Information Content of Process Signals in Quality Control

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Abstract—***Role of quality control in industry, software engineering and government. Control schemes: measurement and actuator signals. Process time constants: material balance control, production control, quality control. The concept of quality assurance and quality culture. Control system as a simple teleonomic, purposeful, system. Kolmogorov definition of teleonomic entropy. Realization restrictions in Kolmogorov entropy calculation. Shannon entropy as system uncertainty measure. Carnap entropy measure by calculating Voronoi diagram of respective signal. Introducing one-dimensional Carnap entropy measure of quantitative signals. Pragmatic question in calculating Shannon entropy: lack of data in some data classes. Problems of Carnap 2D entropy: multiple data points, split-down of signal space, data resolution and calculation duration, interpretation of obtained results. Pragmatic questions in calculating one-dimensional Carnap entropy of control signals: equivalency of entropy measures for slightly different signal shapes, cases of short decision intervals. Delta modulated 1D Carnap entropy. Calculation of measurement and actuator signal entropies of a two-stage laboratory heat exchanger. Entropy contents of quality control signals in highly automated ceramic tile plant.***

Index Terms—***one-dimensional Carnap entropy, process control, signal entropy, tile production quality***

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1. Introduction

Quality of products is of primary interest to industrial producers. Basic quality control actions in plants nowadays take place in well-defined structure known as quality evaluation system [1]. In order to ensure adequate decisions in functioning of quality control system, the most appropriate signal processing with respect to resulting quality should be installed.

There are two main obstacles in this system thinking: nature of the signal and signal-to-quality relation. According to the signal nature there are quantitative or qualitative (mark) signals, and relating signal-to-quality attributes there exist more direct or more remote connections between signal and searched quality mark of the product. The development of quality inspection must take all these aspects into account.

The development of specific quality control schemes in ceramic tiles production includes overall inspection of basic process parameters such as: grinding materials for ceramic body, milling, spray-drying, pressing-drying, glazing and decoration, firing, as well as quality control of suppliers. Some automatic devices have been developed for final inspection of ceramic tiles, but the final quality decision is still on the side of skilled workers [2, 3].

Nevertheless, nowadays ceramic tiles plants are highly automated and robotized processes. The more automated the process the more the final quality depends on quality of respective work power. Thus there exists an overall quality control scheme as depicted in Fig. 1 [4]. It can be observed from Fig. 1 that there is an opposite flow of material and control signals. While the flow of material is to the right side of the scheme, the quality corrections are towards the left side of the scheme. The role of a worker is to interact as soon as possible with the production process, changing actual control signals and implemented actions as indicated by the measurement quality signal from the plant.

2. Signals and Control Schemes in Ceramic Tile Quality Control

Classical quality control scheme for blending can be applied to, spray-drying, glazing and decoration, and glazing preparing, in ceramic tile production. Quality process on press depends on input material and many press parameters:

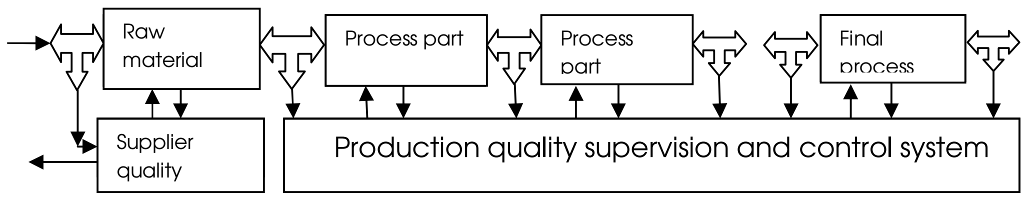


Fig. 1. Overall quality control scheme of the production plant. Measurement and control signals are indicated with arrows from and to the processing parts.

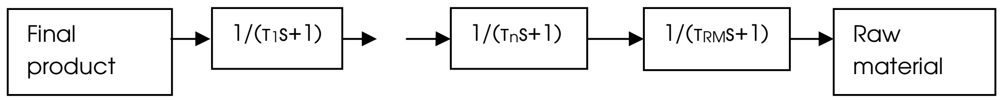


Fig. 2. Signal flow diagram for Fig. 1.

temperature of press tools, dimension of press tiles, power pressure, etc.

Quality of firing process depends on complex interaction of temperature profiles and materials for glazing and decoration. General strategy of process quality control demands as many controls as possible before material processing takes part. With blending the control has to be performed after each processing stage, thus pointing to the signal flow diagram as presented in Fig. 2.

Signal processing time constants $\tau_1,\cdots,\tau_{RM}$ are specific for each quality control part. Integrating control signal flow with time constant τ_s after each quality control step that possesses signal process time constant τ_p enables better quality than in the case of only each consecutive quality control step with approximated improvement factor B equal to $B = {\tau_s}/{\tau_p}$ [5]. Time constant of the quality control signal τ_p should be set to at least five to ten times as large as the largest time constant in the dynamics of the measurement and control equipment [5]. Time constant of the quality control signal τ_p should be set to at least five to ten times as large as the largest time constant in the dynamics of the measurement and control equipment [5]. Thus integration quality scheme as for instance given in Fig. 3 used for spray-drying, glazing and decoration, firing, should be preferred.

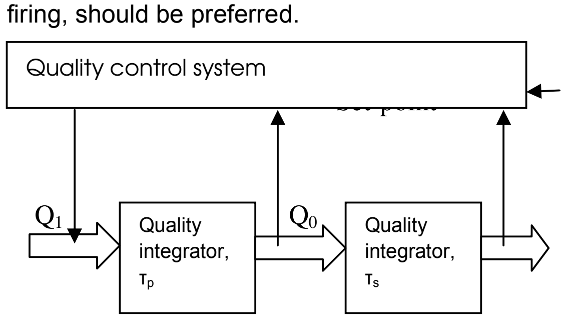


Fig.3. A two-step quality control system

3. Information Measure in the Quality Control System

Quality control system of the ceramic tile plant produces signals that are of quantitative, measurable form, stemming from various plant process parts. They are mostly manually collected by QC crew on the daily bases. The information content of such a measurement points to the actual state of the product quality, because any deviation from the required quality can result in production losses.

Hence, not only overall signal information content but also its required value is of importance to process control. Unfortunately, Shannon's information measure cannot cover such a case because of its insensitivity to signal shape and also to the lack of information measure for data missing cases of the entropy distribution classes [6]. Therefore, Carnap entropy concept [7] has been proposed.

3.1 Signal Information Measure of Basic 1D Carnap-Type Distributions

As a zero-type of Kolmogorov entropy [8] there exist several different topological entropies. Rudolf Carnap (1956) introduced an *n*-dimensional system space using its *n* system variables within their theoretic limits R^{μ}, *μ* being at maximum equal to *n* [7]. Each system variable u_i is given within its minimum and maximum values of its space Φ_i.

A two, three, or more – dimensional spaces are produced with respective Voronoi diagrams [7]. The relevant occupation space e_j is defined for each measurement point $b_j\,(u_{i1},u_{i2})$ according to minimum distance criterion from neighboring points. Because of the possibility to relate the relative ratio of each occupation space and theoretic limit, the dual logarithm of the relation is named Carnap entropy:

$$I_C = -\sum_j \frac{e_j}{R^{\mu}}\, ld\, \frac{e_j}{R^{\mu}} \qquad \text{(bit/volume)} \quad (1).$$

The time-space trajectory of the system produces a dynamic Voronoi diagram that can be traced and compared to the desired, teleonomic system trajectory. Thus Carnap entropy can measure system entropy of dynamic teleonomic systems.

A problem not specified in the Carnap concept is the case of equal measurement points and the case of extreme reduction of the dimensions to only one. One-dimensionality of the Carnap entropy is forced whenever overall plant data are examined whose mutual difference values do not indicate any system dynamics or just do not carry explainable information content. According to the above premises the following theorems can be set:

Theorem [6]: Carnap-based entropy is invariant to event occurrence instant.

Proof: The change of element ordering in the relation (1) does not produce any change in the sum value.

Corrolary: One-dimensional Carnap entropy as stated in relation (1) is applicable also to time-independent data series.

Periodic scanning *N* times of a constant process value *L* should yield a negligible one-dimensional Carnap entropy H because the space that occupies first measurement is total space *L* with $H = \frac{L}{L} ld \frac{L}{L} = 0$ and all the other data occupy a negligible space with $H = \frac{0}{L} ld \frac{0}{L} = 0$. Space of equal measurement values should not be subdivided among equal measurement points. It should be taken as a unique, albeit small space e_j and its content multiplied by the number of equal measurements. This statement is valid for Carnap entropy of any dimension.

1D uniformly distributed Carnap entropy with *N* events in *k* classes of the total measuring span *L*, of the width Δ_i and starting from *x* to $x + 2\Delta y$, $\Delta_i = \frac{2\Delta y}{k}$, with $\frac{N}{k} = \lambda$, possesses entropy equal to [6]

$$C_U = -\lambda\left(\frac{x}{L} ld \frac{x}{L} + (k-1)\frac{\Delta_i}{L} ld \frac{\Delta_i}{L}\right),$$

(bit/measurement unit) (2).

Usually the number of classes *k* is approximated with $= \sqrt{N}$, and relation (2) depends dominantly on the value x.

Gaussian distributed 1D Carnap entropy involves measured values distributed into classes with different number of events with values distributed according to given parameters (μ, σ^2) of mean value and variance. With *Δ* as interval between measurements, *N* as total number of measurements, total occurrence space *L* is given by $L = \max_{\mu,\sigma}(N)$. Event occurrence classes are usually of the width $c \cong \frac{\sigma}{2}$. Left flank of the distribution can be calculated from the binominal distribution as [9]

$$f_{ri} = N\, p_i = 1 \qquad (4),$$

where f_{ri} is the expectation of one event in the minimum value class. With N as known value, the correspondent deviation from mean value for unity normal distribution can be calculated from Gaussian unity distribution as $\varphi\left(\frac{1}{N}\right)$.

For $N = 100$ the distribution is between $(-3{,}54\sigma \text{ and } 3{,}54\sigma)$ and data can be subdivided into 14 classes. Thus one-dimensional Carnap entropy of the Gaussian measurement starts from $\mu - \varphi\left(\frac{1}{N}\right)$. The number of classes is $k = \left|\frac{2c\varphi\left(\frac{1}{N}\right)}{\sigma}\right|$. Therefore total 1D Carnap entropy of the Gaussian distribution is

$$C_G = -\frac{f_{ri}\left(L - 2\varphi\left(\frac{1}{N}\right)\right)}{L} ld \frac{f_{ri}\left(L - 2\varphi\left(\frac{1}{N}\right)\right)}{L} - \sum_{i=2}^{k} \frac{f_{ri}c}{L} ld \frac{f_{ri}c}{L}$$

+err, (bit/measurement range) (5).

The error term has to be added because of volatility of distribution that can change upper and lower flank of the actual distribution. The inclusion of actual data for f_{ri} and with $f_{r1} = 1$ gives

$$C_G = -\frac{L - 2\varphi\left(\frac{1}{N}\right)}{L} ld \frac{L - 2\varphi\left(\frac{1}{N}\right)}{L} - \sum_{i=2}^{k} \frac{Nc^2\varphi(u_i)}{\sigma L} ld \frac{Nc^2\varphi(u_i)}{\sigma L}$$

+err, (bit/measurement range) (6).

3.2 *Information content of the Δ-modulated process signal*

Generally any measurement signal can be presented as a *Δ*-modulated signal, which is a signal composed from a series of unit step changes. This signal approximation can be used as a basic approximation of the quality control signal changes in the plant.

Let us suppose that quality control signal is of value *L* in the first observation cycle. If it remains constant then its Carnap 1D information content is zero. If it changes for a positive unit step *Δ* the information content is $I^+ = I^{L+\Delta}(L + \Delta/2) + I^{L+\Delta}(\Delta/2)$, and for a negative unit step *Δ* the information content is $I^- = I^L(L - \Delta/2) + I^L(\Delta/2)$. Thus for two consecutive unit steps the information content will be

$$I^{++} = I^{L+2\Delta}(L + \Delta/2) + I^{L+2\Delta}(\Delta) + I^{L+2\Delta}(\Delta/2)$$

or $\quad I^{+-} = I^{L+\Delta}(L + \Delta/2) + 2I^{L+\Delta}(\Delta/2)$

or $\quad I^{-+} = I^L(L - \Delta/2) + I^L(\Delta/2)$

or $\quad I^{--} = I^L(L - \Delta - \Delta/2) + I^L(\Delta) + I^L(\Delta/2)$, (7),

depending on the signal step sign sequence. Actual information calculation is performed using the formula

$$I^x(y) = \frac{y}{x} ld \frac{y}{x}, \text{ (bit/x)} \qquad (8).$$

1D Carnap information measure of the Δ-modulated signal after three steps still possesses difference in the signal shape that is

$$I^{+++} \neq I^{++-} \neq I^{+-+} \neq I^{-++} \neq I^{--+} \neq I^{-+-} \neq I^{+--} \neq I^{---} \quad (9).$$

Nevertheless signal information measure after four steps is equal for two different cases

$$Case\ 1: I^{++--} = I^{+-++}, and\ Case\ 2: I^{-+--} = I^{--++} \quad (10),$$

as indicated by the signal spectra from which 1D Carnap entropy is calculated and which are given in Fig. 4 for both cases in expressions (10). Because of obvious entropy identity for two different event sequence there exists a problem of signal teleonomic measure. Generally the expansion of a signal results in entropy decrease and the contraction of a signal results in entropy increase. Thus for $L = 10$, $\Delta = 1$, $I^{++++} = 1{,}3024\ bit$ and $I^{----} = 1{,}617\ bit$.

4. Information content of the fully-expanded Δ-modulation signal

The other approach to the problem of signal teleonomic measure is to perform a full expansion of the signal into a Δ-matrix, such as for a given signal data $(L_1, L_2, L_3, \ldots, L_k)$ the triangle matrix of data differences can be calculated among all set members, i.e. [10]

$$\begin{bmatrix} \Delta L_{1,2} & \cdots & \Delta L_{k-1,k} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Delta L_{1,k} \end{bmatrix} \quad (11).$$

Such matrices for Case 1 data from Fig. 4 exhibit the following results:

$$I^{M(++--)} = \begin{bmatrix} \Delta & \Delta & -\Delta & -\Delta \\ - & \Delta & 0 & -\Delta \\ - & - & \Delta & -\Delta \\ - & - & - & 0 \end{bmatrix}$$

$$I^{M(+-++)} = \begin{bmatrix} \Delta & -\Delta & \Delta & \Delta \\ - & 0 & 0 & \Delta \\ - & - & \Delta & \Delta \\ - & - & - & \Delta \end{bmatrix} \quad (12).$$

Taking each subsequent row of the matrix (12) as a new extension of the 1D Carnap information signal the extended graphs of Fig 4 can be obtained as given in Fig. 5.

5. Calculations of Shannon, 2D Carnap and 1D unexpanded Carnap entropy from the two-stage quality control model

A laboratory two-stage control system has been used as information source that models the two stage quality control process. It consists of heating and mixing unit that feed a larger container with warm water. Its conceptual structure is similar to the scheme in Fig. 3 except for two control signals that can improve final quality requirement of the unit. Two control and two measurement signals were used in the sequence of unit step temperature change of the container.

The total time span was 5000 seconds. Four 30-sample equidistant measurement data have been taken for the calculation of respective entropies. A data sample of the starting sample is given in Table 1. Obtained entropies of the Shannon, 2D Carnap and 1D unexpanded Carnap type are given in Table 2.

Entropy data on quality control signals in the two-stage kiln from the KIO Keramika d.o.o. plant in Orahovica are given in Table 3 [6].

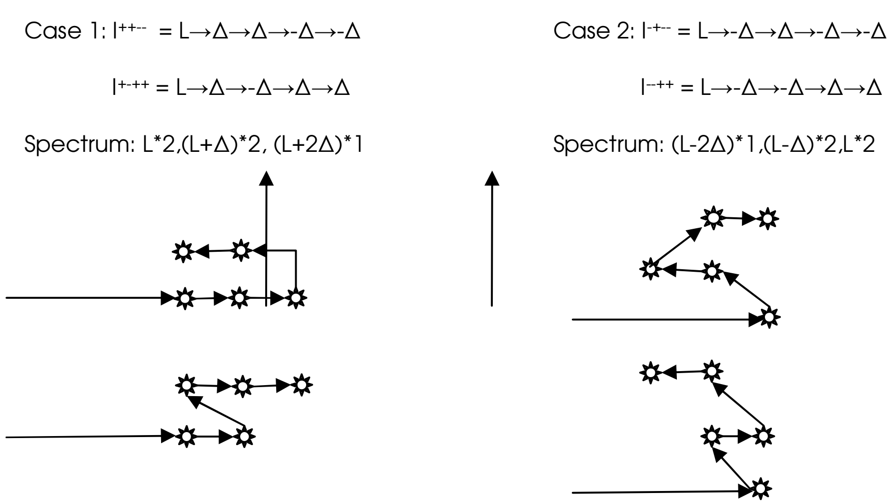
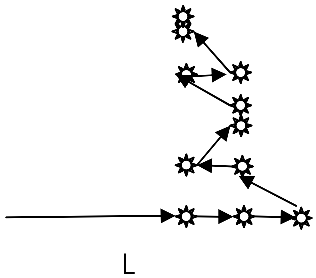


Fig. 4. Identical signal spectra for Case 1 and Case 2 from equation (10) in spite of different signal event flow

$I^{M(++--)}$:

L→Δ→Δ→-Δ→-Δ→Δ→0→-Δ→Δ→-Δ→0

Spectrum: L*5,(L+Δ)*5, (L+2Δ)*1.



$I^{M(+-++)}$:

L→Δ→-Δ→Δ→Δ→0→0→Δ→Δ→Δ→Δ

Spectrum: (L)*2,(L+Δ)*2,(L+2Δ)*3,

(L+3Δ)*1,(L+4Δ)*1,(L+5Δ)*1,(L+6Δ)*1.

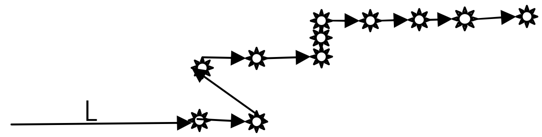


Fig. 5 Carnap 1D entropy of the full expanded *Δ*-modulated process signals from Case 1 as an example of signal teleonomic solution: there are visible essential signal path and entropy content differences

Table 1. Data set for the entropy analyses of the laboratory step quality change – initial observation after the set-point step change, uc - pump control signal, ug - heater control signal, umd – tank water temperature signal, umg – heater water temperature signal

t	uc	ug	umd	umg
0	0,00000	0,00000	19,50000	22,00000
4	0,40001	-0,00097	19,50000	21,99919
8	0,38208	-0,01397	19,50000	21,99616
12	0,44110	9,74745	19,50000	22,12211
16	0,45618	9,92590	19,50000	23,36624
20	0,46787	9,97358	19,50000	25,33188
24	0,44961	9,96402	19,50001	27,60515
28	0,46314	9,94945	19,50001	30,00832
32	0,47041	9,96284	19,50002	32,46636
36	0,46894	9,95965	19,50003	34,94855
40	0,47304	9,96373	19,50005	37,44115
44	0,47051	9,97195	19,50006	39,93704
48	0,46237	9,94937	19,50008	42,43520
52	0,47628	9,96171	19,50011	44,93371
56	0,46576	9,97690	19,50013	47,43147
60	0,48826	9,97856	19,50017	49,92741
64	0,48249	9,95944	19,50022	52,41624
68	0,45541	9,95758	19,50030	54,90607
72	0,46546	9,96551	19,50040	57,40015
76	0,45334	9,91252	19,50051	59,89777
80	0,45653	7,69572	19,50062	62,33556
84	0,45932	5,59170	19,50073	64,46369
88	0,45308	3,88978	19,50086	66,15638
92	0,44594	2,58772	19,50101	67,42535
96	0,44477	1,65228	19,50117	68,33156
100	0,45833	1,03354	19,50134	68,95203
104	0,45217	0,60127	19,50151	69,36033
108	0,48704	0,35816	19,50168	69,61718
112	0,43826	0,15675	19,50186	69,76960
116	0,44238	0,06975	19,50205	69,85784

6. Discussion and Conclusion

Quality control in all human activity requires correct data about the process. Although not easily acquired, data can be collected from process industrial production, software production facility or government office. They significantly differ in volume, content, and impact. The purposefulness of the data is relevant for system owner and user. Thus only teleonomic quality data can be finally regarded as completely socially useful.

Theoretical consideration of information content of signal data is covered by Kolmogorov concept of entropy. Information system theory as proposed by Vladimir S. Lerner [11] is conceived to build a bridge between the mathematical systemic formalism and information technologies dealing with transformation of information. The aim of such a procedure is to obtain system models that reveal information laws and specific object codes and patterns. General evaluation of informational aspect of stochastic data is enabled by application of Kolmogorov entropy that measures the extent of chaos in a system. According to Shannon's interpretation, system uncertainty is closely related to system information content we possess [12].

When applying Shannon’s concept, two distinctive data manipulation problems appear: it cannot cover for deterministic data because of lacking data classes for entropy calculation as indicated in Tables 2 and 3, and it can operate strictly only on qualitatively enumerated data sets such as letters or signs. Quantitative data do not automatically apply for Shannon entropy calculation as indicated in Tables 2 and 3. Both conceptual features are inappropriate for quality control in industrial signal environment that is based on quantitative signals from measurement transducers and that possess a given stochastic component. In addition, Shannon’s concept does not consider any system teleonomy at all.

Application of 2D Carnap entropy is favorable for quantitative data but it sometimes takes extreme calculation time for its Voronoi diagram calculations [6], as indicated in Table 2 as well.

Table 2. Shannon, 2D Carnap and 1D Carnap entropies calculated for four data series: 1-120, 120-240, 240-480 and 480-1120 seconds after step change of the set-point in the laboratory two-stage control system

Data Series	Entropy type	Pump control signal entropy, bit/unit	Heater control signal entropy, bit/unit	Tank water temperature entropy, bit/unit	Heater water temperature entropy, bit/unit
1	Shannon	None*	15,1826	18,2839	12,4693
	1D Carnap	2,2654	3,1688	4,5909	4,5257
	2D Carnap	2,2830	3,0209	4,1475	4,477
2	Shannon	None	14,6696	13,1876	13,0684
	1D Carnap	2,7049	1,5239	2,9164	2,9879
	2D Carnap	2,2896	Not calculable**	2,6843	Not calculable
3	Shannon	9,6729	None	5,0361	9,9045
	1D Carnap	4,6459	3,0487	4,8879	4,1634
	2D Carnap	4,5099	2,5943	4,8352	4,1209
4	Shannon	13,0747	7,7612	6,3382	9,9122
	1D Carnap	4,4152	4,5034	4,6746	3,5847
	2D Carnap	4,0905	4,1877	4,6299	3,8030

* - some data classes missing, ** - calculation on a quad-core Pentium IV machine greater than 4 hours

Table 3: Entropy calculations of 90-day temperature control profiles in the two-channel kiln in KIO KERAMIKA tile factory, Orahovica, Croatia

Control signal name	Temperature lower bound, ˚C	Temperature upper bound ,˚C	Shannon entropy, bit	1-D Carnap entropy, bit/unit	2-D Carnap entropy, bit/unit
Signal 44	1106	1166	None*	17,0464	5,82848
Signal 45	1119	1151	2,89715	20,4124	6,96516
Signal 46	1106	1166	None	16,8702	5,44023
Signal 47	1106	1166	2,9183	19,6232	6,66346
Signal 48	1107	1169	None	15,7453	5,5408
Signal 49	1114	1150	2,74091	18,5061	5,98589
Signal 50	1107	1170	None	16,0646	5,584
Signal 51	1114	1150	2,7879	18,7413	6,5268

* one or more empty data classes

It is also unresolved for multiple data points, in which case a reasonable improvement has been proposed in the form of multiple entropy impact [6]. Its teleonomy is obtainable only for final data point arrangement in the given Voronoi diagram, but the in-between step order teleonomy is completely disregarded.

1D Carnap entropy as proposed in [6] is applicable for quantitative data. It possesses short calculation time but its teleonomy is like its 2D counterpart and only obtainable for final data point arrangement.

A modified 1D Carnap entropy, named *Δ*-modulated 1D Carnap entropy or *Δ*-Carnap entropy, is analysed in this paper as well and proposed for further investigation because of the teleonomy feature improvement over the other Carnap entropies. This entropy installs data differences among all signal data. Thus it promotes also the introduction of qualitative data into entropy calculation. Obtained information content as expressed in Table 2, that is very similar for different quality control sequences, requires such an approach.

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