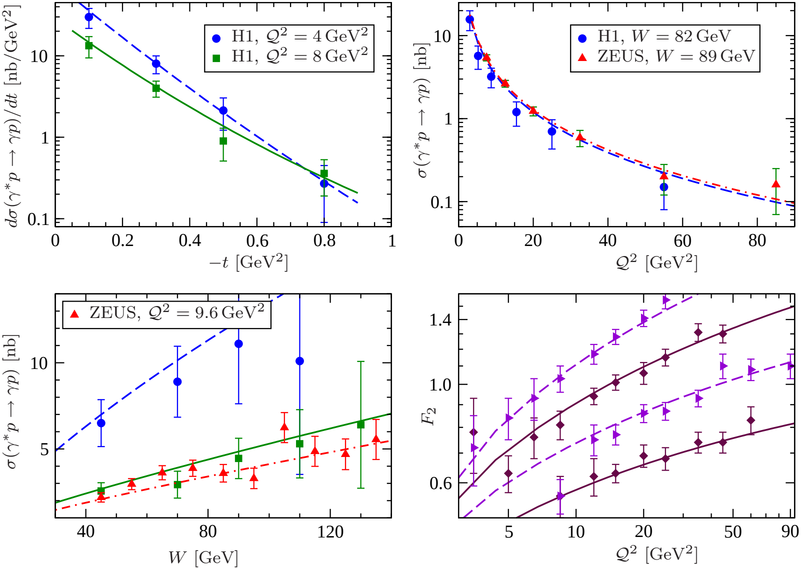
$$\delta^P K = \frac{\left|\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\right|}{\left|\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\right|} - 1\,, \qquad \delta^P \varphi = \arg\left(\frac{\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}}{\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}}\right)$$

resummation of $\alpha_s ln(1/\xi)$ needed ? - situation maybe worse for meson production [Diehl, Kugler, Ivanov, Szymanowski, Krasnikov]

Fast fitting routine (GeParD)



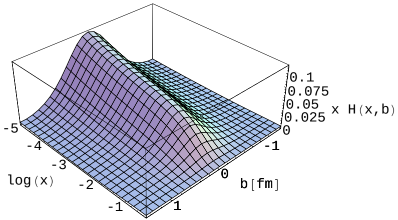
NNLO fit to HERA DVCS+DIS data

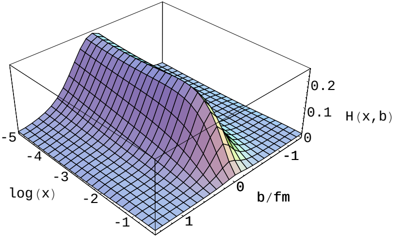


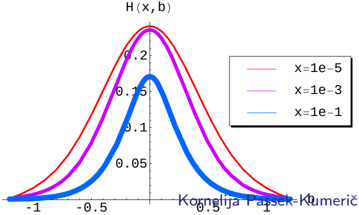
Three-dimensional image of a proton

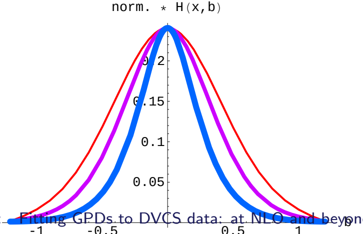
Probalistic interpretation of GPDs [Burkardt '00, '02]

Quarks: Gluons:



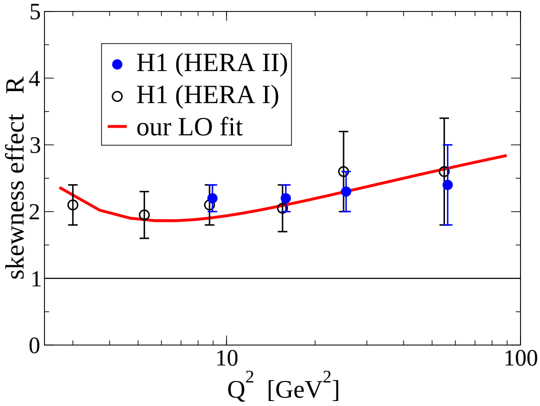






Skewness effect (I) — R

$$R \equiv \left. \frac{\mathcal{I}m\, A_{\rm DVCS}}{\mathcal{I}m\, A_{\rm DIS}} \right|_{t=0} \stackrel{LO}{=} \frac{H(\xi,\xi)}{H(2\xi,0)}$$ [Shuvaev et al. '99]

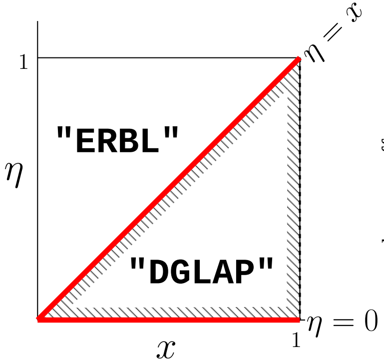


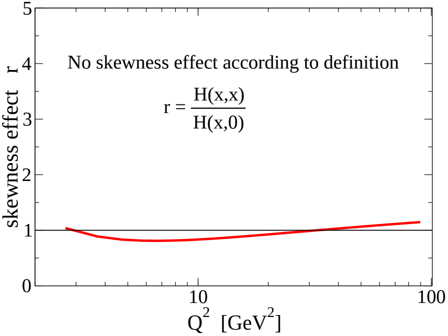
- Significant skewness effect?

Skewness effect (II) — r

- Skewness effect is naturally defined by ratio of GPDs $H(x,\eta)$ at two physically relevant trajectories: $\eta = x$ and $\eta = 0$

$$r = \frac{H(x,x)}{H(x,0)} \overset{LO}{\approx} \frac{1}{2^\alpha} R \qquad \text{for} \quad q(x\to 0) \sim x^{-\alpha} \qquad \alpha \approx 1$$





Skewness effect (III)

- To get the correct normalization and t-dependence, one has to compensate "natural" DVCS-to-DIS enhancement factor [Shuvaev et al. '99]

$$\left.\frac{2^{j+2}\Gamma(j+5/2)}{\sqrt{\pi}\Gamma(j+3)}\right|_{j=\alpha-1\approx 0.2} \approx 1.5$$

  - at NLO radiative corrections take care of that
  - at LO resummed subleading partial waves have to give negative contribution:

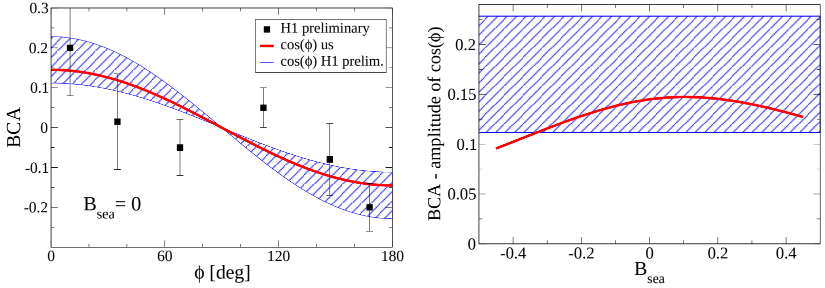
$$\mathbf{H}_j(\eta,t) = \begin{pmatrix} N'_\Sigma F_\Sigma(t)\,\mathrm{B}\left(1+j-\alpha_\Sigma(0),8\right) \\ N'_G F_G(t)\,\mathrm{B}\left(1+j-\alpha_G(0),6\right) \end{pmatrix} + \underbrace{\begin{pmatrix} s_\Sigma \\ s_G \end{pmatrix}\left(\text{subleading partial waves, } \eta\text{-dependence!}\right)}_{<0}$$

negative "intrinsic skewness"

Beam charge asymmetry

$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{\mathcal{A}_{\rm Interference}}{|\mathcal{A}_{\rm DVCS}|^2 + |\mathcal{A}_{\rm BH}|^2} \overset{\rm LO}{\propto} F_1 \mathcal{H} + \frac{|t|}{4M^2} F_2 \mathcal{E}$$

- Model $E_{\rm sea}$ as $\kappa_{\rm sea} H_{\rm sea}$ and take $\mathrm{B}_{\rm sea} \equiv \int dx\, x E_{\rm sea}$ as parameter



- H1 data enable exclusion only of very negative $\mathrm{B}_{\rm sea}$

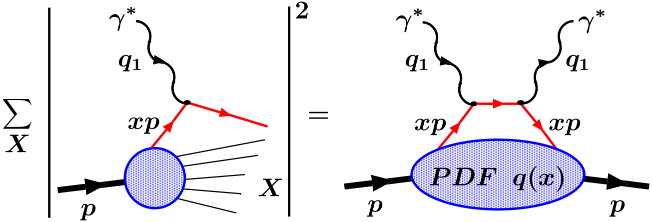
Summary

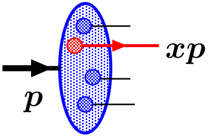
- Using conformal moments of GPDs has several advantages, including
  - elegant approach to NLO and NNLO corrections to DVCS amplitude
  - providing convenient framework for GPD modelling
- NLO corrections can be sizable, and are strongly dependent on the gluonic input.
- NNLO corrections are small to moderate, supporting perturbative framework of DVCS.
- Scale dependence is not so conclusive: large NNLO effects for $\xi \lesssim 10^{-3}$ signaling breakdown of naive perturbation series.
- Fits to available small-x DVCS and DIS data work well at (N)NLO and give access to transversal distribution of partons.
- In order to get good LO fits, one needs sophisticated GPD modelling

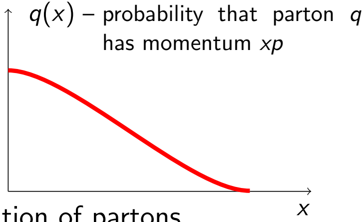
The End

Parton distribution functions

- Deeply inelastic scattering, $-q_1^2 \to \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p\cdot q_1} \to \mathrm{const}$

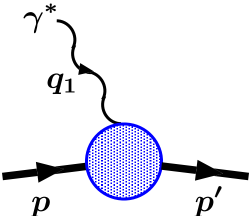






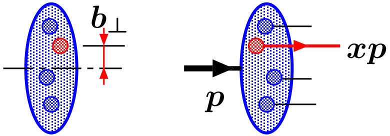
- no information on spatial distribution of partons

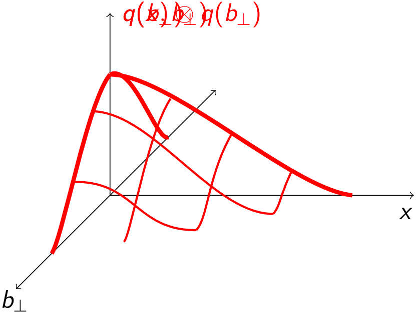
Electromagnetic form factors



- Dirac and Pauli form factors:

$$q(b_\perp) \sim \int \mathrm{d}b_\perp \, e^{iq_1 \cdot b_\perp} F_{1,2}(t = q_1^2)$$

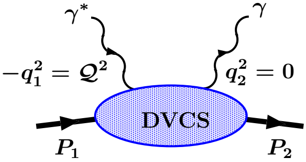




- "skewless" GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp \, e^{i\Delta \cdot b_\perp} q(x, b_\perp)$

Probing the proton with two photons

- Deeply virtual Compton scattering [Müller '92, et al. '94]



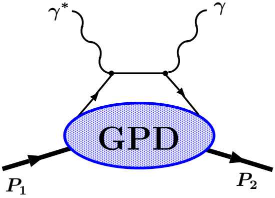
$P = P_1 + P_2 \qquad q = (q_1 + q_2)/2$

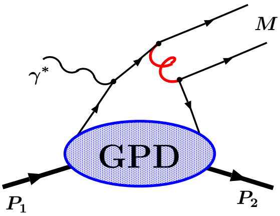
Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

- QCD: factorization of short- and long-distance physics





Alternative probe:
meson production

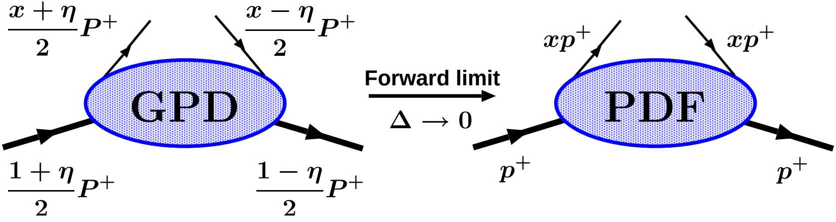
- more difficult, but access to flavours

Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$



$$P = P_1 + P_2\ ; \qquad \Delta = P_2 - P_1\ ; \qquad \eta = -\frac{\Delta^+}{P^+}\ \text{(skewedness)}$$

Properties of GPDs

- Decomposing into helicity conserving and non-conserving part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

- Forward limit ($\Delta \to 0$): $\Rightarrow$ GPD $\to$ PDF

$$F^q(x,0,0) = H^q(x,0,0) = \theta(x) q(x) - \theta(-x)\bar{q}(-x)$$

- Sum rules:

$$\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x,\eta,\Delta^2) \\ E^q(x,\eta,\Delta^2) \end{array} \right. = \left\{ \begin{array}{l} F_1(\Delta^2) \\ F_2(\Delta^2) \end{array} \right.$$

- Possibility of solution of proton spin problem

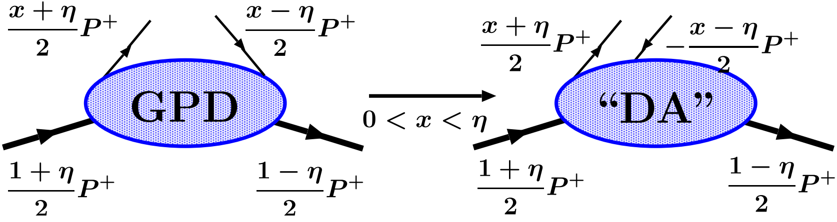
$$\frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\eta,\Delta^2) + E^q(x,\eta,\Delta^2) \right] = J^q(\Delta^2) \qquad \text{[Ji '96]}$$

Relation to distribution amplitudes

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

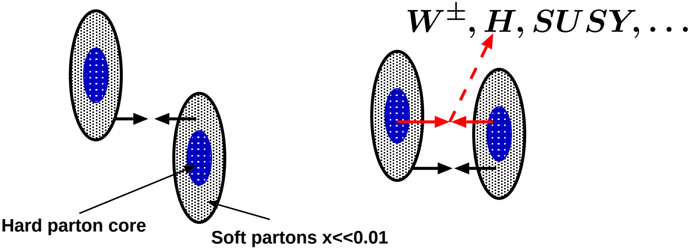
$$F^q(x,\eta,\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$



$P = P_1 + P_2\,;\qquad \Delta = P_2 - P_1\,;\qquad \eta = -\frac{\Delta^+}{P^+}$ (skewedness)

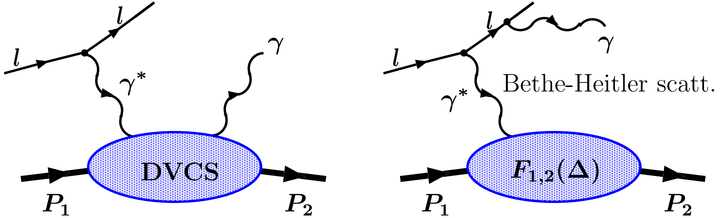
Relevance of GPDs for collider physics



- heavy particle production $\Rightarrow$ collision is more central
  $\Rightarrow$ larger probability for multiple parton collisions
- [Frankfurt, Strikman and Weiss '04]
- No influence on total inclusive cross sections, but event structure is sensitive to transversal parton distributions.
- Relevant for LHC?

Deeply virtual Compton scattering (I)

- Measured in leptoproduction of a real photon:



- There is a background process but it can be used to our advantage:

$$\sigma \propto |\mathcal{T}_{\rm DVCS}|^2 + |\mathcal{T}_{\rm BH}|^2 + \mathcal{T}^*_{\rm DVCS}\mathcal{T}_{\rm BH} + \mathcal{T}_{\rm DVCS}\mathcal{T}^*_{\rm BH}$$

- Using $\mathcal{T}_{\rm BH}$ as a referent "source" enables measurement of the phase of $\mathcal{T}_{\rm DVCS}$ → proton "holography" [Belitsky and Müller '02]

Conformal algebra

- Conformal group restricted to light-cone $\sim O(2,1)$

$$L_+ = -iP_+ \qquad L_- = \frac{i}{2}K_- \qquad L_0 = \frac{i}{2}(D + M_{-+})$$

$$[L_0, L_\mp] = \mp L_\mp \qquad [L_-, L_+] = -2L_0$$

Casimir:

$$L^2 = L_0^2 - L_0 + L_- L_+$$

conf.spin j:

$$[L^2, \mathbb{O}_{n,n+k}] = j(j-1)\mathbb{O}_{n,k}$$

(D — dilatations, K_- — special conformal transformation (SCT))

Operator Product Expansion

$$J_{\rm em}(x) J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \left(\frac{1}{x^2}\right)^2 x_-^{n+k+1} C_{n,k} O_{n,k}$$

$$k = 0: \qquad O_{n,0} \equiv \qquad \bar{\psi}\, \gamma^+ (i \overset{\leftrightarrow}{D}_+)^n \psi \qquad i\partial_+ \overset{\text{M.E.}}{\rightarrow} -\Delta_+$$

$$\overset{\leftrightarrow}{D}_+ \equiv \vec{D}_+ - \overset{\leftarrow}{D}_+$$

- $C_{n,0}$ and γ_n of $O_{n,0}$ are well known from DIS up to NNLO.
- But $C_{n,k}$ and $\gamma_{n,k}$ are not so well known.
- $\gamma_{n,k} \neq 0 \;\Rightarrow$ operators $O_{n,k}$ mix under evolution.
- Choosing operator basis in which $\gamma_{n,k}$ is diagonal would help. But how to diagonalize unknown matrix?!
- (At least) to LO answer is: use conformal operators.

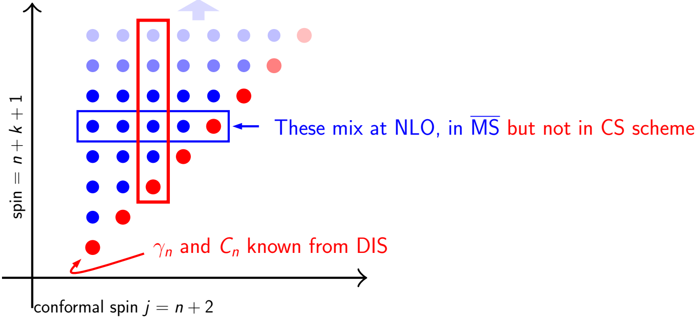
Conformal operators

$$\mathbb{O}_{n,n+k} = (i\partial^+)^{n+k}\, \bar{\psi}\, \gamma^+\, C_n^{3/2}\left(\frac{\overleftrightarrow{D}^+}{\partial^+}\right)\psi$$

- they have well-defined conformal spin $j = n + 2$
- massless QCD is conformally symmetric at the tree level
  $\Rightarrow$ conformal spin is conserved
- mixing of operators with different n is forbidden by conformal symmetry, while mixing of those with different $n + k$ is forbidden by Lorentz symmetry $\Rightarrow \mathbb{O}_{n,n+k}$ don't mix at LO
- conformal symmetry broken at the loop level (renormalization introduces mass scale, dimensional transmutation) $\Rightarrow$
  - running of the coupling constant $\partial g / \partial \ln \mu \equiv \beta \neq 0$
  - anomalous dimensions of operators $\gamma_{jk} = \delta_{jk}\gamma_j + \gamma_{jk}^{\rm ND}$

$\Rightarrow$ $\mathbb{O}_{n,n+k}$ start to mix at NLO

Conformal Towers



- Diagonalize in artificial $\beta = 0$ theory by changing scheme

$$\mathbb{O}^{\text{CS}} = B^{-1}\mathbb{O}^{\overline{\text{MS}}} \qquad \text{so that} \qquad \gamma^{\text{CS}}_{jk} = \delta_{jk}\gamma_k$$

- $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0} \qquad \Rightarrow$ summing complete tower

$\beta \neq 0$ (I)

- In full QCD $\beta \neq 0$ and NLO diagonalization is spoiled:

$$\gamma_{jk}^{\mathrm{CS}} = \delta_{jk}\gamma_k + \frac{\beta}{g}\Delta_{jk}$$

- However, there is also ambiguity in $\overline{\mathrm{MS}} \to \mathrm{CS}$ rotation matrix:

$$B = B^{(\beta=0)} + \frac{\beta}{g}\delta B$$

- By judicious choice of δB one can "push" mixing to NNLO ($\overline{\mathrm{CS}}$ scheme, [Melić et al.]).
- But how to calculate rotation matrix B? This is problem equivalent to calculation of $\gamma_{j,k}$.

$\beta \neq 0$ (II)

- The $B^{(\beta=0)}$ is constrained by conformal Ward identities . . .

$$B_{jk}^{(\beta=0)\mathrm{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi}\theta(j>k)\frac{\gamma_{jk}^{\mathrm{SCT,\ LO}}}{a_{jk}}$$

(a_{jk} — known matrix) [Müller '93]

SCT $\equiv$ special conformal transformation

- . . . and, as a consequence

$$^{\overline{\mathrm{MS}}}\gamma_{jk}^{\mathrm{ND},(1)} = \frac{\left[\gamma^{\mathrm{SCT,\ }(0)} - \beta_0 \frac{b}{g}, \gamma^{(0)}\right]_{jk}}{a_{jk}}$$

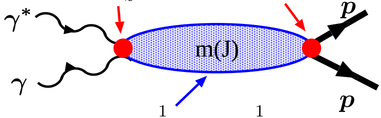
- Final result:
  n-loop DIS (diagonal) result + $(n-1)$-loop SCT anomaly =
  n-loop non-diagonal prediction

Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$, as well as trivial crossing properties of Wilson coefficients C_j, leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

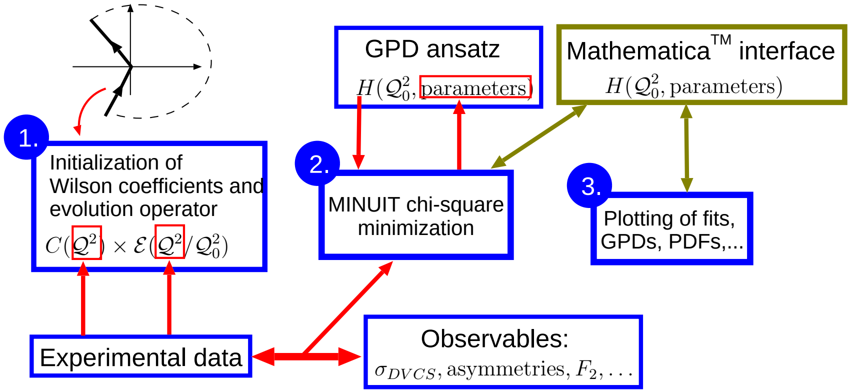
- and t-channel partial waves are modelled as:



$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p} \eta^{j+1-J} d^J_{0,\nu}$$

- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

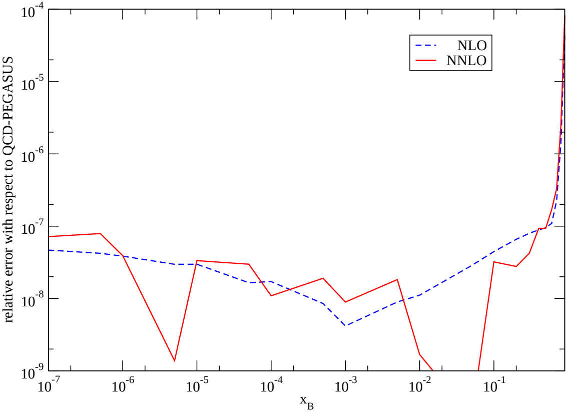
Fast fitting routine (GePardD)



- Observable $= \int \mathrm{d}j\ C_j(\mathcal{Q}^2) \times \mathcal{E}_j(\mathcal{Q}^2, \mathcal{Q}_0^2) \times H_j(\mathcal{Q}_0^2)$

Check

- Check by comparison to QCD-PEGASUS [Vogt '04]
- evolution of Les Houches benchmark PDFs:



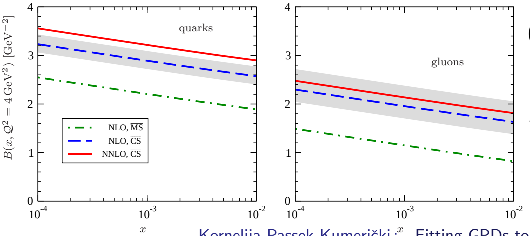
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b (conjugated to $\Delta \equiv P_2 - P_1$) [Burkardt '00, '02]

$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2)\,,$$

- Average transversal distance :

$$\langle \vec{b}^2 \rangle (x, \mathcal{Q}^2) = \frac{\int d\vec{b}\, \vec{b}^2 H(x, \vec{b}, \mathcal{Q}^2)}{\int d\vec{b}\, H(x, \vec{b}, \mathcal{Q}^2)} = 4B(x, \mathcal{Q}^2)\,,$$



(at $\mathcal{Q}^2 = 4\,\mathrm{GeV}^2$)

$$\langle \vec{b}^2 \rangle_{\mathrm{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04}\,\mathrm{fm}^2$$

New LO fit to H1 and ZEUS data