

Toward a closed life support system for interplanetary missions

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Abstract

A theoretical model of a life support system, composed of a cascade of well-mixed continuous culture reactors, is investigated (e.g. photoautotroph, herbivore and carnivore). Simple one limiting nutrient reactor equations provide a starting point for the analysis. Optimum surface illumination intensity for a given reactor depth and population density has been calculated. Linking reactors in a cascade enables decoupling of equations, easier analysis, control and optimisation. Greater stability is thereby also achieved. Maximum sustainable yield for a given nutrient input is found in a general cascade. Needed nutrient input for desired output from each reactor has been calculated as well. In contrast to a single reactor food chain, we found linear dependence of the highest trophic level in each reactor on the nutrient input. Model provides a theoretical basis for acquiring desired yield from each reactor, thus enabling formation of balanced diet for users (e.g. a spaceship crew). Nutrient cycling is achieved with bacterial degradation. A buffer, which acts as a low pass filter, is inserted in a cycle. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Model; Batch reactor; Continuous culture reactor; Food chain; Cascade; Cycle

1. Introduction

Long-duration space missions are no longer a distant future. According to NASA (1999) a man who will walk on Mars might already be born. One of the critical systems in such a mission is a

life support system, particularly food supply. Daily needs of an average human, without grey water, are O₂, 0.84 kg; dry food, 0.62 kg; and water, 2.77 kg (Wieland, 1994), or 4.23 kg per person per day. The mission to Mars will take 500 days and six crew members (Drake, 1998). Therefore, approximately 37 tonnes of supplies is needed. This weight makes the mission very cumbersome and costly, even if one builds the ship in the orbit. The price of \$4000 for bringing 1 kg into the Earth orbit (Bugbee and Salisbury, 1985) is of the least importance.

Life support systems today are physicochemically designed. For example, space station Mir, although discarding CO₂, purifies the water and

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produces O₂ by water electrolysis (Wieland, 1994). We are not aware of any means for generating food physicochemically. Thus food supply must be re-generated biologically. Such systems are called Controlled Ecological Life Support Systems (CELSS, Schneegurta et al., 1995).

Little can be found about experiments in Russia although scientists must have gathered enormous experience during Salyut and Mir space station projects. A series of ground experiments (BIOS) have been conducted. The last one (BIOS 3) utilised higher plants and algae to support three crew members up to 180 days (Wieland, 1994).

NASA (National Aeronautics and Space Agency) research concentrates on higher plants like wheat, potato, lettuce, radish, strawberry, tomato, rice, soybean, sweet potato and peanut (Marsacademy, 2000) and includes several aspects of life support systems. However, in modelling closed systems, they have a problem with negative CO₂ values (Fleisher and Ting, 1998). Influence of changing environmental conditions on higher plants (Fleisher et al., 1998) using hydroponics is investigated. Machines for food preparation, able to give different food textures, are also researched (Lee and Zasytkin, 1998).

There is an effort to develop 'space cultures', i.e. higher plants optimised for growth in space. For example, 'super dwarf rice', which is only 0.2 m high and with 70 days generation time (Bugbee et al., 1999). The longest CELSS experiment lasted for 418 days in Kennedy Space Centre (KSC) with a potato culture which provided enough oxygen and 55% of needed caloric content for one human (Braukus and Malone, 1995). Among other experiments are those made during 'Freedom' space station project and Biosphere experiments. Biosphere II, the most advanced isolated environment experiment, had 12 800 m² but conclusions drawn from it are more applicable to terraforming than to a semiportable life support system.

European Space Agency (ESA) is working on Micro-Ecological Life Support System Alternative (MELiSSA; Lasseur and Fedele, 1999). A simple ecological system is being constructed using a cycle of continuous flow reactors. Both NASA and ESA researchers view blue-green algae popularly named *Spirulina* (MELiSSA, *Spirulina platensis* ATCC

strain 29408 (Lasseur and Fedele, 1999) and NASA, *Cyanothece* sp. ATCC strain 51142 (Arieli et al., 1996) as a candidate for oxygen recycling and food production.

Spirulina is very nutritious, protein- and vitamin-rich (Mitchell et al., 1996), relatively easy to grow and resistant to changes in environmental conditions (Arieli et al., 1996). *Spirulina* caloric contents are, 4.82 kcal/gdw (gram of dry weight); shortest generation time, 18.9 h; typical concentration, $2-5 \times 10^7$ cells per ml = $2-5 \times 10^{13}$ cells per m³ and production of O₂, 1051 g/m³ day \approx 33 mol/m³ day. From the last two data, the number of cells needed for one person per day is about 4×10^{13} .

From the fact that cells were cryogenically preserved at concentrations of 2.65×10^{10} cells per ml (Arieli et al., 1996), the average volume of a cell is smaller than 3.8×10^{-16} m³. This leads to the minimum volume of 15.2×10^{-3} m³ (i.e. 15 l) per person for oxygen recycling. Measurements suggest average yield in semicontinuous reactor with added NO₃ of 150 gdw/m³ day (723 kcal/m³ day), that is roughly a yield of 206×10^{-13} kcal per cell, or 17×10^{13} cells per person per day (i.e. 68 l) for a 3500 kcal/day diet. Although all numbers are rough, we can conclude that by producing enough food for crew members, their need for oxygen is more than satisfied. However, oxygen needed for decomposition has to be considered as well.

Note that, as no optimisation for growth or oxygen production had been made, the above numbers are underestimates of possible yields. Furthermore, numbers for oxygen were obtained in diazotrophic medium while yield of 150 gdw/m³ day was obtained with added NO₃.

NASA predicts approximately 30 m² growing surface weighing 2.15 tonnes per person (Campbell and Moore, 1993). Even though this is a significant improvement in comparison to 6 tonnes of supplies that would have to be taken without CELSS, we believe that algae, because of greater growth rates, are a better alternative. Furthermore, after only 4 days without proper lightning, higher plants need 2 days to regain productivity (Muhlestein et al., 1999). Regeneration time after a serious hazard is approximately 60 days. On the other hand, algae can regenerate fully from inoculum in a few days, offering an advantage compared with higher plants.

Although models on various compartments of CELSS exist (Fleisher and Ting, 1998; Lasseur and Fedele, 1999), we have not found evidence of a model for a whole cycle. In addition, existing models are made to comply with experiment setup instead of modelling different life support system alternatives and making experiments to determine critical parameters.

The aim of this work is to,

1. investigate whether continuous flow or batch reactor should be used in a life support system;
2. devise a simple model for a life support system in order to investigate its feasibility in terms of weight and stability, and introduce possible higher trophic levels through a model of reactors in a cascade;
3. investigate steady-state behaviour of a cascade;
4. identify lacking vital parameters and suggest further research, both experimental and theoretical;
5. investigate whether a life support system based on algae is more portable than a higher plant based system.

2. Yield in a batch versus continuous reactor

In experiments on growth of *cyanotheca* sp. strain 51142 by Arieli et al. (1996) a semincontin-

uous reactor had a greater yield than batch reactor (250 vs. 79.2 gdw/m³ day). Let us investigate in general whether batch or continuous reactor has a greater yield per unit volume. All equations will be formulated for a reactor size of 1 volume unit.

Since it is often used, we will assume Michaelis–Menten kinetics (Michaelis and Menten, 1913). Let us calculate, based on equations in Table 1, the difference between batch and continuous reactor,

Yield in a batch reactor is defined as, biomass at the end of observation – biomass at the beginning, because after harvesting is performed, the batch reactor is refilled with starting values of N_0 and S_0 . Yield in a continuous reactor is the amount of biomass, which has flown out of the reactor in time interval T (in its optimum, $(I - DK/v - D)T$, where T is the time interval of observation).

Consider the example of *Thalassiosira pseudo-nana* (TP, chosen because of availability of parameters). From Jörgensen (1979), we read half saturation constant for nitrogen uptake of TP to be 0.5 $\mu\text{mol/l}$. Similarly, for TP maximum growth rate, we find values 1.14–2.77 1/day. Staying on the safe side (underestimating maximum growth rate), we choose 2 1/day.

Its volume is 55 $\mu\text{m}^3 = 55 \times 10^{-15}$ l, which gives the maximum theoretical density 1.8×10^{13} cells per l.

From Jörgensen (1979), we calculate that the theoretical maximum biomass in N is $N_{\max} = 0.772$ mol/l (1.5×10^6 greater than half-saturation constant).

The lower initial population concentration (N_0) is, the higher initial nutrient concentration needs to be, the longer it will take for the population to reach N_{\max} and higher the yield. However, time between harvests (T) in a given reactor is longer. Therefore, yield in unit of time is higher for higher initial population concentrations (lower initial nutrient concentrations) and as short time between harvests as possible (Fig. 1). In the limit $T \rightarrow 0$, we get a continuous reactor.

For example, if we use batch reactor with exactly the generation time (1 day) between harvesting, the harvest would be 0.666 mol/l day from

Table 1
Equations for batch and continuous reactor

	Batch	Continuous
Nutrient, S	$\frac{dS}{dt} = -\frac{vS}{K+S}N$	$\frac{dS}{dt} = (I-S)D$
Population, N	$\frac{dN}{dt} = -\frac{vS}{K+S}N$	$\frac{dN}{dt} = -\frac{vS}{K+S}N$
Yield, Y	$N_0(e^{vT} - 1)$	$\left(I - \frac{DK}{v-D}\right)DT$
Y in unit of time	in lin. approx. $\frac{N_T - N_0}{T}$	$\left(I - \frac{DK}{v-D}\right)D$

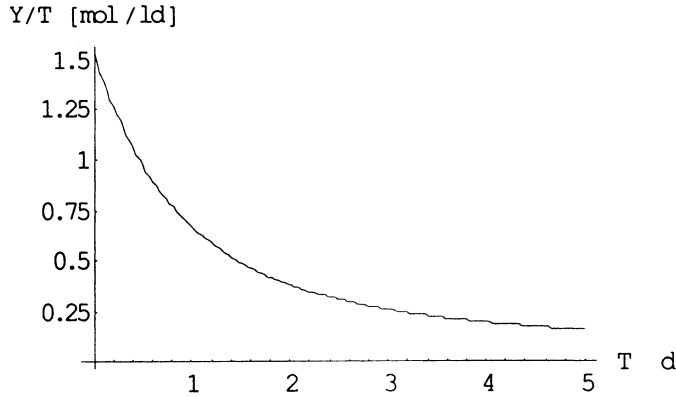


Fig. 1. Dependence of yield in unit of time on time between harvests.

the batch reactor, and 1.58 mol/l day (130% more) from a continuous reactor running in the optimum mode. The optimum flushing rate for a continuous reactor is $D_{opt} = v(1 - 1/\sqrt{1 + I/K})$. In our example, $D_{opt} = 1.998$ l/d.

Furthermore, in a continuous reactor, the average number of cells is greater and, thus, oxygen production benefits as well. For 1 day harvesting cycle, according to the model in (Table 1), oxygen production is also $\approx 130\%$ greater in a continuous reactor.

Both reactors are more efficient with higher nutrient concentrations. However, contrary to a batch reactor, a continuous reactor does not make full use of available nutrients. That is not a problem, because in a closed system, these nutrients will not be wasted.

3. A cascade

For the sake of menu diversity, one may wish either to combine several phytoplankton reactors or add second, third or any number of reactors where each subsequent reactor contains one more trophic level.

A cascade is defined as a chain of $n > 1$ linked reactors (Fig. 2). Output from the first reactor is transferred to the second, output from the second is transferred to the third and so on to the n -th reactor. Input of nutrients to the first reactor is considered as the input in the cascade.

The k -th reactor is described by following equations,

$$\frac{dS^k}{dt} = D^{k-1}S^{k-1} - \frac{v_1S^k}{K_1 + S^k}N_1^k - D^kS^k$$

$$\frac{dN_i^k}{dt} = D^{k-1}N_i^{k-1} + \frac{v_iN_{i-1}^k}{K_i + N_{i-1}^k}N_i^k - \frac{v_{i+1}N_i^k}{K_{i+1} + N_i^k}N_{i+1}^k - D^kN_i^k$$

$$\frac{dN_k^k}{dt} = \frac{v_kN_{k-1}^k}{K_k + N_{k-1}^k}N_k^k - D^kN_k^k$$

Using a cascade of reactors (instead of one reactor with all species in it) has following benefits.

1. There is no influence of predator in the second on predator in the first reactor.
2. Possibility of chaotic behaviour is diminished severely by ruling out feedback, which may lead to large oscillations.

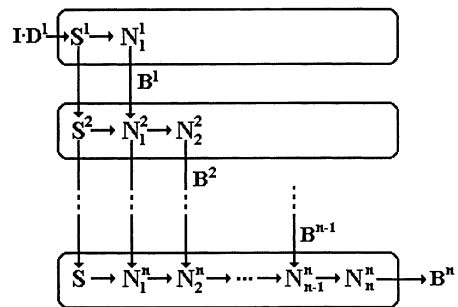


Fig. 2. A cascade of n reactors.

4. The effect of nutrient input on the highest trophic level in a reactor

The inflow of i -th trophical level into k -th reactor from $(k-1)$ st reactor, measured in biomass, is $B_i = D^{k-1}N_i^{k-1}$, $i < k$.

Dropping the upper index, equations for the k -th reactor are,

$$\frac{dS}{dt} = B_0 - \frac{v_1 S}{K_1 + S} N_1 - DS$$

$$\frac{dN_i}{dt} = B_i + \frac{v_i N_{i-1}}{K_i + N_{i-1}} N_i - \frac{v_{i+1} N_i}{K_{i+1} + N_i} N_{i+1} - DN_i$$

$$\frac{dN_k}{dt} = \frac{v_k N_{k-1}}{K_k + N_{k-1}} N_k - DN_k$$

Theorem 1. *In steady state, only the last trophic level of any reactor in a cascade depends on the nutrient input, I .*

Lema 1. *In the n -th reactor, steady states of $N_{k < (n-1)}$ depend only on species-specific parameters, inflow $B_{i \leq k}$, D and state variable S .*

Proof 1. Equations for the first three trophic levels in a reactor,

$$\frac{dS}{dt} = B_0 - \frac{v_1 S}{K_1 + S} N_1 - DS = 0 \tag{1}$$

$$\frac{dN_1}{dt} = B_1 + \frac{v_1 S}{K_1 + S} N_1 - \frac{v_2 N_1}{K_2 + N_1} N_2 - DN_1 = 0 \tag{2}$$

$$\frac{dN_2}{dt} = B_2 + \frac{v_2 N_1}{K_2 + N_1} N_2 - \frac{v_3 N_2}{K_3 + N_2} N_3 - DN_2 = 0 \tag{3}$$

If the dependence on state variables (S and N_i) is traced, from expression (Eq. (1)), N_1 follows as a function of S ; from expression (Eq. (2)), N_2 follows as a function of N_1 ; from expression (Eq. (3)), N_3 follows as a function N_2 .

For every following equation of the kind, we can see that the steady state of the state vari-

able $N_{k < (n-1)}$ depends on inflows from the previous reactor ($B_{i < k}$) and S only.

Equations for the last three trophic levels in a reactor,

$$\begin{aligned} \frac{dN_{n-2}}{dt} &= B_{n-2} + \frac{v_{n-2} N_{n-3}}{K_{n-2} + N_{n-3}} N_{n-2} \\ &\quad - \frac{v_{n-1} N_{n-2}}{K_{n-1} + N_{n-2}} N_{n-1} - DN_{n-2} = 0 \end{aligned} \tag{1a}$$

$$\begin{aligned} \frac{dN_{n-1}}{dt} &= B_{n-1} + \frac{v_{n-1} N_{n-2}}{K_{n-1} + N_{n-2}} N_{n-1} \\ &\quad - \frac{v_n N_{n-1}}{K_n + N_{n-1}} N_n - DN_{n-1} = 0 \end{aligned} \tag{2a}$$

$$\frac{dN_n}{dt} = \frac{v_n N_{n-1}}{K_n + N_{n-1}} N_n - DN_n = 0 \tag{3a}$$

Up to Eq. (1a), the upper rule is valid, i.e. N_{n-2} is a function of $B_{i < (n-2)}$ and S . Since N_{n-3} is a function of $B_{i \leq (n-3)}$ and S , the only unknown state variables in Eq. (1a) are S and N_{n-1} . From Eq. (3a), $N_{n-1} = DK_n/v_n - D$. Hence, N_{n-1} is not dependant on any other state variable except S . It follows that all trophic levels below $N_{(n-1)}$ are dependant on $B_{k < (n-1)}$, species parameters, D and S only, which had to be proven. Notice that Lema 1 is applicable to one reactor with n trophic levels if we set appropriate $B_{i > 0} = 0$. In case of two linked food chains A and B, where there is transition of certain trophic levels from A to B and no transition from B to A, the Lema 1 is also applicable to the food chain B if we put corresponding inflows $B_i \neq 0$.

Proof 2. Theorem 1 is proved by mathematical induction.

Base — For the reactor with one species, the steady states are $S^* = DK/v - D$, $N_1^* = I - DK/v - D$, and the statement from the Theorem 2 is obviously valid.

By the assumption of induction, only the last inflow from the previous reactor depends on initial nutrient concentration I ,

$$B_{i \neq (k-1)} \neq f(I)$$

$$B_{k-1} = f(I)$$

Step of the induction — If the statement is valid for $(n-1)$ st reactor, we have to prove that it is valid for the n -th reactor.

The last two equations for the n -th reactor are

$$\begin{aligned} \frac{dN_{n-1}}{dt} &= B_{n-1} + \frac{v_{n-1}N_{n-2}}{K_{n-1} + N_{n-2}} N_{n-1} \\ &\quad - \frac{v_n N_{n-1}}{K_n + N_{n-1}} N_n - DN_{n-1} = 0 \end{aligned} \quad (3)$$

$$\frac{dN_n}{dt} = \frac{v_n N_{n-1}}{K_n + N_{n-1}} N_n - DN_n = 0 \quad (4)$$

Only B_i which is dependant on I is, by assumption, B_{n-1} . From Eq. (3) $\Rightarrow N_{n-1} = DK_n/v_n - D$. From Eq. (4) $\Rightarrow N_n = (B_{n-1}/D) + (v_{n-1}N_{n-2}/K_{n-1} + N_{n-2})(K_n/v_n - D)$

$-(DK_n/v_n - D)$ i.e. $N_n = f(v_n, K_n, D) +$ linear function of $I + f(v_{n-1}, K_{n-1}, N_{n-2})$. According to Lema 1, N_{n-2} can be obtained recursively from lower trophic levels ($N_{i < (n-2)}$) and N_{n-1} , none of which (by assumption) is dependant on I , hence only N_n depends on I .

Corollary 1. *In steady state in any reactor of the cascade, the abundance of trophic level before the last does not depend on total number of levels in a reactor.*

Corollary 2. *The dependence in Theorem 1 is linear.*

Theorem 1 highlights another important advantage of our approach. Whereas an ordinary food chain reacts to increased nutrient input in increased abundance of the highest trophic level and then alternating decrease and increase in abundance of lower trophic levels (Legović, 1979), in our system increasing the nutrient input increases abundance of all trophic levels. Thus, the need for higher or lower food production is readily controlled by changing nutrient input in the first reactor.

Theorem 1 and Lema 1 are valid for any functional response, which is (a) directly proportional to predator; and (b) dependant on prey only.

5. Considering two top trophic levels in a reactor

Considering only two top trophic levels (the most important ones) in every reactor would significantly simplify the analysis. Let us investigate conditions under which such an approximation is justifiable. Calculating steady states for the second reactor in a cascade gives

$$\begin{aligned} N_1^2 &= \frac{K_2 D^2}{v_2 - D^2} \\ S^2 &= \frac{1}{2D^2} \left[\left(\frac{D^1 K_1}{v_1 - D^1} - v_1 N_1^2 - D^2 K_1 \right) \right. \\ &\quad \left. + \sqrt{\left(v_1 N_1^2 + D^2 K_1 - \frac{D^1 K_1}{v_1 - D^1} \right)^2 + 4 \frac{D^1 D^2 (K_1)^2}{v_1 - D^1}} \right] \\ N_2^2 &= \frac{K_2 + N_1^2}{v_2 N_1^2} \left(\left(\frac{v_1 S^2}{K_1 + S^2} - D^2 \right) N_1^2 + I - \frac{D^1 K_1}{v_1 - D^1} \right) \end{aligned}$$

Steady states for the case with approximation (S^2 is neglected) are.

$$\begin{aligned} N_1^2 &= \frac{K_2 D^2}{v_2 - D^2} \\ N_2^2 &= \frac{K_2 + N_1^2}{v_2 N_1^2} \left(-D^2 N_1^2 + I - \frac{D^1 K_1}{v_1 - D^1} \right) \end{aligned}$$

Note that steady states for the $(k-1)$ st trophic level in k -th reactor are the same whether we use the approximation or not. This decreases the influence of the approximation, because the last two trophic levels are the ones that are most important in the next reactor.

Furthermore, note that steady states would be equivalent if $v_1 S^2 / K_1 + S^2 \rightarrow 0$. This is the case when $S^2 \rightarrow 0$. The condition is fulfilled since the concentration of nutrients decreases through two reactors to the concentration S^2 .

6. A cascade of two reactors

Let us investigate the first two adjacent reactors in a cascade of any number of reactors. Consumer in the first reactor becomes a prey in the second reactor. Taking mortality (m) into account, the equations are given in Table 2.

We assume that the first reactor is in steady state when the second reactor is switched on.

Table 2
Two reactors in a cascade

First reactor	Redefinition	Second reactor
$\frac{dS^1}{dt} = (I - S^1)D$	$N_1^1 \rightarrow I^2$	$\frac{dS^2}{dt} = (I^2 - S^2)D$
$\frac{dN_1^1}{dt} = \frac{-\frac{v_1 S^1}{v_1 S^1 + S^1} N_1^1}{K_1 + S^1} N_1^1 - DN_1^1$	$N_1^2 \rightarrow S^2$	$\frac{dN_2^2}{dt} = \frac{-\frac{v_2 S^2}{v_2 S^2 + S^2} N_2^2}{K_2 + S^2} N_2^2 - DN_2^2$

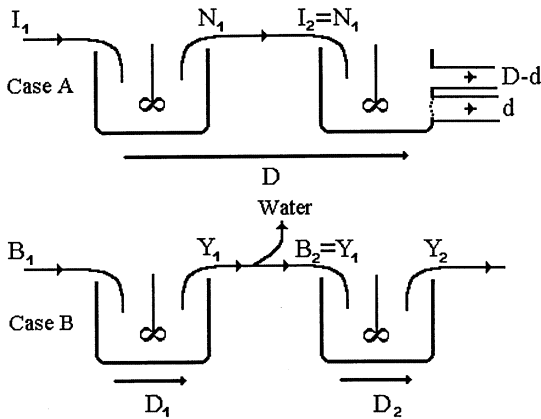


Fig. 3. Two ways of having different flushing rates in reactors.

Consequently, a redefinition is made of N_1^1 to input nutrient concentration in the second reactor (I^2) and first species in the second reactor (N_1^2) to nutrient concentration in the second (S^2).

Using the approximation in a cascade of reactors enables us to treat yield per unit time in the first reactor as nutrient inflow to the second, leading to the same equations and stability analysis in all reactors (under the assumption that previous reactors are kept in steady state).

We assumed that the prey from the $(k - 1)$ st reactor does not play a significant role in the k -th reactor. The assumption is justifiable if,

1. one operates the $(k - 1)$ st reactor under optimum conditions;
2. growth of predator due to increased prey biomass which consumed nutrient remaining from the previous reactor and not spent on

respiration are second- and third-order effects. In our example, corrections due to three-species interaction in the second reactor are somewhat less than 5%, not taking respiration into account (which would diminish the difference even further).

As an illustration, we take *Thalassiosira pseudonana* as a prey ($K_1 = 0.5 \mu\text{mol/l}$, $v_1 = 2 \text{ 1/day}$, $m = 0.2 \text{ 1/day}$ (Jørgensen, 1979) and *Daphnia* as a predator ($K_2 = 147.6 \mu\text{mol/l}$, $v_2 = 0.4 \text{ 1/day}$, $m = 0.005 \text{ 1/day}$). Parameters taken for *Daphnia* are converted from 12.4 mg(C)/l (mean value) to mg(N)/l by using C to N ratio by weight (5–7, we take 6) and then to $\mu\text{mol/l}$ by considering molar mass of *N*.

Since the optimum flow is a function of species-specific parameters, there is no single flushing rate that would let all reactors in a cascade operate under optimum conditions. Therefore, we have to allow different flushing rates. There are at least two ways of doing that (Fig. 3) which are described below.

7. Case A

In the Case A (Fig. 3), by keeping a part of the population in the reactor we decrease the effective biomass outflow. Hence, we are able to define effective flushing rate, $(D - d)$. The equation describing the predator in the second reactor has to be modified to $-\frac{dN_2^2}{dt} = v_2 S^2 / (K_2 + S^2) N_2^2 - (D - d) N_2^2$. Since $(D - d)$ exerts the influence on the population as D before, by controlling d , we control the effective flushing rate $(D - d)$ without influencing nutrient input $D \times I^2$. Optimisation of effective flushing rate cannot be done (Fig. 4, dotted line). When we ignore mortality, the smaller the effective flushing rate $(D - d)$ is, the greater is the yield obtained. The reason for this is a rapid increase of predator population density in the reactor, and as the flushing rate is lowered (hence reducing the yield), increased density more than compensates that effect. However, the residence time (Legović and Cruzado, 1995) increases and predator mortality cannot be ignored. By introducing mortality, optimum is readily found (Fig. 4, solid line).

Appropriate equations are given in Table 3.

Influence of predator mortality on yield is shown on Fig. 5.

Mortality reduces yield, but does not change the qualitative behaviour of the reactors. If nutrient concentration in the inflow falls below the critical value, $I^2 \leq K_2 m / v_2 - m$, there are no

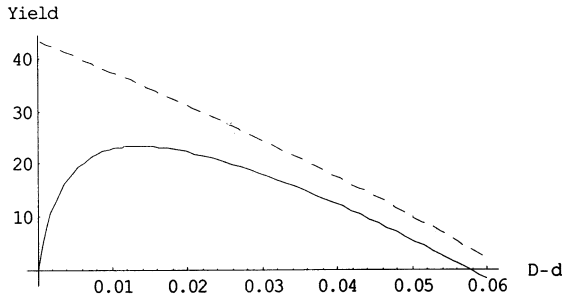


Fig. 4. Optimising effective flushing rate for $m = 0.005$ l/day (solid) and $m = 0$ (dashed).

Table 3
Model of the case A

First reactor	Redefinition	Second reactor
$\frac{dS^1}{dt} = (I - S^1)D$	$N_1^1 \rightarrow I^2$	$\frac{dS^2}{dt} = (I^2 - S^2)D$
$-\frac{v_1 S^1}{K_1 + S^1} N_1^1$		$-\frac{v_2 S^2}{K_2 + S^2} N_2^2$
$\frac{dN_1^1}{dt} = \frac{v_1 S^1}{K_1 + S^1} N_1^1$	$N_1^2 \rightarrow S^2$	$\frac{dN_2^2}{dt} = \frac{v_2 S^2}{K_2 + S^2} N_2^2$
$-(D - m_1)N_1^1$		$-(D - d - m_2)N_2^2$

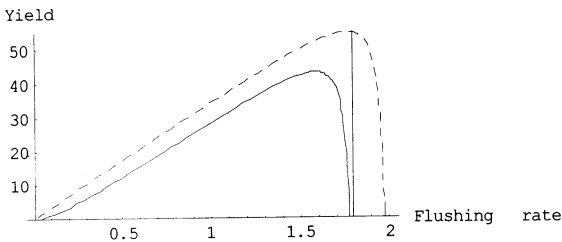


Fig. 5. Influence of predator mortality on yield, $m = 0.2$ l/day (solid), $m = 0$ (dashed).

Table 4
Model of the case B

First reactor	Redefinition	Second reactor
$\frac{dS^1}{dt} = B_1 - S^1 D^1$	$N_1^1 D^1 \rightarrow B_2$	$\frac{dS^2}{dt} = B_2 - S^2 D^2$
$-\frac{v_1 S^1}{K_1 + S^1} N_1^1$		$-\frac{v_2 S^2}{K_2 + S^2} N_2^2$
$\frac{dN_1^1}{dt} = \frac{v_1 S^1}{K_1 + S^1} N_1^1$	$N_1^2 \rightarrow S^2$	$\frac{dN_2^2}{dt} = \frac{v_2 S^2}{K_2 + S^2} N_2^2$
$-(D^1 - m_1)N_1^1$		$-(D^2 - m_2)N_2^2$

steady-state solutions with positive biomass concentrations.

8. Case B

In the Case B, a constant nutrient input (B_i) is considered with flushing rate in each reactor controlled independently by adding or distracting water. Therefore, both reactors can be kept in optimum at the same time. Optimised flushing rate for each reactor independently gives maximum sustainable yield in both reactors (Table 4).

9. Comparison of case A and B

For $I = 35$ $\mu\text{mol/l}$, Table 5 summarises the maximum yield in steady state with mortality taken into account.

The case of optimised joint flushing rate ($D_1 = D_2 = D$) is given for comparison.

If one takes all three species into account in the second reactor (in the case A), yield of 40.4 $\mu\text{mol/l}$ day is computed. If we can choose the individuals to be harvested (e.g. fish), case A offers better description. For homogenous cultures like algae and zooplankton, case B is obviously more appropriate.

In reality, not all taken nutrients convert to biomass. When a conversion coefficient is added modified equations for the second reactor are.

Table 5
Comparison of yields per day for different handling of the flushing rate

	First reactor ($\mu\text{mol/l day}$)	Second reactor ($\mu\text{mol/l day}$)
$D^1 = D^2 = 0.04 \text{ l/day}$	1.4	0.7
Case A in Fig. 3, optimum D in first (1.585 l/d), optimum ($D-d$) in second (0.0135 l/day) reactor	43.4	23.4
Case B in Fig. 3, optimum in both reactors ($B_2 = 43.4 \mu\text{mol/l day}$)	43.4	38.5

Equations	Steady state solutions
$\frac{dS^2}{dt} = B_2 - S^2 D^2$	$S^{*2} = \frac{(D^2 + m_2)K_2}{c_2 v_2 - (D^2 + m_2)}$
$-\frac{v_2 S^2}{K_2 + S^2} N_2^2$	
$\frac{dN_2^2}{dt} = c_2 \frac{v_2 S^2}{K_2 + S^2} N_2^2$	$N_2^* = \frac{c_2 B_2}{D^2 + m_2}$
$-(D^2 + m_2) N_2^2$	$-\frac{c_2 D^2 K_2}{c_2 v_2 - (D^2 + m_2)}$

Compared with the system with complete conversion, the yield is smaller. Otherwise, qualitative behaviour of the system with conversion efficiency taken into account is the same as the one described by equations in Table 4.

10. Yield in a cascade of n reactors

The maximum yield in all reactors in a cascade may not define the ideal proportion of food for the crew. It would be useful to know the required nutrient input in a cascade as in Fig. 6 in order to obtain desired output from each reactor.

Assume that b_2 units of biomass are needed per day from the second and b_1 from the first reactor. In order to obtain b_2 from the second reactor, solving equations in (Table 6) gives the needed input of.

$$B_2 = \frac{(D^2 + m_2)}{c_2 D^2} \left(b_2 + \frac{c_2 D^2 K_2}{c_2 v_2 - (D^2 + m_2)} \right)$$

Total output of the first reactor should be $B_2 + b_1$, since we need b_1 for consumption by the crew and B_2 for transport to the second reactor. Required input for this output is.

$$B_1 = \frac{(D^1 + m_1)}{c_1 D^1} \left(b_1 + B_2 + \frac{c_1 (D^1)^2 K_1}{c_1 v_1 - (D^1 + m_1)} \right)$$

Let us discuss the results in some detail.

1. The leading coefficient $(D + m)/cD$ points to an intuitively clear conclusion — if mortality is greater or conversion coefficient smaller, more influx to the reactor is needed.
2. Required input is a linear function of needed output, $B_n \propto (b_n + B_{n+1})$.
3. Required input also has to compensate for food outflow. By rearranging the solution and

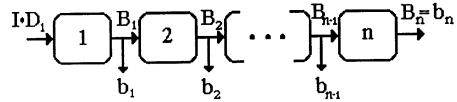


Fig. 6. A cascade of n reactors with b_i taken from the i th reactor.

Table 6
Equations of the production–decomposition loop

Reactor		Equation
First	Anorganic	$\frac{dS^1}{dt} = B - S^1 D^1 - \frac{v_1 S^1}{K_1 + S^1} N_1^1$
First	Organic	$\frac{dN_1^1}{dt} = \frac{v_1 S^1}{K_1 + S^1} N_1^1 - D^1 N_1^1$
Second	Organic	$\frac{dS^2}{dt} = D^1 N_1^1 + D^2 N_2^2 - S^2 D^2$ $-\frac{v_2 S^2}{K_2 + S^2} N_2^2$
Second	Bacteria	$\frac{dN_2^2}{dt} = r \frac{v_2 S^2}{K_2 + S^2} N_2^2 - D^2 N_2^2$
Buffer	Anorganic	$\frac{dN_3^3}{dt} = \frac{1}{r} D^2 N_2^2 - B + S^1 D^1$

comparing with steady-state values for food concentration, makes that point clearer, $-c(D)^2K/D + m - cv = c(D)^2K/cv - (D + m)$, this being the second term in steady state solution to the system (multiplied by D because this is yield and solutions are for concentrations).

Generalization for n reactors is straightforward.

$$B_i = \frac{(D^i + m_i)}{c_i D^i} \left(b_i + B_{i+1} + \frac{c_i (D^i)^2 K_i}{c_i v_i - (D^i + m_i)} \right)$$

Ignoring sinks ($c = 1$, $m = 0$) results in readily comprehensible solution,

$$B_i = \left(b_i + B_{i+1} + \frac{(D^i)^2 K_i}{v_i - D^i} \right)$$

i.e. needed input = required output + input needed in the subsequent reactor + nutrients which are not used.

Theorem 2. Let the reactors in Fig. 6 be described by

$$\frac{dS^i}{dt} = B_i - S^i D^i - X(S^i) N_i^i \quad (5)$$

$$\frac{dN_i^i}{dt} = Y(S^i) N_i^i - (D^i - m_i) N_i^i \quad (6)$$

$X(S)N$ is an uptake function, and $Y(S)N$ growth function. Given b^i , $i = 1, \dots, n$, it is possible to find a recursion formula that would calculate needed nutrient input I in the cascade. Specifically, if and only if $X = Y$ and mortality $m = 0$, then the recursion formula is, $B_i = (b_i + B_{i+1} + D^i S^{i*})$, where S^{i*} is the steady state of the i -th reactor.

Proof 3. Since equations in all reactors are the same, only indices change, steady-state solutions to any reactor in the system will have the same form. Consequently, it suffices to prove that B_i is a function of B_{i+1} and b_i (and constants describing the species and D^i).

From Eq. (6), steady state of the system S^{i*} is given by $Y(S^{i*}) = (D^i - m_i)$, which means that S^{i*} is a function of m_i , D^i and species-specific parameters contained within Y .

Substituting S^{i*} into Eq. (5), steady state N^{i*} is obtained as a function of D^i , m_i and B_i as well as

species-specific parameters contained within Y and X .

Since D^i , m_i and species-specific parameters contained within X and Y are determined in advance and are known constants, we can look at N^{i*} as a function of B_i .

Furthermore, $(B_{i+1} + b_i) \equiv D^i N^{i*}$ and N^{i*} is a function of B_i , consequently B_{i+1} is a function of b_i and B_i , i.e. from B_{i+1} and b_i needed B_i can be obtained. Therefore, the general case holds.

Specific case follows from mass conservation law. If all taken, nutrients get converted to biomass, then, needed input $B_i =$ wanted yield $b_i +$ needed input for the next reactor $B_{i+1} +$ unused nutrients $D^i S^{i*}$. If there is any loss of biomass, either to conversion inefficiency or difference of uptake and growth functions, mass is not conserved and the formula does not hold. Conversely, if the formula does not hold, the mass is not conserved and there must exist some sinks. Sinks in the system are introduced through mortality and/or difference of X and Y .

11. Illumination

In order to be as compact as possible, reactor with phytoplankton growth must have as large a number of cells as possible. Consequently, supply of light might be critical.

Consider expression for dependence of growth rate on illumination intensity (Canale, 1976),

$$f(I) = \frac{I}{I_{\text{opt}}} e^{(1 - I/I_{\text{opt}})}$$

with exponential intensity dependence on depth ($I = I_0 \exp(-z \cdot N/c)$), I_{opt} is optimum and I_0 is surface illumination intensity, z is depth at which light intensity (I) is measured. For fixed population density N , cell transparency c and given depth of reactor z_{max} , optimum surface illumination intensity can be found

$$I_0 = I_{\text{opt}} \frac{N z_{\text{max}}}{c} \frac{\exp(N z_{\text{max}}/c)}{(\exp(N z_{\text{max}}/c) - 1)} \quad \text{or}$$

$$I_0 = I_{\text{opt}} \frac{x e^x}{e^x - 1} \quad \text{for} \quad x = \frac{N z_{\text{max}}}{c}$$

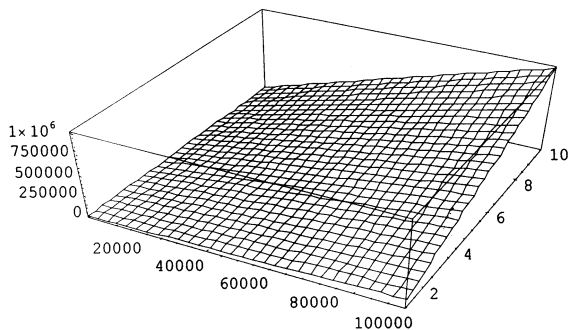


Fig. 7. Qualitative dependence of optimum surface illumination intensity on reactor depth and population density.

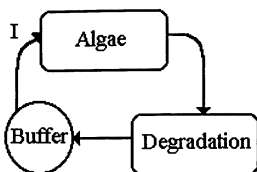


Fig. 8. Simple material loop.

Similarly, for given surface illumination intensity and population density, optimum depth of the reactor can be found. If we let population density to vary, no optimum can be found because any light inhibition is more than compensated by increased density. Illustrating this point, a diagram of optimum surface illumination intensity versus reactor depth and population density is shown on Fig. 7.

12. Degradation

Degradation is a link that closes the material loop.

Data from MELiSSA experiments (Lasseur and Fedele, 1999) were used. Every two days 0.15 l from 1.6 l batch reactor were taken out, analysed and replaced by fresh waste material taken from different people. Degradation efficiency was from 40 to 50%, and was constant after initial settling period of 60 days. From these data, assuming exponential growth, the growth rate can be calculated,

$$N(0) = N(T) \frac{(V_0 - V)}{V_0};$$

$$S(0) = \frac{(IV + S(T)(V_0 - V))}{V_0}$$

$$S(T) = \frac{I}{2}$$

$$N(t) = N(0)(e^{vt} - 1)$$

$$\frac{I}{2} = Ir + \frac{I}{2}(1 - r) + N(T) \Rightarrow N(T) = \frac{Ir}{2}.$$

Considering given values, $v = 0.4$ l/day. This is quite a low value considering that bacteria have doubling time of few hours (compared with 60 h in this experiment). Surely order of magnitude improvements and optimisation can be made.

Furthermore, sample contained material which is not easy to degrade and which probably will not be present in a life support system.

Finally, bacterial growth was $Ir/2$ where $r = V/V_0$ is the ratio of exchanged volume to total volume. This means (providing 50% efficiency of the reactor) that to grow $(Ir/2)V$ bacterial cells $IV/2$ organic matter needs to be decomposed. Such reasoning gives us ‘coefficient of conversion’ r to be ca. 0.1, i.e. the decomposition of one unit of waste material requires increase in bacterial biomass for 0.1 units.

13. Material loop

First step towards a fully operating life support system could be similar to the (Fig. 8).

The role of the ‘buffer’ is to stabilise the system by acting as a low-pass filter. As both reactors are locally stable, any disturbance starting from (say) production reactor is going to change its output and thus propagate to decomposition reactor by changing its input. However, buffer will stop the propagation by supplying production reactor with constant input and allowing it to come back in original steady state. This system assumes that fluctuations are random. Any trend has to be dealt with dynamically. Buffer is also a convenient place to devise ‘shield’ against infection of the production reactor by means of microwave radiation, for example.

Bacteria use organic matter to multiply, thus, anorganic output is proportional to their yield (Table 6).

If we take typical values for bacterial growth and half-saturation constants ($v = 2$ 1/day (Jørgensen 1979) and $K = 0.3$ $\mu\text{mol/l}$), we can make an estimate of other parameters. In order to run production reactor in the optimum mode, we must have $B = 1.538$ mol/l day and $D^1 = 1.998$ 1/day. It follows that the flushing rate in the decomposition reactor must be $D^2 = 0.2$ 1/day, and the system is in the steady state. Since bacterial and phytoplankton densities are similar, the biodegradation compartment must be similar in weight to phototrophic compartment. This would give about 150–200 l per person given that adequate mixing and illumination with such cell densities can be provided. For comparison in aquatic environments on Earth about 4 400 000 l are needed.

14. Conclusion and recommendations

In general, continuous reactors give greater maximum sustainable yield (MSY) than batch reactors. However, a continuous reactor does not make full use of available nutrients. Both batch and continuous reactor are more efficient with higher concentrations of organisms.

Two ways for linking reactors in a cascade have been proposed. The first one is more appropriate where population can be harvested selectively (fish), the second one is more appropriate for homogenous populations (algae, zooplankton).

A cascade of n reactors has been investigated,

1. the MSY of each reactor has been computed and it implies population specific optimum flushing rate;
2. consideration of only the highest two trophic levels in a cascade of reactors has been justified;
3. using Michaelis–Menten kinetics and considering only the highest two trophic levels in a cascade of reactors, general recursive relation giving needed input in a cascade for desired output from each reactor has been found,

$$B_i = \frac{(D^i + m_i)}{c_i D^i} \left(b_i + B_{i+1} + \frac{c_i (D^i)^2 K_i}{c_i v_i - (D^i + m_i)} \right);$$

4. Theorem 1 and Corollary 1 and Corollary 2 have been proven, stating that considering all trophic levels in every reactor of a cascade, yield of the top predator in every reactor depends linearly on nutrient input in the cascade, while concentrations of other trophic levels are independent on nutrient input to a cascade.
5. Theorem 2 has been proven, stating that the recursion formula can be found for other kinetics as well (not only for Michaelis–Menten).

Using a model of a batch reactor, ratio of yield of bacteria to production of anorganic matter has been calculated, $r \approx 0.1$. Closed material loop has also been modelled, with the usage of buffer to stop the propagation of perturbations. This model is the basis for calculation of volume needed for CELSS based on algae. Using the data from literature, it has been calculated that it should be possible to construct CELSS of 200 l per person. That would be a significant improvement compared with size of a CELSS based on higher plants ($30 \text{ m}^2 \times 0.2 \text{ m} = 6 \text{ m}^3$).

For optimum operation, apart from nutrient and optimum temperature, the optimum illumination is needed. Since the expected phytoplankton culture will be very dense, the surface illumination intensity that will give MSY is computed, $I_0 = I_{\text{opt}}(xe^x/e^x - 1)$ where $x = N \cdot z_{\text{max}}/c$. MELiSSA report does not mention optimisation of surface illumination on these variables. It is probable that this optimisation has not been carried out, and I_{opt} used instead, I_{opt} being optimum illumination of a single cell, but not of the whole reactor.

NASA, in its report 1324 (Wieland, 1994), mentions the need for maximising yield per area in a CELSS. We believe that yield should be optimised on volume, time and weight. If the optimisation is carried out using the aforementioned variables, algae are by far more suitable for CELSS than higher plants, enabling use of smaller volume through higher efficiency.

Depending on how fast the waste material can be decomposed, the size of the decomposition

compartment can be computed. Although, in this paper, the decomposition is taken as a single process, it is clear how decomposition compartments would look like if more processes were to be taken into account.

The discussion above leads us to conclude that, at least theoretically, the idea of a life support system based on algae is feasible.

Further advance in giving more precise estimate of the size of a closed life support system is hindered by absence of experimental results. In the following, we discuss the most urgent sets of needed data.

1. The optimum menu must be established and the combination of algae satisfying these needs should be found.
2. Growth of algae as a function of nutrients is based on experiments with very low densities of cells found in water bodies on earth. A life support system for interplanetary missions must be as small as possible. This means that density of cells must be as large as possible, surely several orders of magnitude higher than natural water bodies on earth. The same is valid for the nutrient concentration in the input, hence appropriate experiments are needed to investigate the highest subtoxic nutrient concentration that can be administered to the phytoplankton growth compartment. Needed are mortality and respiration at very high concentrations of any biological entity that is grown.
3. In models considered above, due to lack of parameters, uptake and growth are treated as a single process. We know that this is not the case. Uptake (predation) and growth parameters are needed for very high concentration of every biological entity from the menu. As a consequence, to reach the maximum sustainable yield, more than one continuous reactor, or a combination of continuous and batch reactors, may be needed for a population grown.
4. Decomposition stands for a number of processes. There are indications that degradation could be improved by using few compartments (e.g. aerobic and anaerobic, Lasseur and Fedele, 1999) and industrial applications. The

slowest process must be given the highest attention because it is on this process that the size of the life support system will depend.

5. Uptake rates for individual compounds (like NO₃, NO₂ or NH₃ not just total N) are needed.

Future models should consider several nutrients and distinguish uptake from growth (Legović and Cruzado, 1997). That approach would, besides being more realistic description, enable greater densities. One well-mixed reactor, with small residence time, could be utilised for nutrient uptake, while growth could be allowed in non-mixed, thin and easily illuminated stripes.

Once the parameters are known, numerical analysis would be possible, and thus, respiration as a function of nutrient availability can be taken into account. These models could be very useful guidelines as to what characteristics of species are desirable and indicate directions for possible genetic manipulations. Needed reserves and controls could also be estimated.

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