DOPPLER EFFECT IN NON-GSO SATELLITE PROPAGATION

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INTRODUCTION

Recent technological advances have accelerated a renaissance in usage of non-geostationary (non-GSO) communication satellites in provision of various services domestically and globally. Early ventures into communication satellites have used satellites in low Earth orbits. The main reasons for this mode of operation were that the booster technology was not available to lift satellites into a higher orbit, the satellite technology was not sufficiently mature to achieve higher performance such as power generation and signal bandwidth. Also, there was a problem with reliability due to operation in hostile space environment. After that early stage, more than 30 years the most dominant domain of communication using satellites has been in circular geostationary equatorial Earth synchronous orbit. As the satellite system technology developed, it became possible to provide services by the use of satellites in various orbital regimes like low Earth orbit (LEO at about 1000 km altitude), medium Earth orbit (MEO at about 10000 km altitude), geostationary orbit (GSO) and the highly elliptical orbit satellites (HEO).

One of the problems ground transceivers are facing when communicating with non-GSO satellites is significant Doppler frequency shift. This time varying phenomenon is caused by the line of sight (LOS) component of the relative velocity vector evolving from the rapid movement of the satellite in its orbit relative to the ground transceiver including satellite velocity and the relative velocity due to the Earth's rotation. This paper deals with analytic derivation of the Doppler frequency shift observed by a user on the surface of the Earth as a function of time. Previous researches in this area mainly focused on efficient methods for compensation of this effect. Expression for obtaining radial velocity of the satellite using measurement of Doppler frequency shift is derived in [1]. In [2] the authors study Doppler frequency shift at the point of the Earth equator from the non-GSO satellite using circular equatorial orbit. The LEO satellites Doppler estimation algorithm for implementation in the terminal phase-lock loop, compensating frequency shift have been studied in [3].

This paper gives mathematical derivation of the trajectory coordinates of non-GSO satellites using inclined circular orbits. Based on this, LOS relative velocity component is deduced giving explicit equations for the frequency shift observed by a transceiver situated anywhere on the Earth. Furthermore S-shaped Doppler frequency shift – time curves are derived (with maximum elevation angle as parameter). Possible applications of Doppler frequency shift for satellite navigation and positioning system, compensation of time varying frequency shift in terminal PLL processor and aircraft monitoring, positioning, search and rescue are proposed and analysed.

NON-GSO SATELLITE TRAJECTORY COORDINATES

Trajectory of the non-GSO satellite (S) in circular inclined orbit using spherical longitude – latitude coordinates in the geocentric orbital plane coordinate system (O) (see Fig. 1) can be simply written as [1]

$$r_{S,O} = R_E + H_S = r_S \quad ; \quad j_{S,O} = 0 \quad ; \quad l_{S,O} = w_S \cdot t + f_0 \tag{1}$$

where R_E is mean equatorial radius (6378.144 km) of the Earth, H_S is satellite height above the Earth surface, w_S is angular velocity of the satellite, t is time and f_0 is the initial phase of the satellite comparing to the right ascension of the ascending node. Fig.1 gives the geometry of coordinate systems used in transformations for calculation of satellite coordinates and LOS velocity and main parameters which specify satellite orbit. Rotating the orbital plane coordinate system (O) around X_O axis by inclination angle (i) in the right hand direction, satellite trajectory is located in the geocentric equatorial plane coordinate system (E). As seen in Fig.1, Z_E axis coincides with the Earth's axis of rotation and extends through geographic north pole. The X_E and Y_E axes lie in the equatorial plane where X_E lies in the direction of the line of nodes. To have coordinates of the non-GSO satellite expressed in the reference coordinate system which translates as the Earth revolves around the Sun, but do not rotate in time, we transform equatorial plane coordinate system (E) by rotating around Z_E, in such way that X axis points toward the first point of Aries (X axis points from the centre of the Earth through the centre of the Sun at the vernal equinox when the subsolar point crosses the equator from south to north), see Fig.1. The coordinates of the non-GSO satellite orbit in the geocentric equatorial plane vernal point coordinate system (EV) are obtained transforming coordinates in (1) from O through E to EV coordinate system.

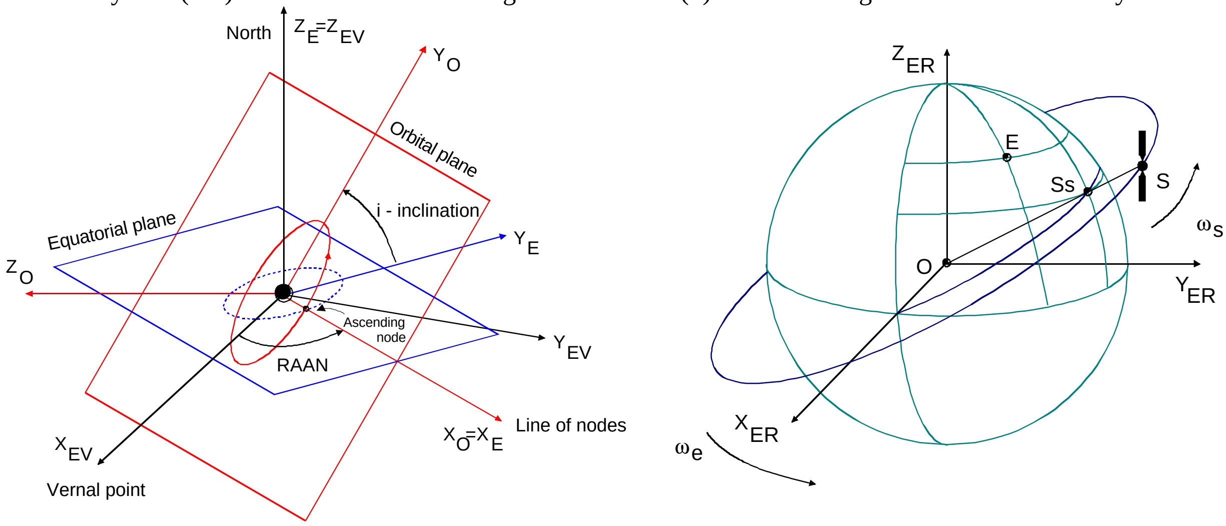

Fig.1. Geometry of coordinate systems used in transformations

Coordinates of the non-GSO satellite in EV coordinate system can be written as

$$r_{S,EV} = r_{S,O} = r_S \quad ; \quad j_{S,EV} = \arcsin[\sin i \cdot \sin(w_S \cdot t + f_0)] \quad ; \quad l_{S,E} = \arctan[\cos i \cdot \tan(w_S \cdot t + f_0)] + RAAN \tag{2}$$

where *RAAN* is right ascension of the ascending node which determines the orientation of the non-GSO satellite orbital plane (ascending node) regarding direction of the vernal equinox.
To determine the effects of the non-GSO satellite motion to the transceivers located on the surface of the Earth, it is necessary to present trajectory of the satellite in the coordinate system which rotates synchronous with Earth. Transforming the non-GSO coordinates from the EV to the ER coordinate system (geocentric equatorial plane Earth rotating coordinate system, Fig.1) equations for spherical (3) and rectangular (4) non-GSO satellite coordinates are obtained. They are shown in Fig.2 as function of time (remark: Figs. are presented for non-GSO satellite system with T=2h, i=60°, r_S=8052.61km, $RAAN$=-20°..20°, f_0=-30°..30°, e=0).

$$\begin{gathered} r_{S,ER} = r_S \quad ; \quad j_{S,ER} = \arcsin[\sin i \cdot \sin(w_S \cdot t + f_0)] \\ l_{S,ER} = \arctan[\cos i \cdot \tan(w_S \cdot t + f_0)] + RAAN - RAGM - w_E \cdot t + kp \end{gathered} \tag{3}$$

$$\begin{bmatrix} X_{S,ER} \\ Y_{S,ER} \\ Z_{S,ER} \end{bmatrix} = \begin{bmatrix} r_S \cdot \cos(w_S \cdot t + f_0) \cdot \cos(w_E \cdot t + RAGM - RAAN) + r_S \cdot \cos i \cdot \sin(w_S \cdot t + f_0) \cdot \sin(w_E \cdot t + RAGM - RAAN) \\ -r_S \cdot \cos(w_S \cdot t + f_0) \cdot \sin(w_E \cdot t + RAGM - RAAN) + r_S \cdot \cos i \cdot \sin(w_S \cdot t + f_0) \cdot \cos(w_E \cdot t + RAGM - RAAN) \\ r_S \cdot \sin i \cdot \sin(w_S \cdot t + f_0) \end{bmatrix} \tag{4}$$

In (3) and (4) *RAGM* is right ascension of the Greenwich meridian at 0h UT (universal time) at an observed Julian date, and w_E is angular velocity of the Earth. Using relations form triangles ESC, OES and ESsB in Fig.5 and introducing (3), (6) in relations, $e = \arcsin\left[\left(r_S{}^2 - R_E{}^2 - D^2\right)/2R_E\right]$, and $, a = \arctan(\tan \Delta l / \sin \Delta j)$, we obtain time dependent expressions and curves for azimuth and elevation seen from Earth transceiver shown in Fig.3.

ESTIMATION OF DOPPLER FREQUENCY SHIFT

Doppler frequency shift is caused by LOS component of the relative velocity vector which can be calculated by differentiating expression for distance of a non-GSO satellite from a point on the Earth which is given by

$$D = \left[(X_{S,ER} - X_{E,ER})^2 + (Y_{S,ER} - Y_{E,ER})^2 + (Z_{S,ER} - Z_{E,ER})^2\right]^{0,5} = \left(R_E{}^2 + r_S{}^2 - 2 \cdot R_E \cdot r_S \cos d\right)^{0,5} \tag{5}$$

$$D = R_E^{\ 2} + r_S^{\ 2} - 2 \cdot R_E \cdot r_S \cdot \cos(\Delta\, l_{S,E}) \cdot \cos(\Delta j_{\ S,E}) - 2 \cdot R_E \cdot r_S \cdot \sin(j_{\ S,ER}) \cdot \sin(j_{\ E,ER}) \cdot [1 - \cos(\Delta\, l_{S,E})] \tag{6}$$

where $X_{E,ER}$, $Y_{E,ER}$, $Z_{E,ER}$ are coordinates of the Earth terminal, $d$is geocentric angle between the Earth terminal and non-GSO satellite, $D\, l_{S,E}$ and $D j_{\ S,E}$ are longitude and latitude differences between satellite and Earth station coordinates.

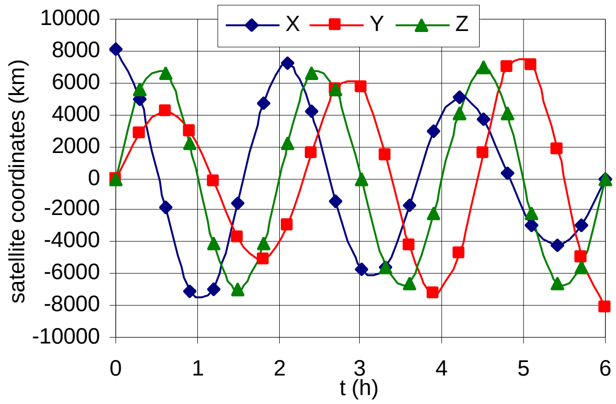

Fig.2. Rectangular NON-GSO satellite coordinates in ER

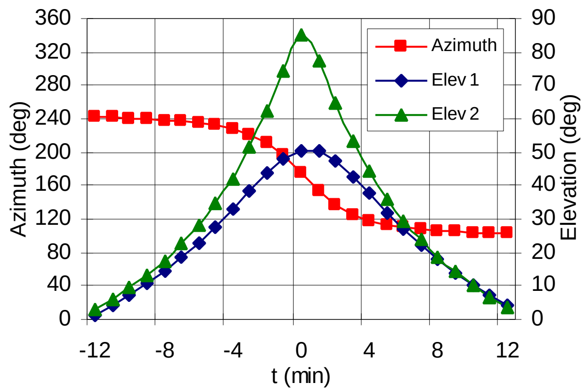

Fig.3. Azimuth and elevation curves

After introducing (4) in (5), time differentiating and substituting expression for maximum elevation (e_{m}), we obtain expression for the line of sight component of the relative velocity vector.

$$\frac{dD}{dt} = \frac{R_E \cdot r_S \cdot \sin\left[e_m + \arcsin\left(\frac{R_E}{R_E + r_S} \cdot \cos e_m\right)\right] \cdot \sin[(w_S - w_E \cdot \cos i) \cdot t] \cdot (w_S - w_E \cdot \cos i)}{\left\{R_E^{\ 2} + r_S^{\ 2} - 2 \cdot R_E \cdot r_S \cdot \sin\left[e_m + \arcsin\left(\frac{R_E}{R_E + r_S} \cdot \cos e_m\right)\right] \cdot \cos[(w_S - w_E \cdot \cos i) \cdot t]\right\}^{0,5}} \tag{7}$$

Introducing LOS relative velocity (7) in expressions, $v_{LOS} = dD/dt$, $f_d = -(v_{LOS}/c) f_c$, where c is speed of light, and f_c is carrier frequency, expression for Doppler frequency shift is obtained. Normalised Doppler frequency shift curves (f_d/f_c) with maximum elevation angle as a parameter are shown in Fig.4. Frequency shift curves have characteristic S shape with zero Doppler shift occurring in the moment when NON-GSO satellite is closest to the Earth transceiver station ($D=D_{min}$, $e=e_{max}$). Curves have an inflection point at zero Doppler shift and slant of the curves is determined by the maximum elevation angle.

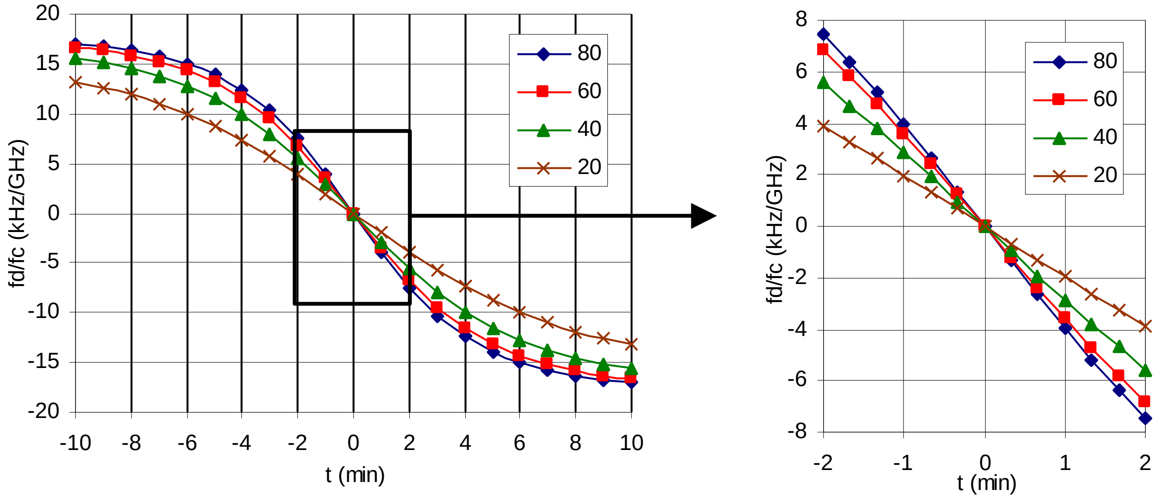

Fig.4. Normalised Doppler frequency shift – time curves with maximum elevation as parameter

Applications

Expression (7) can be used in Earth transceivers for the compensation of the time varying frequency shift in the terminal PLL processor, thus eliminating harmful effect to modulated signal. Because of their rapid movement relative to the ground transceiver, the signals provided by non-GSO satellites can be used for satellite based positioning and navigation system. Using relations form triangles ESsB, ESsA and OES in Fig.5 we obtain expressions for determination the position of the Earth terminal using one satellite.

$$l_{E,ER} = l_{S,ER} - \arctan\{\tan(L)\cdot\cos[\arcsin(\cos i/\cos j_{S,ER})]\} \quad \text{where} \quad L = p/2 - e_m - \arcsin(R_E \cdot \cos e_m / r_S) \quad (8)$$

$$l_{E,ER} = l_{S,ER} - \arctan\{\tan[\arcsin(D_{\min}\cdot\cos e_m/r_S)]\cdot\cos[\arcsin(\cos i/\cos j_{S,ER})]\} \quad (9)$$

$$j_{E,ER} = j_{S,ER} \pm \arcsin[\cos i\cdot D_{\min}\cdot\cos e_m/(r_S\cdot\cos j_{S,ER})] = j_{S,ER} \pm \arcsin[\cos i\cdot\sin(L)/\cos j_{S,ER}] \quad (10)$$

$$H = \{D_{\min}{}^2 + r_S{}^2 - 2\cdot D_{\min}\cdot r_S\cdot\sin[e_m - \arcsin(D_{\min}\cdot\cos e_m/r_S)]\}^{0.5} - R_E \quad (11)$$

Earth terminal can determine its position with measuring slant angle of Doppler frequency shift which determines maximum elevation angle, calculating pseudorange distance from propagation time, using satellite coordinates received and with known parameters of satellite constellation. Second satellite can be used for obtaining better precision of time reference and for resolving sign ambiguity in (10). Advantages of using Doppler effect in position determination is that non-GSO communication systems could provide additional positioning service with very small investments. Additional advantages comparing to existing satellite navigation systems based only on pseudorange measurements [4], [5], are that they requires significantly smaller number of satellites above horizon for position determination (two comparing to four), reducing the necessary number of satellites in constellation. In applications where height value is not needed (e.g. maritime navigation), (8) and (10) can be used. In these applications, synchronism of satellites and Earth receiver clocks is not necessary, which additionally reduces complexity of the system. Disadvantages of such system are smaller precision of satellite location determination, and more monitoring time required for position determination.
Doppler frequency shift presented here, with slight modifications depending on implementation can be used in various positioning, search and rescue applications where two objects with large velocity difference exist (e.g. aircraft monitoring, positioning, search and rescue of objects/people on ground).

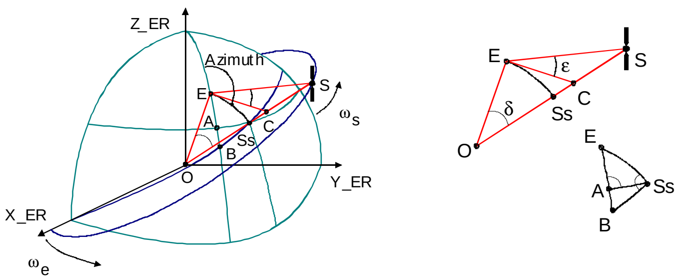

Fig. 5 Geometry of Earth station and satellite

CONCLUSIONS

In this paper effect of Doppler frequency shift in non-GSO satellite communications was analysed. Analytic expressions for satellite trajectories, frequency shift and position determination coordinates were derived. Applications for satellite based positioning system and frequency shift compensation were also presented. Future work includes study of orbital perturbations effects, investigation of influence of various orbital parameters on minimisation (maximisation) of the frequency shift, as well as further analysis of possible implementations of this effect in satellite and aircraft systems.

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