Heuristic Parameter Tuning Procedures for a Virtual Potential Based AUV Trajectory Planner ⋆

Matko Barisic * **Nikola Miskovic** * **Zoran Vukic** *

* *University of Zagreb,*
Faculty of Electrical Engineering and Computing, Unska 3,
HR-10000 Zagreb, Croatia
(e-mail: matko.barisic@fer.hr, nikola.miskovic@fer.hr,
zoran.vukic@fer.hr).

Abstract: This paper deals with the study and analysis of heuristics and tradeoffs incipient in the choice of method-independent parameters in an algebraic trajectory planner based on virtual potentials. The real-time trajectory planning framework for autonomous underwater vehicles (AUVs) is described in detail in Barisic et al. (2007), Barisic et al. (2008) and based on the referenced, as well as previous work of the authors. Of special interest among the parameters are those governing the strength of repulsion of obstacles (Barisic et al. (2007), and that of the strength of the rotary field (Barisic et al. (2008), Healey (2006)) that is introduced in order to eliminate obstacle-attached local minima present in classical virtual potential based methods (Healey (2006)).

Keywords: Unmanned systems, Intelligent control systems, Decentralised control

INTRODUCTION

Algebraic planners and algebra-based mathematical frameworks are prolific in contemporary path-planning and trajectory-planning solutions to AUV navigation problems (Healey et al. (2007), Sepulchre et al. (2005), Kalantar, S. and Zimmer, U. (2007)). This is due to the fact that modern embedded computing systems, used to run the suite of necessary control algorithms to steer AUVs are very efficient at performing algebraic operations. Also, algebraic operations are present at almost any level of abstraction when programming control systems – all the way down to processor assembler mnemonics.

In contrast to the methods used to evaluate inputs and arrive at commands, the input data of algebraical methods are generally at a high level of abstraction. This represents a natural counter-tendency to the control system engineer's choice of an algebraical method as the basis for trajectory planning. However, the strengths of algebraical methods must not be disregarded – high levels of abstraction that makes them easily "readable" by human code designers and system engineers, cross-layer design, great levels of leverage that existing coded capabilities represent for further modules exhibiting even more complex and desirable behavior etc. In order to make a certain algebraic system implementable in a physical AUV system, the demands on high levels of pre-processing or operator deliberation on the nature, range and values of input data need to be addressed.

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In the broadest sense the inputs of algebraical real time trajectory planning systems can be divided into:

(1) Method-external data: Data dependent on the current real time situation of the AUV system: relative position and orientation of obstacles, AUV's own pose and location etc.
(2) Method-independent data: This data arises in the form of general numbers, like factors or additive terms, or alternatively as a choice of a constrained class of functions (upper and lower bounds vs. insensitivity range, vs. a three-level relay etc.).

There is a large volume of ongoing research dedicated to robust, non-intermittent, efficient, fast and memory-conservative ways to deal with the abstraction or extraction of perceptive and proprioceptive sensory data, i.e. the former. Control engineer's familiarity, leverage that a certain recipe's strengths bring to the overall system design of the whole AUV, and whose weaknesses are in turn leveraged against by other components and capabilities of the chosen systemic design. Some of the candidates for this "recipe" are: Kalman filtering techniques, sequential Monte Carlo methods, Markovian systems, various voting-based systems, Bayesian systems, neural networks, fuzzy logic inference systems, etc.

The latter type of inputs are generally much more difficult to ascertain with any level of consistency. Engineering constraints of the system, including sensor speed, bandwidth, accuracy, actuator dynamic range, energy balance, absolute limits of delivered strength, and stability lead to the formulation of necessary *ranges of values* for these data. However, the procedures to consistently link the values of these data to certain "hard" specifications on the performance of the AUV, or to arrive at a form of inverse reasoning from desired performance indices to parameter values, are difficult. They turn to be entirely unavailable, prohibitively computationally expensive, intractable, or at the very least extremely reliant on the modeling of the AUV and therefore precariously non-robust.

This problem is exacerbated if generality is pursued, i.e. if the algebraic trajectory planner is being developed as a general platform regardless of the craft's kinematics (configuration of actuators, deliverable thrust, holonomic constraints etc.). This is the case with the virtual potentials based trajectory planner developed by Barisic et al. (2007), Barisic et al. (2008), which is being deployed in systems as varied as an AutoMarine module (Stipanov et al. (2007)) -equipped VideoRay (Miskovic et al. (2007)) as opposed to a Remus off-the-shelf tactical submersible (Healey et al. (2007)). It is critical that an initial heuristic analysis be undertaken. This will serve to narrow the range of values provided by the stability analysis further, so that the available range is non-prohibitive for a system engineer overseeing the deployment of one of the aforementioned craft. The motivation is to arrive at a small number of recipes of values can be deployed to foreseeable use scenarios of certain broad categories of AUV craft.

In Section 1, the proposed virtual potentials based real time trajectory planning method (Barisic et al. (2007), Barisic et al. (2008)) is briefly revisited, examining the method-independent parameters that regulate the influence of the stator and rotor classes of potentials on the trajectory. In Section 2, a heuristic criterion based on actual considerations in operating AUVs in constrained waterspaces and waterways is proposed. In Section 3 an analysis based on MATLAB simulation is undertaken for a set comprised of the most critical independent parameters. In Section 4 conclusions are reached and heuristic recommendations given based on the performed simulation. Work aimed at further optimization of the recommended values is suggested.

1. THE METHOD

Within the proposed virtual potential based trajectory planner, the influence of n obstacles on the AUV trajectory is represented by a summation $E_s(\boldsymbol{p}) = \sum_N^{i=1} f_s^{(i)}$, where each $f_{obs}^{(i)}(\cdot), i = 1 \ldots N$ is given by:

$$f_s(\boldsymbol{p}) = \exp\left\{A_s^+ / r\left[\boldsymbol{p}_{obs}\right](\boldsymbol{p})\right\} \tag{1}$$

Where:
- $\boldsymbol{p}$ is the vector of point coordinates in the missions space whereat potential is being sampled,
- $r\left[\boldsymbol{p}_{obs}\right](\mathbb{R}^n)$ is a function returning the distance between the current AUV's location and the obstacle, parameterized by a representative point of the obstacle $\boldsymbol{p}_{obs}$ (geometric barycenter or some other typical point of the obstacle, e.g. a polygon vertex). These functions vary depending on the shape of the obstacle: a circle, an ellipse, a rectangle or a triangle.
- A_s^+ is a *method-independent static repulsion amplification.*

In Barisic et al. (2008), a modification including rotor potentials was proposed, along the following reasoning:

$$\begin{aligned} & E_s(\boldsymbol{p}) \rightarrow E(\boldsymbol{p}) \\ \therefore\ & E(\boldsymbol{p}) \doteq E_s(\boldsymbol{p}) + E_r(\boldsymbol{p}) \\ & \quad = \sum_N^{i=1} f_s^{(i)}(\boldsymbol{p}) + \sum_N^{i=1} f_r^{(i)}(\boldsymbol{p}, \boldsymbol{p}_{AUV}) \\ & \quad = \sum_N^{i=1} \left[f_s^{(i)}(\boldsymbol{p}) + f_r^{(i)}(\boldsymbol{p}, \boldsymbol{p}_{AUV})\right] \\ \therefore\ & f_{obj} \doteq f_s + f_r \end{aligned} \tag{2}$$

The redefinition of $f_{obj}(\cdot)$ from (2), based on Barisic et al. (2008) leads to:

$$f_{obj}(\boldsymbol{p}) = f_s(\boldsymbol{p}) + f_r\left[A_r^+, \boldsymbol{p}_{obs}\right](\boldsymbol{p}, \boldsymbol{p}_{AUV}) \tag{3}$$

Where:
- $f_r\left[A_r^+, \boldsymbol{p}_{obs}\right](\mathbb{R}^n, \mathbb{R}^n)$ is a rotor potential function parameterized by A_r^+ and $\boldsymbol{p}_{obs}$,
- A_r^+ is a *method-independent rotor amplification*,
- $\boldsymbol{p}_{AUV}$ is the coordinate vector of the AUV's current location.

Note that in accordance with Barisic et al. (2008):

$$f_r(\boldsymbol{p}_{AUV}) \equiv 0$$
$$\Rightarrow$$
$$f_{obj}(\boldsymbol{p}_{AUV}) \equiv f_s(\boldsymbol{p}_{AUV})$$

The potential field gradient $E(\boldsymbol{p}) = \sum_N^{i=1} f_{obs}^{(i)}$ is numerically approximated on the basis of a finite set of ordered pairs of the evaluations of $E(\boldsymbol{p})$ at p_{AUV} and at each of the points in $\mathcal{P}$, a set of radially spaced equidistant sample points around $\boldsymbol{p}_{AUV}$ at sample distance ϵ and angular increments $2\pi/n_\gamma$: $\mathcal{P} = \left\{\boldsymbol{p}_\epsilon^{(i)}\right\}$ (Barisic et al. (2007)). This leads to the an expression for the *controlling force*. The intention of the AUV's control system being fed by the trajectory planner is thereupon that the subsequent control allocation and the dynamic quality of actuator control reproduce this force as well as possible. The controlling force is bounded (in norm) by a *method-independent maximum force* F_{max}:

$$\boldsymbol{F} = \text{bound}\left\{\max_i \left[E(\boldsymbol{p}) - E\left(\boldsymbol{p}_\epsilon^{(i)}\right)\right] - \mu \cdot \boldsymbol{v}, F_{max}\right\} \tag{4}$$

Where:
- $\text{bound}\left\{\mathbb{R}^n, \mathbb{R}_0^+\right\}$ is a function of bounding a vector's norm, $\text{bound}\left\{\boldsymbol{a}, a_{max}\right\} = \boldsymbol{a}/\left|\boldsymbol{a}\right| \cdot a_{max}$.

Likewise, the helm speed command, v_c is bounded by a *method-independent maximum speed* v_{max}:

$$\begin{aligned} v_c &= |\boldsymbol{v}_c| \\ \phi_c &= \text{atan2}(\boldsymbol{v}_c) \\ \boldsymbol{v}_c &= \text{bound}\left\{T/2 \cdot (\boldsymbol{F}(k) - \boldsymbol{F}(k-1)) + \boldsymbol{v}_c(k-1), v_{max}\right\} \end{aligned} \tag{5}$$

The technical maxima of F_{max} and v_{max} are of course although independent from the method used to plan the trajectory, dictated by the AUV's actuators and their configuration.

From the above analysis of the equations used in trajectory planning, it can be determined that the greatest influence on trajectory shape, dictated by equations (1, 3)is in the choice of *method-independent parameters* of the type of repulsion and rotor amplifications, namely:

(1) The static repulsion amplification A_s^+,
(2) The rotor amplification A_r^+.

2. THE HEURISTIC CRITERION

In order to adjudicate between heuristic choices for the values of A_s^+ and A_r^+, the following qualitative analysis is performed:

(1) *The closest distance from any obstacle during cruise,* $\underline{d}_{obs}$

$$\underline{d}_{obs} = \min_{j=1\ldots m} d_{obs}^{(j)} \left[\boldsymbol{x}_{obs}^{(j)}\right] (\boldsymbol{p}(k)\forall k)$$

Where:
- j is the ordinal number of the obstacle,
- m is the total number of obstacles encountered by the AUV during the simulation,
- $d_{obs}^{(j)} \left[\boldsymbol{x}_{obs}^{(j)}\right] (\mathbb{R}^2)$ is a function returning the distance of a vector $\boldsymbol{x} \in \mathbb{R}^2$ to the j-th obstacle (parameterized by a coordinate vector of a representative point of the obstacle – e.g. a barycenter or a geometric center, or a vertex of a polygon, $\boldsymbol{x}_{obs}^{(j)}$), which depends on the *class* i.e. type of the j-th obstacle.

This measure is included in the third quadratic criterion term, $\mathcal{I}_d$, in the criterion function $\mathcal{I}\left(A_s^+, A_r^+\right)$, in the following manner:

$$\mathcal{I}_d = w_d \cdot \left(\underline{d}_{obs} - \underline{d}_{obs}^{(nom)}\right)^2 \tag{6}$$

Where:
- w_d is a weight such that the expression:

$$\left(\underline{d}_{obs} - \underline{d}_{obs}^{(nom)}\right)^2 \overset{\text{id}}{=} 49$$
$$w_d = 49/\underline{d}_{obs}^{(nom)\ 2} \tag{7}$$

- $\underline{d}_{obs}^{(nom)}$ is identically equal to 2m, which is a rational average distance at which an AUV should circumnavigate all obstacles.

(2) *Relative duration of time while cruising with maximum speed,* T_v

$$T_v = \left(\text{card}\left\{k : v(k) = v_{max}\right\} \cdot T\right) / T_{exp}$$

Where:
- card $\{\cdot\}$ is the cardinality operator, i.e. the number of elements of a set.
- T_{exp} is the total duration of the experiment.

This, being an expression dependent on $\left(A_s^+, A_r^+\right)$ is used as one of the weighted quadratic terms that constitute the criterion function $\mathcal{I}\left(A_s^+, A_r^+\right)$, which is (numerically) minimized in order to heuristically fine-tune the $\left(A_s^+, A_r^+\right)$.

The term dependent on T_v, $\mathcal{I}_v$ has the following weighted quadratic form:

$$\mathcal{I}_v = w_v \cdot \left(T_v - 0.05\right)^2 \tag{8}$$

Where:
- w_v is a weight such that the expression:

$$\left(T_v - 0.05\right)^2 \overset{\text{id}}{=} 49$$
$$w_v = 49/0.05^2 \tag{9}$$

- 0.05 is the nominal value of a rational expectation that in a congested environment, with gratuitous manoeuvres necessary to circumnavigate obstacles, only 5% of time will be spent cruising at maximum allowable velocity.

(3) *Relative duration of time while manoeuvering with maximum thrust,* T_F

$$T_F = \left(\text{card}\left\{k : F(k) = F_{max}\right\} \cdot T\right) / T_{exp}$$

This measure is used as another one of the weighted quadratic terms, this one designated by $\mathcal{I}_F$ that constitute the criterion function $\mathcal{I}\left(A_s^+, A_r^+\right)$:

$$\mathcal{I}_F = w_F \cdot \left(T_F - 0.01\right)^2 \tag{10}$$

Where:
- w_F is a weight such that the expression:

$$\left(T_v - 0.01\right)^2 \overset{\text{id}}{=} 49$$
$$w_v = 49/0.01^2 \tag{11}$$

- 0.01 is a nominal value, taking into account a rational assumption that thruster wear-and-tear should be avoided, as well as the energy budget being as much conserved as possible, thereby using only the 1% of time as optimal for effecting full-thrust manoeuvres.

(4) *The time of cruise,* T_c This is a parameter that is measured during the simulation. The termination conditions are that the commanded speed is less than 0.1 m/s for more than $5T$. It too is used in a quadratic criterion term, $\mathcal{I}_T$, in the criterion function $\mathcal{I}\left(A_s^+, A_r^+\right)$:

$$\mathcal{I}_T = w_T \cdot \left(T_c - 1.15\frac{d_{gp}(0)}{v_{max}}\right)^2 \tag{12}$$

Where:
- w_T is a weight such that the expression:

$$\left(T_c - 1.15 \cdot d_{gp}(0)/v_{max}\right)^2 \overset{\text{id}}{=} 49$$
$$w_T = 49\frac{v_{max}^2}{1.15^2 d_{gp}^2(0)} \tag{13}$$

- 1.15 is a nominal figure included under the assumption that in a congested environment of a simulation, it is optimal if the cruise to the goal point takes 15% more than the amount of time it would take an AUV homing in on the goal point with the maximum allowable speed, v_{max}, in a completely empty environment,
- $d_{gp}(0)$ is the beginning distance of an AUV from the goal point, at $k = 0$.

From equations (6 – 13) a criterion, based on the performance characteristics to be expected of AUVs operating in realistic closed-water, harbor or installation waterspaces is arrived at as:

$$\begin{aligned}\mathcal{I} &= \mathcal{I}_d + \mathcal{I}_v + \mathcal{I}_F + \mathcal{I}_T \\ &= w_d\left(\underline{d}_{obs} - \underline{d}_{obs}^{(nom)}\right)^2 + w_v\left(T_v - 0.05\right)^2 \\ &\quad + w_F\left(T_F - 0.01\right)^2 + w_T\left(T_c - 1.15\frac{d_{gp}(0)}{v_{max}}\right)^2\end{aligned} \tag{14}$$

3. THE ANALYSIS OF THE SIMULATION

Two sets of simulations were performed in order to numerically evaluate the criterion function $\mathcal{I}\left(T_v, T_F, \underline{d}_{obs}, T_c\right)$ in equation (14) so that a simple search for a minimum of the tabulated values will center on the pseudo-optimal area of the (A_s^+, A_r^+) space. This will allow for the heuristic fine-tuning of the potential method trajectory planner proposed in Barisic et al. (2007), Barisic et al. (2008), summarized above in Section 1.

The first set of simulations was performed for purposes of qualitative inspection and visualization of the influence of A_s^+ and A_r^+, respectively on the geometrical shape of the trajectory – the path taken through the simulated waterspace. Both method-independent parameters were through a logarithmical equidistant series $(0.04 - -20.00)$. The nominal parameters are listed in table 1. The series is listed in table 2.

Table 1. Constant values of parameters

A_s^+	A_r^+	F_{max}	v_{max}
2.1389	2.1389	4.0	2.0

Table 2. Varying values of parameters A_s^+ and A_r^+ respectively

Simulation 1 $A_s^+(i)$	Simulation 2 $A_r^+(i)$
0.40000	0.40000
0.52895	0.52895
0.69947	0.69947
0.92496	0.92496
1.2232	1.2232
1.6175	1.6175
2.1389	2.1389
2.8284	2.8284
3.7402	3.7402
4.946	4.946
6.5405	6.5405
8.649	8.649
11.437	11.437
15.124	15.124
20.000	20.000

The results of the first simulation, where A_r^+ was kept constant at 2.1389 while A_s^+ was changed in accordance with table 2, are displayed in figure 1.

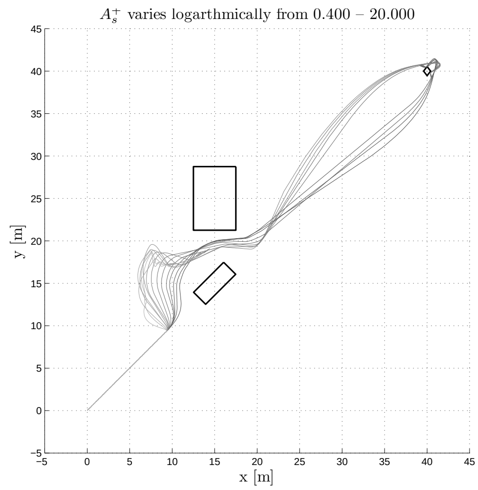

Fig. 1. The plot of paths for varying values of A_s^+

The results for the second simulation, where A_s^+ was kept constant at 2.1389 while A_r^+ was changed in accordance with table 2, are displayed in figure 2.

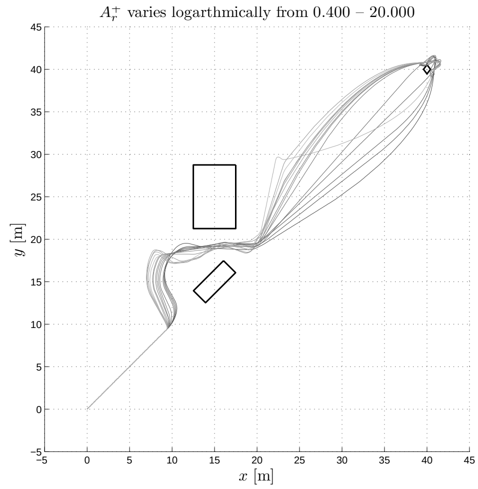

Fig. 2. The plot of paths for varying values of A_r^+

In both cases, it can be observed that the range of values is rational and sensible, since no collisions occur, nor does the planned trajectory diverge without bound.

In the second set of simulation, an exhaustive simulation series was performed for all ordered pairs of values (A_s^+, A_r^+) from table 2. The second set of simulations was used to narrow down the area by simple inspection of a thus constructed manifold of $\mathcal{I}(A_s^+, A_r^+)$ criterion. Some interesting results of the second set of simulations are displayed in figures 3 – 6. The resulting suboptimal but narrowed down ordered pair of values of (A_s^+, A_r^+) is given in table 3.

Table 3. Results of the inspection of $\mathcal{I}(A_s^+, A_r^+)$

A_s^+	A_r^+
1.62	11.44

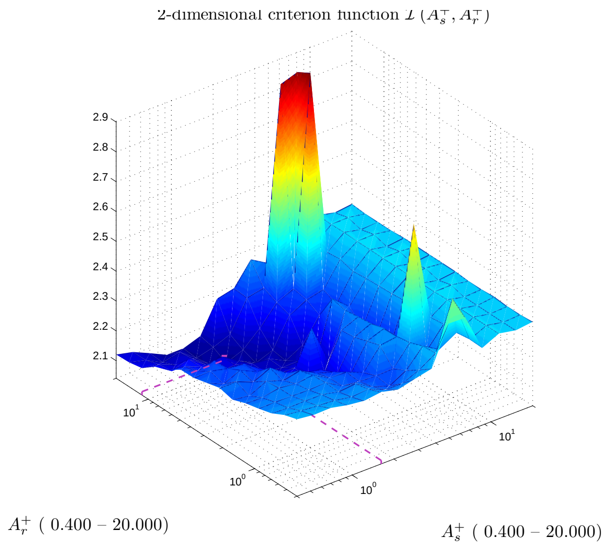

Fig. 3. The plot of the criterion $\mathcal{I}(A_s^+, A_r^+)$

As can be observed from figure 3, the shape of the criterion manifold is complex and features multiple local extremals.

The criterion value axis of the plot shows that the values of $\mathcal{I}$ are in between the orders of magnitude $10^2 - -10^3$ which is to be expected of a linear combination of terms that take on values of $0.5 \cdot 10^2$ in a relatively small interval of values of processed variables – $\{T_v, T_F, \underline{d}_{obs}, T_c\}$. The position of the minimum point $\underline{\mathcal{I}}$, which testifies to the fact that A_r^+ is more important to minimizing $\mathcal{I}$ than A_s^+ is unsurprising due to the fact that the simulated waterspace (in figure 1 or 2) features an obstacle that is right on the initial separatrix of the AUV and the goal-point. The correct circumnavigation of such an obstacle, which would, without *rotor* potentials cause the appearance of a local minimum and the faulty termination of the planned trajectory in front of the obstacle, is highly dependent on the values of A_r^+.

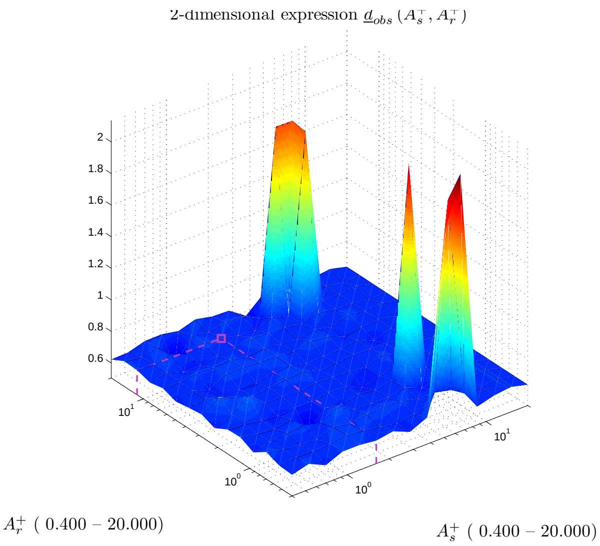

Fig. 4. The plot of the term $\underline{d}_{obs}(A_s^+, A_r^+)$

Figure 4 displays the manifold of the $\underline{d}_{obs}(A_s^+, A_r^+)$. A large section of the manifold lies at relatively low values of the minimum distance, because the simulation waterspace includes a narrow strait between two obstacles which, for all but the most conservative settings of (A_s^+, A_r^+), represents an optimum path towards the goal point. Those conservative settings are, in turn, signified by the two disjoint elevated regions in the plot. However, while navigating this strait, coming in closer than 2m to at least one of the obstacles is impossible.

Figure 5 displays the manifold of the $T_v(A_s^+, A_r^+)$. There is a prominent part of the manifold that includes values of $\approx 2\%$. This section represents all trajectories resolving into the strait in between the two obstacles. In the strait, two opposed influences are at play:

- the interaction of repulsive static potentials of both obstacles. This would ordinarily create a "ridge" that would represent a saddle-like local finite maximum.
- the interaction of counter-rotating rotors of both obstacles. This creates a "slipstream" or "current" in the strait allowing for the saddle-like local finite maximum to be overcome *albeit* by careful and *slow* navigation.

The region around the peak-value represents those trajectories resolving in a very conservative wide circumnavigation of the entire surrounding area of the two obstacles. Once outside this area, the trajectory approaches the goal point at maximum speed for a significant amount of time (covering ground lost to conservative circumnavigation of the obstacles).

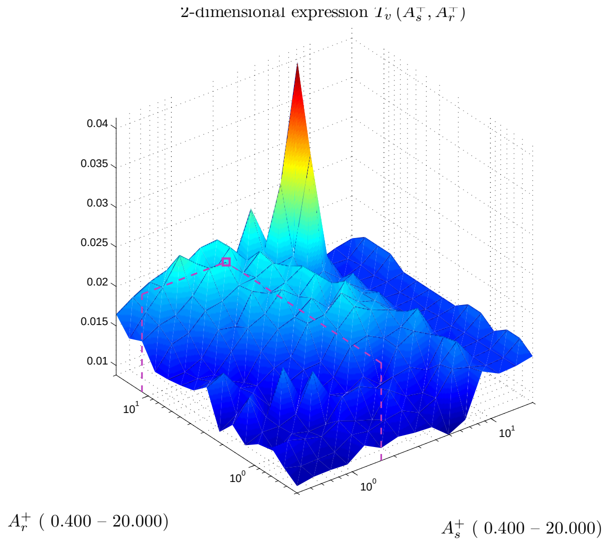

Fig. 5. The plot of the term $T_v(A_s^+, A_r^+)$

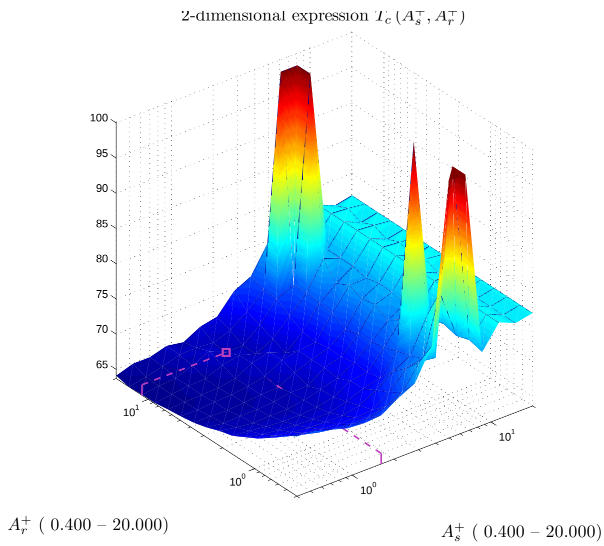

Fig. 6. The plot of the term $T_c(A_s^+, A_r^+)$

The region of interest of the manifold of the $T_c(A_s^+, A_r^+)$ displays values of ≈ 60s. Taking into account that $d_{gp}(0) = 40\sqrt{2}$, and $v_{max} = 2$m/s from table 1, it turns out that the nominal term $1.15 \cdot d_{gp}(0)/v - max = 35.53$s. The typical T_c-s are ≈ 2.3 times longer than the ideal straight-line navigation through unobstructed waterspace with maximum allowable forward speed. However, this is to be expected due to the congestion of the waterspace.

In contrast, note that the elevated regions of the $T_c(A_s^+, A_r^+)$ manifold – signifying even longer navigation to the goal point, coincide with the regions of the manifold of $T_v(A_s^+, A_r^+)$ in figure 5 that represent greater proportion of the trajectory taken at the maximum speed. At first glance, this might be counter-intuitive. However, one must be aware of the fact that the conservative settings of

(A_s^+, A_r^+) will result in the trajectory "shying away" from the strait in between the two obstacles along the direct path to the goal point. This steering clear of the strait, although allowing for a final approach along a clear section of the waterspace, wherein maximum speed can safely be developed, will induce an irrecoverable net loss of covered ground. Subsequent navigation at speed high to the north or away to the east of the region of obstacles (in figure 1) will fail to make up for the lost time.

The trajectory planned for the identified values (A_s^+, A_r^+) in table 3 is presented in figure 7.

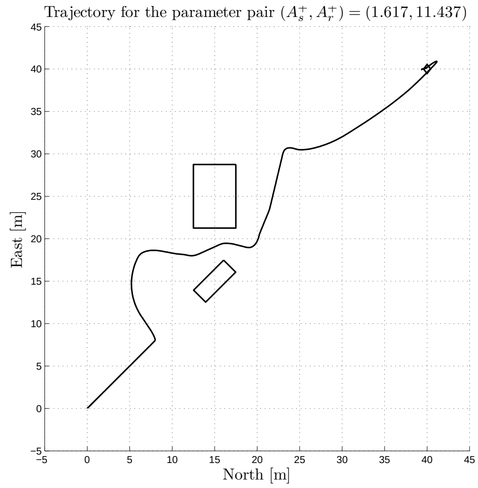

Fig. 7. The plot of the trajectory for identified values of parameters (A_s^+, A_r^+)

4. CONCLUSION

A heuristic simulative search through the parameter-space $\mathbb{A} = \{A_s^+, A_r^+\} = \mathbb{R} \times \mathbb{R}$ was carried out. A sensible criterion was chosen based on the actual performance characteristics that an AUV in a real waterspace would need to conform to. The trajectory planned by the heuristically identified parameter set is sensible, safe, and rational from the aspect of the energy budget of the autonomous vehicle.

Further work will include the development of a VRML-based simulation suite that will include the dynamical model of several AUVs, most notably the OceanServer Iver 2, the Hydroid Remus 100 and the in-house developed autonomous variant of the VideoRay Pro III XE GTO ROV. The criterion will be modified to "smooth out" the trajectory in the part where it exits straits or other types of congested areas.

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[1] The Ministry of Science, Education and Sports of the Republic of Croatia.

[2] a non-for-profit non-government civil organization of underwater system engineers and scientists registered in Zagreb, Croatia.