Phase transitions in the asymmetric exclusion process with long-range hopping

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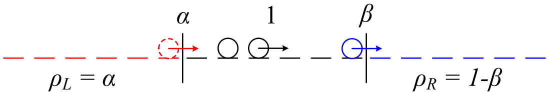
Motivation

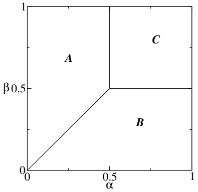
- scope: to study phase transitions in simple systems far from the equilibrium
- critical phase transitions in equilibrium systems
  - universal feature: collective behavior
    (divergence of the correlation length)
  - universality: some properties independent of the details of the system
  - only few key ingredients: symmetries of the Hamiltonian, spatial dimension, range of the interactions
- systems maintained far from the equilibrium
  - violation of detailed balance $\Rightarrow$ dynamics play an important role
  - nonequilibrium steady state: not the Gibbs one
  - generic feature: long-range correlations
  - phase transitions: key ingredients?

Example: asymmetric exclusion process

- exactly solvable 1d model of driven diffusion with hard-core interactions [MacDonald et al. (1968), Spitzer (1970), Derrida et al. (1992, 1993)]
- nonequilibrium state maintained by contact with two reservoirs at different densities



- presence of the net current ⇒ boundary conditions important
- phase transitions: sudden change in the particle density as one varies ρ_L and ρ_R



Phase diagram

- characteristic length ξ: determines the deviation $\phi_n \equiv \langle\tau_n\rangle - \rho$ of density profile from its bulk value near the boundaries

- phase diagram
  - low-density phase (A)

    $$\langle\tau_n\rangle \approx \rho_L, \qquad \phi_n \sim e^{-n/\xi}$$

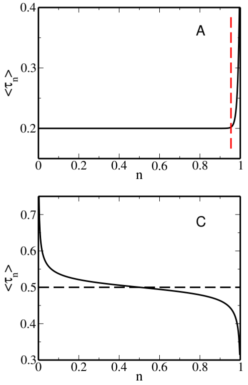
  - high-density phase (B)

    $$\langle\tau_n\rangle \approx \rho_R, \qquad \phi_n \sim e^{-n/\xi}$$

  - maximum-current phase (C)

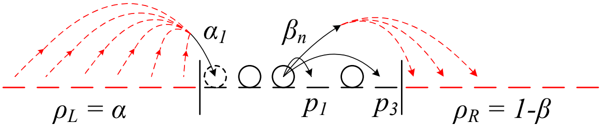
    $$\langle\tau_n\rangle \approx \frac{1}{2}, \qquad \phi_n \sim n^{-1/2}$$

exponent $1/2$ robust to various modifications



Generalization to long-range hopping

- particles may jump any distance r with the probability that decays as $p_r \propto r^{-(1+\sigma)}$ [Szavits-Nossan and Uzelac (2006)]



- "nonlocal boundary conditions": exchange of particles with reservoirs takes place at each site
- dynamics:

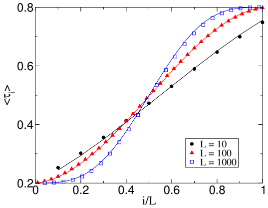
$$\text{hopping} \qquad 1_n 0_{n+r} \underset{p_r}{\longrightarrow} 0_n 1_{n+r}$$

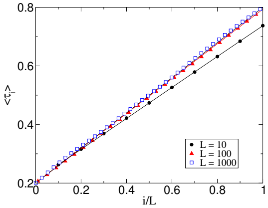
$$\text{exchange with left reservoir} \qquad 0_n \underset{\alpha_n}{\longrightarrow} 1_n$$

$$\text{exchange with right reservoir} \qquad 1_n \underset{\beta_n}{\longrightarrow} 0_n$$

Phase diagram

- main result: for $\sigma > 1$ phase diagram remains the same, different effects at the transition lines
- first-order phase transition [Szavits-Nossan and Uzelac (2006)]
  - phase separation for $1 < \sigma < 2$
  - short-range limit for $\sigma > 2$





- continuous phase transition
  - σ-dependent exponent for $1 < \sigma < 2$ of the mean-field type
  - short-range exponent for $\sigma > 2$

Hydrodynamic approach

- many-body problem: lattice equation for $\langle \tau_n \rangle$ includes unknown correlation function $\langle \tau_n \tau_m \rangle$
- mean-field approach: $\langle \tau_n \tau_m \rangle \approx \langle \tau_n \rangle \langle \tau_m \rangle$
- continuous limit: coarse-graining of density $\phi_n(t) \to \phi(x,t)$ in the limit when the lattice spacing $a = 1/L \to 0$
- short-range case: $\phi(x,t)$ satisfies Burgers' equation

$$\frac{\partial \phi}{\partial t} = -\kappa \phi \frac{\partial \phi}{\partial x} + a \cdot \nu \Delta \phi \qquad \text{solution:} \quad \phi(x) \propto \frac{a}{x}$$

- long-range case on infinite lattice [Szavits-Nossan and Uzelac (2008)]:

$$\frac{\partial \phi}{\partial t} = -\kappa \phi \frac{\partial \phi}{\partial x} + \begin{cases} a^{\sigma-1} \cdot \nu_\sigma \Delta_\sigma \phi & 1 < \sigma < 2 \\ a \cdot \nu_2 \Delta \phi & \sigma > 2 \end{cases}$$

fractional Laplacian: $\mathcal{F}\{\Delta_\sigma \phi(x)\} = -|k|^\sigma \hat{\phi}(k).$

Results: steady state

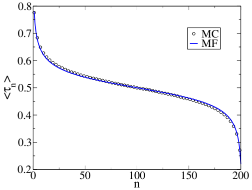
- boundary conditions? take asymptotic ansatz:

$$\phi(x) = \begin{cases} \text{const.} & x < 0 \\ \frac{a^\mu}{x^\mu} & x \gg 1 \end{cases}$$

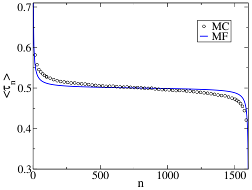
- funny thing about fractional derivatives

$$\Delta_\sigma \phi(x) \sim \underbrace{\phi(0) x^{-\sigma}}_{\text{left boundary}} + O(x^{-\sigma-\mu})$$

- correct exponent for $1 < \sigma < 2$, fails for $\sigma > 2$ (short-range limit)



$1 < \sigma < 2$



$\sigma > 2$

Results: relaxation to steady state

- mapping to fractional Kardar-Parisi-Zhang equation
- dynamical exponent in agreement with the dynamical exponent $z = \{\sigma, 3/2\}$ of the fractional KPZ equation [Katzav, 2003]