IIR FILTERS WITH MAXIMUM IMPULSE RESPONSE SYMMETRY

Mladen Vucic and Hrvoje Babic

Faculty of Electrical Engineering and Computing
Unska 3, Zagreb, HR10000, Croatia

ABSTRACT

A class of optimum IIR discrete time systems based on the impulse response symmetry criterion is presented. Optimization of rational transfer function parameters of the second to the tenth order, with (i) zeros at the origin and (ii) zeros located on the real axis, is carried out. The optimum systems have the smallest symmetry error for a given system order. In the frequency domain they approximate a constant group delay and have a quasi gaussian amplitude response. The filter design procedure is outlined. The impulse invariance method based on continuous time systems with symmetric impulse response [1] is also considered.

1. INTRODUCTION

In the discrete time systems the finite impulse response (FIR) is possible, what enables realization of causal systems with a symmetric impulse response and an ideal linear phase. The realization of FIR filters typically requires quite a large number of memory locations and multiplications. In practical applications, however, an ideal linear phase system is not always necessary. This means that IIR systems with approximately symmetric impulse response or approximate linear phase will have application in particular, because they are computationally more efficient than FIR filters.

The finite order systems based on design requirement in the time are, generally, different from those based on the frequency domain criterion. Therefore we have recently used the symmetric impulse response as the design criterion for continuous systems [1]. In this paper we will optimize IIR filter transfer functions to obtain maximum impulse response symmetry for a given system order.

2. IMPULSE RESPONSE SYMMETRY

The discrete form of impulse response symmetry error is given by

$$E_S = \sum_{n=S}^{\infty}\left[h(n)-h(2S-1-n)\right]^2 , \tag{1}$$

where h(n) is filter impulse response. Symmetry line is placed between samples S-1 and S. Arranging (1), we obtain

$$E_S = \sum_{n=0}^{\infty} h^2(n) - 2\sum_{n=0}^{S-1} h(S+n)h(S-1-n) = E_0 - 2E_a . \tag{2}$$

Summation E_0 represents the energy of the impulse response to which symmetry error is normalized. Thus, the error of the impulse response symmetry will be given by

$$E = 1 - 2\frac{E_a}{E_0} . \tag{3}$$

The criterion E will be expressed by transfer function poles and zeros and used as a goal function in the optimization process. For the N-th order system the transfer function is given by

$$H(z) = K_0 \frac{\prod_{k=1}^{M}(1-c_k z^{-1})}{\prod_{i=1}^{N}(1-d_i z^{-1})} , \qquad K_0 = \frac{\prod_{i=1}^{N}(1-d_i)}{\prod_{k=1}^{M}(1-c_k)} , \tag{4}$$

where d_i and c_k are transfer function poles and zeros. If the poles are simple and M≤N, the impulse response can be expressed as

$$h(n) = \sum_{r=1}^{N} A_r d_r^n , \quad A_r = K_0 \frac{\prod_{k=1}^{M}(1-\frac{c_k}{d_r})}{\prod_{\substack{i=1 \\ i\neq r}}^{N}(1-\frac{d_i}{d_r})} , \tag{5}$$

where the pole residues are A_r, r=1,2,...,n.

Now, E_a and E_0 can be expressed as function of poles, zeros and residues:

$$E_0 = \sum_{i=1}^{N}\sum_{k=1}^{N}\frac{A_i A_k}{1-d_i d_k} , \tag{6}$$

$$E_a = \sum_{i=1}^{N}\sum_{\substack{k=1 \\ k\neq i}}^{N}\frac{A_i A_k}{d_i - d_k} d_i^{2S} + \sum_{i=1}^{N} A_i^2 d_i^{2S-1} S . \tag{7}$$

As the goal function variables, the real and imaginary part of poles and zeros were used. Finally, the optimum poles and zeros were found as

$$\min_{c_k, d_i} E[c_k, d_i] \tag{8}$$

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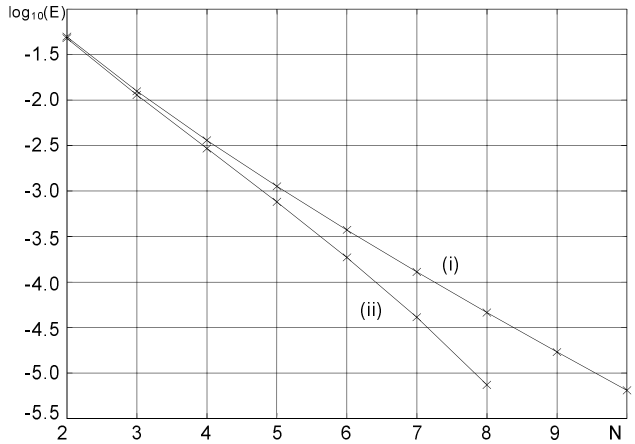

Figure 1. Symmetry error of optimized systems with (i) all zeros at the origin and (ii) N−1 real zeros.

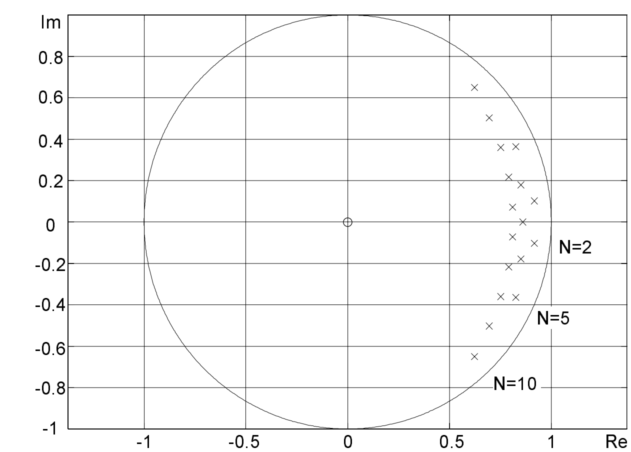

Figure 2. Pole locations of the optimum system with all zeros at the origin, S=10.

For searching minimum a Quasi-Newton method with BFGS formula for Hessian matrix update was used [2]. In each iteration, a bisection type line search was performed followed by quadratic interpolation. In situations when Hessian matrix had irregular inverse, a gradient method was forced by setting Hessian matrix to identity.

The optimization procedure was performed for the second to the tenth order transfer functions, with S=3 to S=30.

3. OPTIMUM FILTERS WITH ALL ZEROS AT THE ORIGIN

First we optimized transfer function with all zeros (M=N) at the origin of the z-plane, which is equivalent to the all pole transfer function in the continuous systems. The results of optimization are numerical values of the transfer function poles. The obtained symmetry error is generally smaller for higher system order as shown in Figure 1., by the diagram (i).

Pole positions, as an example, for order N=2, 5 and 10 and S=10, are given in Figure 2. It is interesting to note that poles are very nearly equidistant in the frequency.

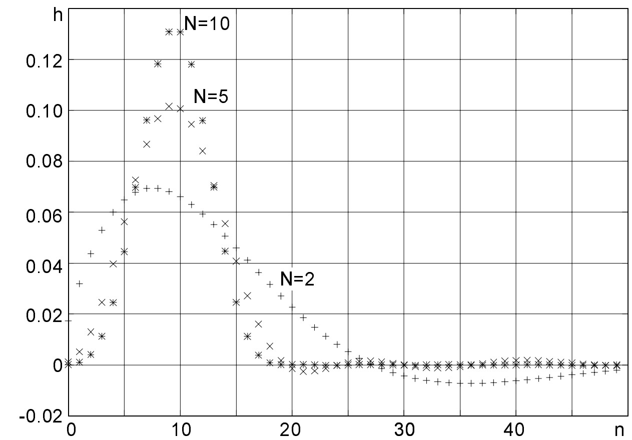

Figure 3. Impulse responses of the optimum systems with all zeros at the origin, S=10.

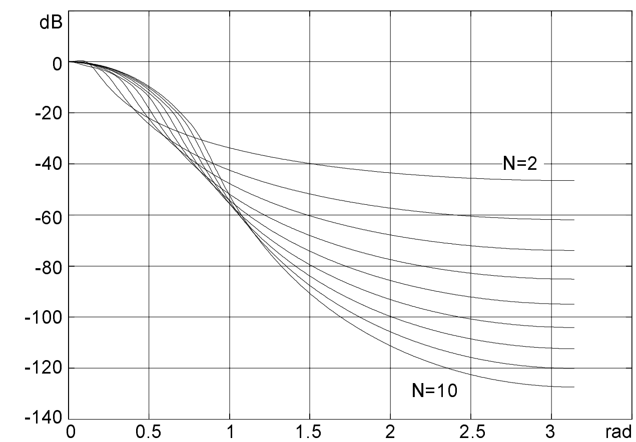

Figure 4. Amplitude response of the optimum system with all zeros at the origin, S=10.

The impulse responses of the optimum systems are given in Figure 3. They are quasi gaussian responses with better symmetry and smaller ringing for larger system order, N. Delay of the response is S-1/2, while the length is practically L=2S-1.

The amplitude response in dB is given versus frequency in linear scale in the Figure 4. It is possible to see parabolic character of the attenuation throughout the band up to $4\omega_{3dB}$. This means that dominant part of the frequency response approximate gaussian response.

The group delay curves approximate a constant with a ripple, as shown in Figure 5. The ripple, however, for a given N is not equal, but it is increasing with the frequency. Apparently, a larger group delay error is tolerable in the frequency region where amplitude attenuation is high.

Generally, properties of the obtained systems are similar to the continuous time systems with symmetric impulse response presented in [1], for the all pole transfer functions. Here we have transfer functions with multiple zeros located at the origin of the z plane. Such position of zeros does not influence the system response.

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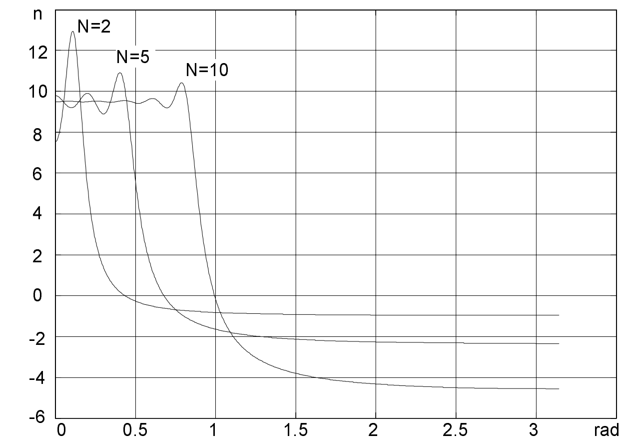

Figure 5. Group delay response of the optimum system with all zeros at the origin, S=10.

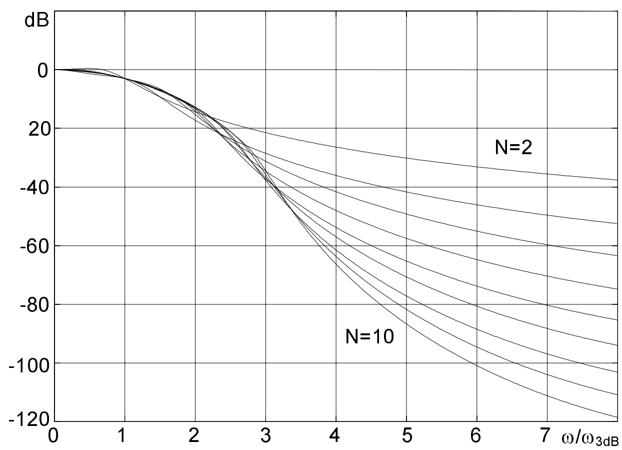

Figure 6. Amplitude responses of discrete time systems with all zeros at the origin.

4. FILTER DESIGN

The filter design starts by selection of the necessary system order N, which might be based on (a) tolerable symmetry error, or (b) attenuation at the stop band frequency ω_s.

(a) System order, N, can be selected from the tolerable symmetry error (3) using diagram (i) in Figure 1. Symmetry line, S, can be selected from the required impulse response length L=2S-1. We may consider such IIR filter equivalent to a gaussian FIR filter with the same length L.

(b) Design in the frequency domain seems to be complex because there are a large number of possibilities to design the filter with prescribed ω_{3dB}. Fortunately, the set of amplitude response curves is practically identical for various S when given versus normalized frequency ω/ω_{3dB}, as shown in Figure 6.

Now, the necessary system order, N, can be found from Figure 6. and the required attenuation at the stop band frequency, or ω_s / ω_{3dB}. The required S can be found from diagrams in Figure 7., using selected N and required ω_{3dB}.

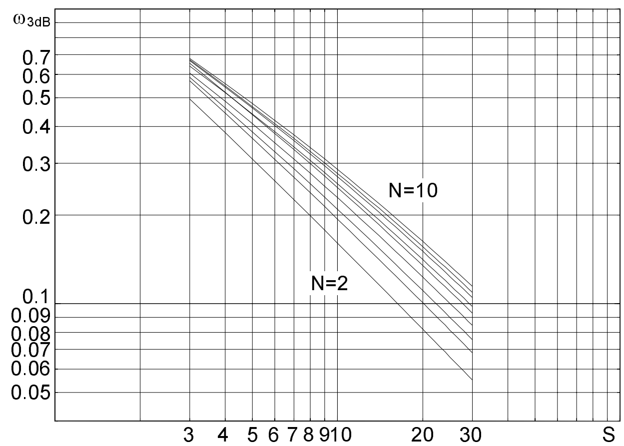

Figure 7. Cutoff frequencies ω_{3dB}, of the optimum systems with all zeros at the origin.

When parameters N and S are known, only one run of the optimization process (8) is necessary to obtain the numerical values of poles.

5. IIR FILTERS OBTAINED FROM CONTINUOUS FILTERS

The recursive digital filters can be designed from continuous prototype using the impulse invariance method [3]. The IIR filters with symmetric impulse response can also be designed from continuous prototype calculated and tabulated in [1]. The pole position of the IIR filters are easily obtained by transformation

$$d_i = e^{p_i T} \quad . \tag{9}$$

By this, where p_i are prototype poles and T is the sampling time, the optimization is not necessary. Because the sampling time can be selected continuously, one may have any distribution of samples within a symmetric envelope and any delay or cutoff ω_{3dB}. The impulse invariance method offers, therefore, more flexibility in the filter design than optimum system based on the criterion (3), where S is an integer.

The application of impulse invariance method on the all pole transfer function of the continuous system gives, an IIR transfer function with zeros spread along the real axis. This causes the higher attenuation in the stop band and at the Nyquist frequency. To illustrate this, the frequency scales of the optimum discrete and continuous prototype filter are adjusted to meet at 3 dB cutoff, Figure 8.

The presence of zeros outside of the origin makes the filter realization more complex than in the case with zeros at the origin. The structure complexity can be somewhat reduced if some zeros are close to the origin because they can be assumed to be at the origin. Also, the zeros far away from the origin can be considered that they at infinity.

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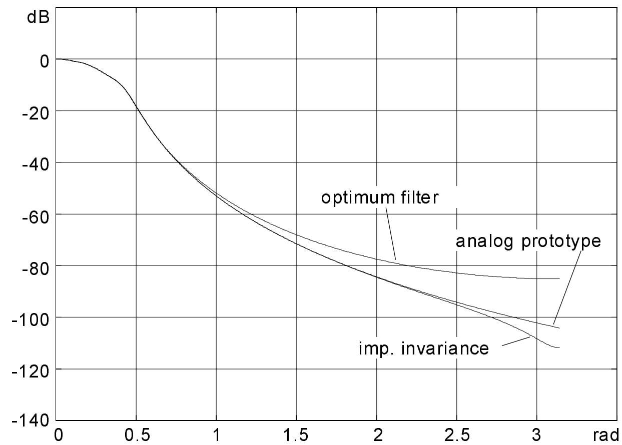

Figure 8. Amplitude response of the optimum filter (i), continuous prototype, and the filter obtained by impulse invariance method, N=5.

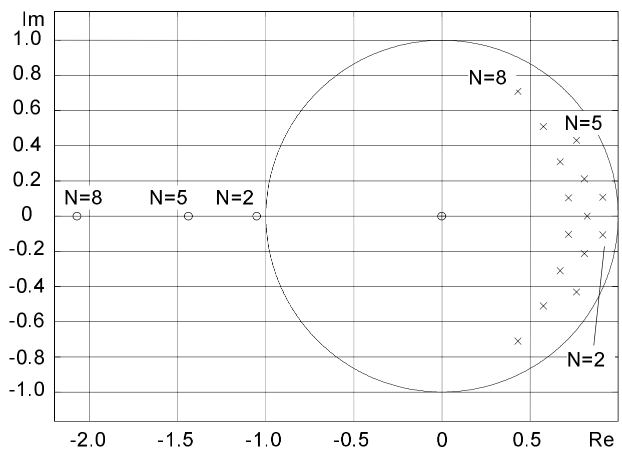

Figure 9. Pole locations of the optimum system with N-1 real zeros.

A good influence of zeros in the impulse invariance method has suggested the optimization of IIR transfer functions with all zeros on the real axis.

6. OPTIMUM IIR FILTERS WITH ZEROS ON THE REAL AXIS

The optimization procedure was also carried out with N-1 zeros on the real axis and one zero at the origin. The optimum locations of zeros are very interesting. All N-1 zeros are very nearly equal and they may be considered as a multiple zero. The zero-pole locations for system orders N=2, 5 an 8, and S=10 are given in Figure 9., where one zero is at the origin, and the second zero is on the real axis with multiplicity N-1, as it is shown in Figure 10.

The filter amplitude and group delay responses are somewhat better than for filter obtained by impulse invariance method, as for example is shown in Figure 10. Generally, impulse invariance method gives Nyquist attenuation half way between both optimum systems (i) with all zeros at the origin and (ii) with zeros on the real axis, as it is shown in Figure 8. and Figure 10.

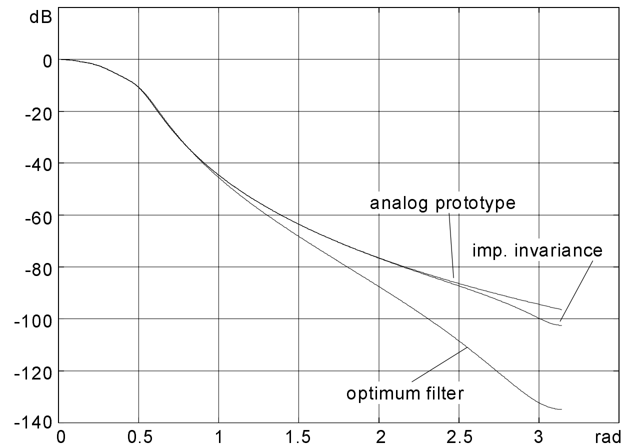

Figure 10. Amplitude response of the optimum filter (ii), continuous prototype, and the filter obtained by impulse invariance method, N=5.

The design of the optimum IIR filter with zeros on the real axis is similar to the design outlined in the section 3 using the diagram (ii) in Figure 1.

7. CONCLUSION

A method for optimum IIR filters design with symmetric impulse response was developed. The transfer function with (i) all zeros at the origin and (ii) N-1 zeros at the real axis were considered. Both optimal filters are interesting because the first enables symmetric response with simple structures. The second is with better properties, but require more complex structure. The used error criterion produces the best symmetry of samples about the symmetry line, for a given system order.

The impulse invariance method, based on optimum continuous systems in [1], can also be used for IIR filters design.

8. ACKNOWLEDGEMENT

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