Comment on "Potts Model with Long-Range Interactions in One Dimension"

In a recent Letter, Bayong *et al.* [1] use the Monte Carlo (MC) simulations to determine the border in the (σ, q) plane dividing the first- from the second-order transition regime in the 1D q-state Potts model with ferromagnetic long-range (LR) interactions decaying as $1/r^{1+\sigma}$. Although the onset of the 1st-order transition is to be expected in this model and was already examined by MC simulations, the proper determination of this border is one of the intriguing and unsolved questions which escapes standard real-space renormalization-group techniques and finite-range scaling, but also presents difficulties within the MC approach. The intention of this Comment is to draw attention to the limitations of the approach used, as well as to include several corrections and additions following from earlier work done on the subject, which considerably modify the proposed (σ, q) diagram.

The (σ, q) diagram shown in Fig. 6 of Ref. [1] summarizes the presented results, obtained by simulations on chains with up to 900 sites. The authors establish the borderline $q_c(\sigma)$ which is equal to $q_c = 2$ for $\sigma < 0.3$, and then increases passing through the points $q_c(0.7) = 5$ and $q_c(1) = 8$, determined up to the integer q.

The horizontal line $q_c = 2$ for low σ is easily understood through the well-known result [2,3], reproduced also in Ref. [1], that the transition becomes of the 1st order for $q > 2$ when the mean-field (MF) approximation applies. However, setting its end at $\sigma = 0.3$ might be misleading. According to all available results, the line in the same plane, separating the classical from the nonclassical regime, increases monotonously with q (i.e., with the number of degrees of freedom). It passes through points ($q = 1, \sigma = 1/3$) (see Ref. [16] of [1]), ($q = 2, \sigma = 0.5$) [4], and ($q = 3$, $\sigma = \sigma_c \geq 0.65$) [5]. This leads to the conclusion that the horizontal line $q_c = 2$ will in fact reach as far as $\sigma = 0.5$. For the analogous SR problem, it was explicitly shown [6] by the $4 - \epsilon$ expansion, that the 1st- to 2nd-order transition borderline $q_c(d) = 2$ goes down to $d = 4$, which corresponds to $\sigma = d/2$ in the LR analog.

For $0.5 < \sigma < 1$, $q_c(\sigma)$ of Ref. [1] appears to be largely overestimated, compared to what can be concluded from existing MC studies. Study on small chains up to 400 sites [7] already shows 1st-order transition for $q > 2$ and small σ, which becomes stronger with increasing q or decreasing σ, suggesting that the threshold of the onset of the 2nd-order phase transition, $\sigma_c(q)$ depends on q. However, its precise determination is a less trivial problem. A more detailed analysis was performed [5] for $q = 3$ by the more efficient cluster algorithm [8]. On chains of sizes up to $L = 3000$ the border was located between 0.6 and 0.7 (while for L up to 400 the estimation was $\sigma_c > 0.5$). This result is taken only as a lower limit of the actual threshold σ_c, although the correspondence between long- and short-range models extended outside the MF regime and to the Potts model, would imply $\sigma_c(q = 3)_{\mathrm{LR}} = 2/d_c(q = 3)_{\mathrm{SR}}$, i.e., close to 0.66 [5]. The problem is that, by approaching σ_c, the transition can become arbitrarily weak, i.e., the correlation length ξ, although finite, becomes arbitrarily large. Since the simulations are always performed on systems of finite size L, sufficiently close to the border σ_c (or q_c) this gives $\xi \gg L$, which makes the 1st-order transition indistinguishable from the 2nd-order one, the problem well known from MC studies of the SR Potts model. (Hence the interest to push the numerics to the largest possible sizes.) This is an important point which was not considered at all in [1]. For instance, the point $q_c(0.7) = 5$ from the diagram which was determined up to the integer q may be reexamined on larger chains. Using the cluster algorithm proposed by Luijten and Blöte [8,9] it takes only several hours on a Pentium II processor to show that the transition for $\sigma = 0.7$ is of the 1st order also for $q = 4$.

A comment should also be made on the critical exponents, mainly given for $\sigma = 1$. This is the line of the defect-mediated transitions, and the nonlinear RG calculations by Kosterlitz for $q = 2$ [10] and by Cardy for general q [11] give in both cases an essential-singularity-type behavior for the correlation length instead of a power law. The scaling relations used in the article are not appropriate for this case. Also, the results of Cardy put at least doubt whether it would be possible to have a 1st-order transition for $\sigma = 1$ at any q, and should the result $q_c(1) = 8$ [in text it stays $q_c(1) = 10$, probably by misprint] be rather attributed to some effective σ different from 1 for finite L.

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