Simple iterative algorithm for image enhancement

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Abstract: - Image enhancement methods can be divided in two groups: the ones that use only one single image and the ones that rely on specific training set or use multiple images. In this paper it is introduced an iterative algorithm based on the quasi-Newton methods, with the objective to enhance resolution only by one single image. In the paper there will be compared results gained depending on the objective histogram: spline histogram and one augmented with Empirical Mode Decomposition.

Key-Words: - Image enhancement, Broyden algorithm, Iterative algorithms, Empirical Mode Decomposition.

1 Introduction

Image-based models for computer graphics lack resolution independence. They cannot be zoomed much beyond the pixel resolution they were sampled at without a degradation of quality. Interpolating images usually results in a blurring of edges and image details. A method to achieve higher resolution views of pixel-based image representations is called super-resolution which aim is to take a set of one or more low-resolution input images of a scene, and estimate a higher-resolution image [1].

Image enhancement methods can be divided in two groups: the ones that use only one single image and the ones that rely on specific training set or use multiple images [3].

The method proposed in this paper belongs to the first group. Up to now, the single image super-resolutions have been achieved in a variety of ways.

Paper [2] focuses on the issue of how to increase the resolution of a single image using only prior information about images in general.

Freeman in [1] follows a supervised approach, learning a low to high resolution patch model (or rather storing examples of such maps), and utilizing a Markov random field for combining them and loopy propagation for inference. Dynamic structure super-resolution [2] provides a technique for resolution enhancement, and provides an interesting starting model which is different from the Markov random field approaches [1], while method proposed in [1] preserve fine details, such as edges, generate believable textures, and can give good results even after zooming multiple octaves, thanks to the overlapping predicted patches at their borders.

Irani-Peleg [3] use the iterative algorithm. The main feature of the Irani and Peleg method is that it iteratively uses the current best guess for the SR image to create LR images and then compare the simulated LR images to the original LR images. These difference images (found by subtracting real LR - simulated LR) are then used to improve the initial guess by "back projecting" each value in the difference image onto the SR image. This results in an improved SR image [3], [4], [5], [6]. In [3] an iterative algorithm to increase image resolution, together with sub pixel accuracy is presented in the paper, and the super resolution algorithm for color images. The theorems in [3] shows that the iterative super resolution scheme is an effective deblurring operator. In [6] an MRI reconstruction using Irani–Peleg super resolution algorithm is shown. An extension of the Irani–Peleg algorithm from 2D to 3D is conceptually straightforward. The paper presents and demonstrates MRI inter–slice reconstruction using super resolution.

According to Pickup at all the aim of super-resolution is in taking a set of one or more low-resolution input images of a scene, and estimate a higher-resolution image [7], [8], [9], [10], [11]. If there are several low resolution images available with sub-pixel displacements, then the high frequency information of the super resolution image can be increased [7]. A novel method for combining super-resolution with image registration and the learning of a Huber edge-preserving image prior has been presented in [11]. Introducing an algorithm to estimate a super-resolution image at the same time as finding the low-resolution image registrations, this simultaneous approach offers visible benefits on results obtained from real data sequences. An

algorithm also incorporates a photometric model to handle brightness changes often present in images captured in a temporal sequence.

In [12] the observed low resolution images are regarded as degraded observations of a real, high-resolution image. These degradations include geometric warping, optical blur, spatial sampling and noise. Given several such low resolution image observations the objective is to determine the super-resolution image from the measured low resolution images given the image formation model. The paper proposes two solutions to this problem. In the first, the determinations of the Maximum Likelihood (ML) estimate of the super-resolution image such that, when reprojected the imaging model, it minimizes the difference between the actual and "predicted" observations. In the second, the determinations of the Maximum a posteriori (MAP) estimate of the super-resolution image including prior information.

The problem of single image super resolution in this paper is treated as nonlinear optimization problem, similar as in Iran-Peleg algorithm. The "Broyden-like" methods for nonlinear optimization are suitable when the large number of unknown parameters have to be estimated, which is the case in super resolution image where the unknown parameters are image pixel values of the sub-pixel level. Some of the Broyden-like methods are very robust to noise which is regularly present in images. Also, such approach threat the system as the whole which differ it from existed approaches that usually treat small patches stitching it together to form super-resolution image.

The rest of paper consists as follows. In Section 2 the applied iterative procedure is presented as well as a typical procedure for image enhancement (EMD) gained depending on the objective histogram, which is the same objective we achieve. The methods implementation are described in Section 3. In Section 4 we have compared the visual plausibility of the described methods, whereas Section 5 concludes the paper.

2 Iterative algorithms

2.1. Broyden algorithm

In this section purely iterative algorithm which was used in uncalibrated visual servoing [13] and which has never been applied to an image enhancement is described.

Let $T : \Re^n \to \Re^n$ be a continuously differentiable function and a fixed point problem is in finding $x \in \Re^n$ such that $T(x) = x$. For the problem of nonlinear systems of equations the solution can be characterized as finding the $x \in \Re^n$ that

$$F(x) = 0 \qquad (1)$$

such that $F : \Re^n \to \Re^n$ is continuously differentiable function.

The most efficient approaches to solve nonlinear systems of equations are based on Newton's idea to replace the nonlinear function by its first order approximation. If the function is too expensive to compute or the first order derivatives are even more expensive or not available, Newton's idea can not be directly used. Therefore Quasi – Newton method consider at each iteration the linear model

$$L_k(x; B_k) = F(x_k) + B_k(x - x_k) \qquad (2)$$

approximating $F(x_k)$ in the neighborhood of x_k and computes x_{k+1} as a solution of the linear system $L_k(x; B_k) = 0$. The solution exists and is unique if B_k is nonsingular. In this case, an iteration of Quasi – Newton method is described by solving

$$B_k p_k = -F(x_k) \qquad (3)$$

and determining

$$x_{k+1} = x_k + p_k \qquad (4)$$

followed by the computation of B_{k+1}. Various Quasi – Newton methods differentiates the way of updating B_k. If $B_{k+1} = J(x_{k+1})$, Newton method is obtained, what implies that B_{k+1} is the Jacobian of F, evaluated at x_{k+1}. Jacobian is a $n \times m$ matrix which entries (i, j) are $\partial F_i / \partial x_j$ [14].

$$B_{k+1} = B(x_{k+1}) = \nabla F(x_{k+1}) \qquad (5)$$

Secant methods avoid computation of derivatives capturing variational information from the following secant equation

$$y_k = B_{k+1}s_k \qquad (6)$$

where $s_k = x_{k+1} - x_k$ and $y_k = F_{k+1} - F_k$. Equation (6) ensures that (3) interpolates $F(x)$ at x_k and x_{k+1}.

Broyden proposed [15] the most successful class of quasi – Newton methods based on the secant equations, imposing the linear model L_{k+1} to exactly match the nonlinear function at iterations x_k and x_{k+1}, that is

$$L_{k+1}(x_k) = F(x_k)$$
$$L_{k+1}(x_{k+1}) = F(x_{k+1})$$
(7)

Subtracting the equations in (7) and taking notice that $s_k = x_{k+1} - x_k$ and $y_k = F_{k+1} - F_k$, the classical secant equation can be obtained (8),

$$B_{k+1}s_k = y_k \qquad (8)$$

which is identical to (6). If the dimension of n is strictly greater than one, there are an infinite number of matrices B_{k+1} satisfying (8). Hence additional criteria must be given to gain one unique solution.

The Broyden algorithm can be described in steps:

1. minimization of $\|B_{k+1} - B_k\|$;

2. using (8) results with "least – change secant update" formula:

$$B_{k+1} = B_k + \frac{(y_k - B_k s_k)}{s_k^T s_k} s_k^T \qquad (9)$$

2.1 Empirical mode decomposition

The EMD of images relies on proper spline interpolation in two dimensions, and the sifting process extended to two dimensions is presented in [16]. To find the first IMF it is necessary to start with the input image itself as the input signal $in_{11}(m,n) = x(m,n)$. The first index is the IMF number, $l = 1...L$, and the second index is the iteration number, $k = 1...K$, in the sifting process. m and n represent two spatial dimensions. To find the next IMF, the residue corresponding to the previously found IMF is then used as input signal $in_{21}(m,n) = r_1(m,n)$. The sifting process to find the IMFs of a signal $x(m,n)$ fallows the next steps:

(a) Find the positions and amplitudes of all local maxima and all local minima in the input signal $in_{lk}(m,n)$.

(b) Create the upper and the lower envelope by spline interpolation of the local maxima and the local minima, respectively. Denote the envelopes $e_{max}(m,n)$ and $e_{min}(m,n)$, respectively.

(c) For each position (m,n) calculate the mean of the upper envelope and the lower envelope:

$$em_{lk}(m,n) = \frac{e_{max}(m,n) + e_{min}(m,n)}{2} \qquad (10)$$

The signal $em_{lk}(m,n)$ is referred to as the envelope mean.

(d) Subtract the envelope mean signal from the input signal:

$$h_{lk}(m,n) = in_{lk}(m,n) - em_{lk}(m,n) \qquad (11)$$

This is one iteration of the sifting process. The next step is to check if the signal $h_{lk}(m,n)$ is the IMF or not. The process stops when the envelope mean signal is close enough to zero:

$$|em_{lk}(m,n)| < \varepsilon, \forall(m,n) \qquad (12)$$

Forcing the envelope mean to zero will give the wanted symmetry of the envelope and the correct relation between the number of zero crossings and the number of extremes that define the IMF.

(e) Check if the mean signal is close enough to zero, based upon the stop criterion. If not, repeat the process from step 1 with the resulting signal from step (d) as the input signal, sufficient number of times.

$$in_{l(k+1)}(m,n) = h_{lk}(m,n) \qquad (13)$$

When the stop criterion is met, $k = K$, the IMF is defined as the last result of (d).

$$c_l(m,n) = h_{lK}(m,n) \qquad (14)$$

After the IMF $c_l(m,n)$ is found, define the residue $r_l(m,n)$ as

$$r_l(m,n) = in_{l1}(m,n) - c_l(m,n) \qquad (15)$$

(f) The next IMF is found by starting over from step 1, now with the residue as the input signal

$$in_{(l+1)1}(m,n) = r_l(m,n) \qquad (16)$$

Steps from (a) to (f) can be repeated for all the subsequent r_j. The EMD is completed when the residue, ideally, does not contain any extrema points. This means that it is either a constant or a monotonic function. The signal can be expressed as the sum of IMFs and the last residue [16]:

$$x(m,n) = r_L(m,n) + \sum_{j=1}^{L} c_j(m,n) \qquad (17)$$

3 Application of iterative algorithms

When image enhancement is based only one image, the target histogram has to be set from the original image as the objective histogram. As the number of pixels has to be increased, the desired histogram has to be augmented. The iterative algorithm proposed in this paper is first applied on objective histogram augmented only by cubic spline interpolation, and afterwards on the objective histogram augmented by Empirical Mode Decomposition and cubic spline interpolation.

3.1 Broyden algorithm applied on image

After performing the cubic spline data interpolation on the original image histogram (HVORG) the signal has to be filtered with zero – phase forward and reverse digital filtering (HV). In this way the desired image histogram is reached. From the original image m, sized $(n \times n)$, it is essential to create a smaller one, $(n-1 \times n-1)$ and preserve the changes of the pixels in a matrix of changed pixels (s_k). On the histogram of the smaller image (HM) cubic spline interpolation and filtration has to be performed as well. The difference of the desired and the small histogram y_k is then reached:

$$y_k = HV - HM \qquad (18)$$

where the minimum value of the difference y_k should not be smaller than zero, what is obvious from the equation (18). If $y_k < 0$ that would mean that the smaller image histogram has more pixels of certain value than the original image histogram, what would be unreasonable. The initial guess of the Jacobian J_0 is obtained from the matrix of changed pixels s_k, what is the difference between the pixels of the original and the smaller image. Furthermore the Jacobian is replaced by its model J_k (19), which is continuously adapting during the iterations, and the matrix of changed pixels, s_k changes as well.

$$J_{k+1} = J_k + \frac{y_k - J_0 * s_k}{s_k^T * s_k} * s_k^T \qquad (19)$$

From the equations:

$$\Delta f = HV - HVORG$$
$$\Delta s = -inv(J_{k+1}) * \Delta f \qquad (20)$$

It is possible to update a new pixel value:

$$M(N,N) = m(n,n) + \Delta s_i \qquad (21)$$

Before updating new pixel value, it is preferable to multiply the change in pixels Δs with the scaled percentage of the wished maximum change (ZMP). If the ZMP is close to one, the change will be maximal. The smaller the value ZMP the smaller the change will be. The histogram of the new (bigger) image becomes new HV. The iterations stop when the error value comes close to threshold, which must be set close to zero. The error function is given by (22):

$$e = \sum_{i=k}^{k+1} \Delta f_i \qquad (22)$$

3.2 EMD applied on image

The EMD has to be performed first on the original input image. The first IMF extracts the locally highest spatial empiquencies in the image, while the second IMF holds the locally next highest spatial empiquencies, etc. The EMD in two dimensions provides a tool for image processing by its special ability to locally separate spatial frequencies that build texture [16]. The EMD sorts the spatial frequency components into a set of IMFs where the highest spatial frequency component of each spatial position is in the first IMF and the next highest spatial frequency component of each spatial position is in the second IMF, etc. The stop criterion is based on the condition that the IMF envelope mean must be close enough to zero. In our tests we set the stop criterion on the value of $\varepsilon = 0.08$. Like in [17], we also decomposed an input image into low and high frequency information by 2D EMD. Then expand the high frequency part, multiplying it with factor k, which must be set in between $1 < k < 3$. For k too big the highest frequency, which is the border, augments too much hence the borders become too ridge. Also, for k smaller than one the loss of borders is eminent. For $k = 1$ nothing changes. The histogram of the image stays the same. Linderhed [16] proposed a trick to solve the border problem by adding extra data points at the borders to the set of extrema points. The only way we solved the border problem was with the correct choosing of factor k.

4 Results

Application of the method was performed in MatLab. Input image sized 16×16 was cut out from Lena's original image and turned to grey using traditional grey transformation method. The image can be seen on figure 1 with the proper original image histogram.
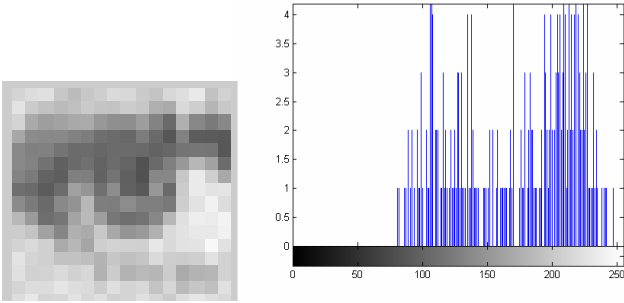

Fig.1. The original input image, *Lena's eye* (16x16), with proper histogram.

After applying the Broyden algorithm, the difference is obvious, figure 2. The number of pixels is doubled in a congruous way, what is obvious from the image histogram.
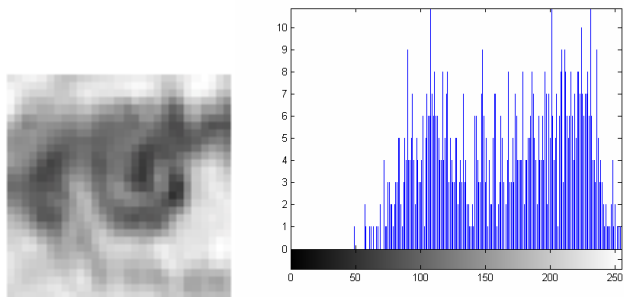

Fig.2. The enhanced original image with her histogram (30x30). The objective histogram is augmented by cubic spline interpolation.

The results of applying objective histogram augmented with EMD method is presented on figure 3.
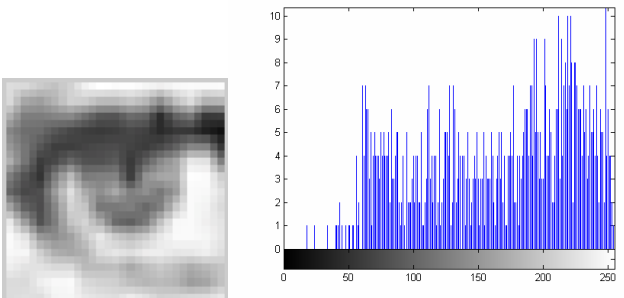

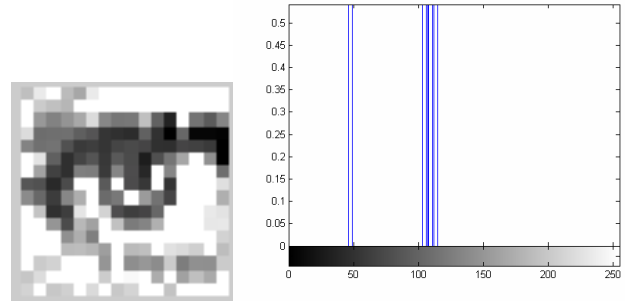Fig.3. The enhanced original image with her histogram (30x30). The objective histogram is augmented by EMD.

Fig.4. The original input image (16x16) after performing only EMD without augmentation by Broyden algorithm.

From the results obtained from our algorithm we can say with certainty that Broyden iterative method augments the number of pixels in congruous way. The result of the method depends mostly on the objective histogram we wish to attain. It is known from the previous research [1] that cubic spline interpolation suffers from blurring of edges and image details, though is a very common image interpolation function. We also tried to solve the border problem applying EMD like in [17], by multiplying the highest empiquencies with factor *k*. On figure 4 can be seen the result attained after applying EMD on original image sized (16x16), (the original image is shown in figure 1), without performing Broyden algorithm. The EMD can increase the differences between the pixels and in that way ameliorate borders, but it can not enhance the image quality without Broyden iterative algorithm. Combination of the two methods, Broyden algorithm and EMD, gives the result, but not in perfect way. The resulting picture is altered from the original, what is the result of the sifting process.

5 Conclusion

Image enhancement method proposed in this paper relies on histogram manipulation. As shown from the multiple image analysis, manipulation of the image histogram results in image manipulation. Hence, if it is possible to manipulate an image in any way, it is possible to manipulate it in a 'making it bigger way'. The main question is how to make augmentation of the number of pixels in congruous way? Our experiments showed that the Broyden iterative method has potential. It does not even call for much iteration. The Jacobian matrix J_{k+1}, which upgrades in every iteration, draws us fast enough to the objective histogram. As the result of the method depends mostly on the objective histogram we wish to attain, the future work will, hence, be focused in finding better interpolation method for gaining the objective histogram.

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