

VARIABLE BIT-RATE LSF CODING USING WAVELET SUB-BAND DECOMPOSITION

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ABSTRACT

Variable bit-rate Wavelet sub-band coding of the Line Spectrum Frequencies (LSF) is presented in this paper. The evolution of the speech power spectrum envelope (PSE) described by the LSF vector sequence is decomposed into several parts, e.g.: very rapid transitions, medium speed transitions and slowly evolving part. All these parts are coded independently, according to the transformed data dependent Weighted Euclidean Distance (WED) measure between the original and the quantized LSF vector. Forward adaptive quantization is realized by employing the dynamic weighting path into the non-uniform filter bank realization of the Discrete Wavelet Transform (DWT). Vectors of the sub-band signals are classified using the nearest neighbor rule, based on the square root weighted sub-band signal magnitudes. Independent vector quantizers are designed for each of the classes and for each of the sub-band signals. Thus the required bit-rate for the speech PSE coding is adapted according to the changing dynamics of the speech signal.

Keywords: speech processing, spectral quantization, inter-frame sub-band coding, adaptive quantization, line spectrum frequencies

1. INTRODUCTION

An LSF transform coding technique based on Discrete Wavelet Transform was introduced in [1]. The basic idea of transform coding is to remove the so called inter-frame correlation between LSF vectors using the DWT. Components of the successive P -dimensional LSF vectors are treated as a P time-varying waveforms and are decomposed into a set of sub-band signals by means of the DWT. The DWT is identical for each of the P components. The sub-band signals have different rates and are critically sampled. The highest level approximation signal, a_J , and the highest level detail signal, d_J , have the lowest rate ($f_s/2^J$), while the first level detail signal, d_1 , has the highest rate, equal to the half of the frame rate ($f_s/2$). By proper choice of the mother Wavelet, the variances of the sub-band signals are inversely proportional to the sub-band signal rates; i.e. d_1

has the lowest variance, while d_J and a_J have the largest variance. Such variance distribution is the basis for bit-rate reduction. Only a few bits are allocated for coding of the quantized value of d_1 , while more bits are reserved for sub-band signals with larger variances. The scalar quantization of sub-band signals was proposed in [1] and was performed independently for each sub-band and each of the components. However it will be shown that additional bit savings can be obtained by combining several components of the same sub-band signal into a sub-band vectors and then performing Vector Quantization on this sub-band vectors.

The bit allocation can be static as in [1], but since LSF vectors exhibit time-varying statistics due to the changing dynamics of the speech signal, even better results can be obtained by adaptive quantization. For example, by performing two level Wavelet decomposition of the LSF vector sequence, the evolution of speech power spectrum envelope (PSE) is decomposed into three parts: very rapid spectral transitions, medium speed transitions and slow spectral evolution. For speech segments with slowly evolving PSE like sustained vowels, detail sub-band signals d_1 and d_2 are close to zero and only the approximation signal a_2 needs to be coded and transmitted (only one output vector for 4 input LSF vectors). On the other hand, for speech segments containing rapid spectral transitions like plosive sounds, more emphasis should be given to detail signals. The concept of such variable bit-rate LSF Wavelet sub-band coding will be presented in this paper.

2. PROBLEM OF CODING DELAY

A short summary of the former results will be given first. An LSF transform coder based on the DWT with biorthogonal Wavelets and optimal scalar quantization of the sub-band signals was presented in [1]. It was shown that the reduction of the average bit-rate required for LSF coding, compared to the case with no inter-frame decorrelation and with the same average spectral distortion, depends on the choice of the mother Wavelet and on the decomposition level J . As can be expected, higher decomposition levels result in lower average bit-rate, but also in exponentially larger delay.

The problem with any kind of inter-frame sub-band coding system is the introduced delay. For some applications the coding delay is not a problem (e.g. voice storage/retrieval, voice-mail etc.), so the level J can be chosen to minimize the average bit-rate. On the other hand, for real-time voice coder implementations, the total delay must be kept below certain limits. For low bit-rate voice coders this limit is usually set to 300ms. Total analysis/synthesis delay of a non-uniform filter bank realization of the J -level DWT is equal to $(2^J-1)L$. Number L is the total delay of a single level decomposition followed by reconstruction and depends on the chosen Wavelet type. The real-time delay requirement automatically restricts the choices of J and Wavelet type only to certain convenient combinations. However it was shown in [1] that a considerable bit-rate reduction can be obtained even with a simple biorthogonal Wavelet 'bior2.2' with $L=3$, while introducing the total delay of only 9 frames for two-level decomposition.

3. VARIABLE BIT-RATE LSF CODING

Using the same theoretical background outlined in [1], a variable bit-rate Wavelet LSF coding technique was developed. Instead of static weighting and fixed bit allocation as described in [1], the sub-band signals are quantized and coded according to the dynamic weighting functions. With such dynamic weighting it is possible to adapt the coding system to the changing statistics of the input speech signal, by varying bit allocation of the sub-band signals. This technique result in the reduction of the average bit-rate required for transparent coding and in the reduction of the percentage of the outlier frames. Since all LSF components are processed using identical decomposition filter-banks, all components corresponding to the same sub-band signal can be reunited to form a P -dimensional sub-band vector. This vector is then quantized using the Classified Vector Quantization (CVQ) with separately optimized low-complexity codebooks of different sizes for each of the classes and for each of the sub-band vectors. The same procedure is performed independently for all $J+1$ sub-band vectors.

3.1. Distortion measure in the LSF domain

Quantization of the LSF vector components introduces distortion in the decoded PSE. This distortion is usually measured by averaging the log spectral distortion, SD_n , between the unquantized and the quantized power spectra $S_n(\omega)$ and $\hat{S}_n(\omega)$, corresponding to the unquantized and the quantized LSF vectors \mathbf{x}_n and $\hat{\mathbf{x}}_n$ as in (1a).

$$SD_n = \left(\frac{100}{\pi} \int_0^\pi [\log S_n(\omega) - \log \hat{S}_n(\omega)]^2 d\omega \right)^{\frac{1}{2}} \quad (1a)$$

Averaging is performed for all N input frames, where n is the frame index:

$$SD = \frac{1}{N} \sum_{n=1}^N SD_n \quad (1b)$$

If the average spectral distortion SD is kept below 1dB and if the percentage of the outlier frames with SD_n greater than 2dB is below 2%, then the perceived distortion is almost negligible. Although this distortion measure is quite appropriate for comparative assessment of different quantization schemes, a much simpler distortion measure is usually used for coder design and implementation. It is a Weighted squared Euclidean Distance (WED) between the original and the quantized LSF vector and it is defined as in (2):

$$d_n^2 = (\mathbf{x}_n - \hat{\mathbf{x}}_n)^T \mathbf{W}_n (\mathbf{x}_n - \hat{\mathbf{x}}_n) \quad (2)$$

Weighting matrix \mathbf{W}_n is a diagonal $P \times P$ matrix that depends on the LSF vector \mathbf{x}_n , with diagonal elements $w_{1n}, w_{2n}, \dots, w_{Pn}$. Distortion definition given in (2) includes several well known data dependent WED measures such as: LPC spectral weights [2], inverse harmonic mean weights [3], local spectral approximation weights [4] and Gardner LSF spectral sensitivity measure [5]. Since the actual quantization is performed in the Wavelet domain, by quantizing $(J+1)$ sub-band vectors, the distortion measure d_n^2 must be converted to the Wavelet domain as well what will be explained next.

To simplify the procedure of WED distortion transformation, the DWT is defined as a linear block transform, transforming the finite sequence of input LSF vectors into the matrix of DWT coefficients. Matrix \mathbf{X} is a LSF matrix with N columns and P rows holding time sequence of N LSF vectors \mathbf{x}_n ($n=1$ to N). Matrix \mathbf{B} is the rectangular DWT transform matrix with N columns and N^+ rows, while \mathbf{B}^{-1} is the left side inverse of \mathbf{B} with N rows and N^+ columns ($N^+ > N$, due to the finite block length). Matrix \mathbf{Y} is the output matrix of DWT coefficients with P rows and N^+ columns. Matrices \mathbf{B} and \mathbf{B}^{-1} are huge but sparse. DWT of the matrix \mathbf{X} , as well as the inverse DWT of the quantized Wavelet coefficients $\hat{\mathbf{Y}}$ can be written as in (3):

$$\mathbf{Y} = (\mathbf{B}\mathbf{X}^T)^T, \quad \hat{\mathbf{X}} = (\mathbf{B}^{-1}\hat{\mathbf{Y}}^T)^T \quad (3)$$

Rows of the input matrix \mathbf{X} : $\mathbf{X}r_1, \mathbf{X}r_2, \dots, \mathbf{X}r_P$, corresponding to LSF components 1 to P , are DWT transformed into rows of matrix \mathbf{Y} : $\mathbf{Y}r_1, \mathbf{Y}r_2, \dots, \mathbf{Y}r_P$. The average LSF WED distortion in the LSF domain can be found by summing up the frame distortions d_n^2 given in (2) for all frames, $n=1, 2, \dots, N$. The same average distortion can also be found by summing up the component distortions e_i^2 for all components, $i=1, 2, \dots, P$, by simply rearranging the order of summation:

$$e_i^2 = (\mathbf{x}_{r_i} - \hat{\mathbf{x}}_{r_i})^T \mathbf{W}_i (\mathbf{x}_{r_i} - \hat{\mathbf{x}}_{r_i}), \quad \sum_{n=1}^N d_n^2 = \sum_{i=1}^P e_i^2 \quad (4)$$

Row vectors $\hat{\mathbf{X}}r_i$ are found from the quantized coefficients $\hat{\mathbf{Y}}r_i$ by performing the inverse DWT (i denotes the LSF component). Matrix \mathbf{V}_i is N by N diagonal matrix with diagonal elements $v_{i1}, v_{i2}, \dots, v_{iN}$, that are actually identical to the weights at the position (i, i) taken from matrices $\mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_N$ ($v_{i1} = w_{i1}$, $v_{i2} = w_{i2}$, ..., $v_{iN} = w_{iN}$). By expressing $\mathbf{X}r_i$ and $\hat{\mathbf{X}}r_i$ in terms of $\mathbf{Y}r_i$ and $\hat{\mathbf{Y}}r_i$ according to the equation (3), the distortion measure e_i^2 is transformed into WED distortion measure between the unquantized and the quantized DWT coefficients of the i^{th} LSF component:

$$e_i^2 = (\mathbf{Y}r_i - \hat{\mathbf{Y}}r_i) \mathbf{U}_i (\mathbf{Y}r_i - \hat{\mathbf{Y}}r_i)^T, \quad \mathbf{U}_i = (\mathbf{B}^{-1})^T \mathbf{V}_i \mathbf{B}^{-1} \quad (5)$$

New weighting matrices $\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_p$ are non-diagonal N^+ by N^+ matrices. Employing such a non-diagonal weighting matrix into the quantization scheme is to complex so some simplifications were necessary. In the case of independent quantization of different Wavelet coefficients (i.e. sub-band vectors) with zero bias quantizers, such that quantization errors of different coefficients are not correlated, the sum of all cross-product terms will be much smaller than the sum of diagonal elements. Therefore, an approximation of the e_i^2 can be used by simple nullification of all non-diagonal elements of \mathbf{U}_i . Such approximation holds as long as the correlation between quantization errors of any two sub-band vectors is much lower than the variance of the quantization error itself. It was shown experimentally that this assumption is valid for the LSF process studied in this paper.

3.2. Weighting functions for the DWT filter bank

Since the actual real-time realization of the DWT LSF coding is based on the continuous sub-band decomposition using non-uniform filter bank rather than on the block transformation presented above, matrix form diagonal weights \mathbf{U}_i , had to be associated with the corresponding sub-band signal weighting functions. An example of a 2-level DWT with dynamic sub-band signal weighting based on the transformed WED distortion

measure is shown in Figure 1 for component i of the LSF vector sequence. Upper-left part of the system is the analysis bank decomposing the input line spectrum frequency signal $xr_i(n)$ into three sub-band signals a_{i2} , d_{i2} and d_{i1} . The time index is intentionally omitted for these signals, since their rates are different. Sub-band signals are critically sampled as shown in Figure 1, such that the total number of samples of all of these three signals is equal to the total number of the input signal samples for any given time interval. Samples of these signals are in fact the Wavelet coefficients in the row vector $\mathbf{Y}r_i$, while their quantized values \hat{a}_{i2} , \hat{d}_{i2} and \hat{d}_{i1} are the elements of $\hat{\mathbf{Y}}r_i$. After quantization, the sub-band signals are reconstructed in the synthesis bank shown on the right side of Figure 1. There is nothing new about these two parts, since they represent the well-known filter bank realization of DWT that can be found in the literature on Wavelets. However, the dynamic weighting-path shown in the bottom of Figure 1 is something that is proposed by this paper, obtained by analyzing the structure of the transformed weighting matrix \mathbf{U}_i . Weighting function signals w_{a2} , w_{d2} and w_{d1} have identical rates as their corresponding sub-band signals and their samples are equal to the diagonal elements of the transformed weighting matrix \mathbf{U}_i . As it was shown in [1], these weighting functions can be derived from the input weights $v_i(n)$, by simple linear filtering. The input weights $v_i(n)$ are diagonal elements of the matrix \mathbf{V}_i . Each sub-band signal has its corresponding weighting filter: $H_{wa2}(z)$, $H_{wd2}(z)$ and $H_{wd1}(z)$. The impulse responses of these FIR filters can be found from the impulse responses of the corresponding synthesis filters $F_{a2}(z)$, $F_{d2}(z)$, and $F_{d1}(z)$ by squaring each sample of the impulse response. The impulse responses of the synthesis filters $F_{a2}(z)$, $F_{d2}(z)$, and $F_{d1}(z)$ are actually the basis functions of the inverse DWT and can be read out from the columns of the reconstruction matrix \mathbf{B}^{-1} , or directly from $F_0(z)$ and $F_1(z)$, by combining interpolation and convolution.

Since the filter order of the analysis filters doesn't necessary have to be equal to the filter order of the corresponding weighting filters, some additional synchronization is required. This is performed by

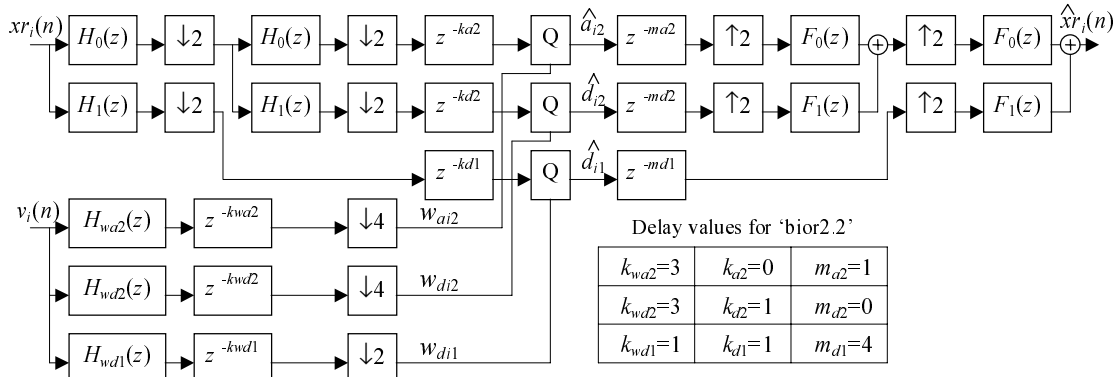


Figure 1. Block diagram of the filter bank realization of 2-level DWT with dynamic weighting and quantization

inserting delay elements k_{wa2} , k_{wd2} , and k_{wd1} in the weighting path before decimation, delay elements k_{a2} , k_{d2} , and k_{d1} in the signal path before quantization and finally m_{a2} , m_{d2} , and m_{d1} in the signal path after quantization. These delay values depend on the chosen mother Wavelet and must be determined for all decomposition levels that are to be used. Delay in the signal path before quantization is necessary for combinations that result in non-causal weighting filters, while delay after quantization is used to balance delay in the low and the high branch. The delay in the weighting path does not increase the total analysis/synthesis delay of the system. On the other hand, the delay in the signal path does, especially the one in the branch of the highest-level approximation signal a_J and detail signal d_J . For example, for the 2-level DWT with the biorthogonal Wavelet 'bior2.2', one step delay is necessary in the d_2 branch increasing the total delay from 9 to 13 samples.

Figure 1 depicts the system structure for only one LSF component, since the remaining $P-1$ are processed by the identical filter banks. Thus, the sub-band signals of all P filter banks form a P dimensional vectors \mathbf{A}_2 , \mathbf{D}_2 and \mathbf{D}_1 , with their corresponding weighting vectors \mathbf{W}_{A2} , \mathbf{W}_{D2} and \mathbf{W}_{D1} , as given in (6). Again, the time index is omitted since the sub-band vector rates are different.

$$\begin{aligned} \mathbf{A}_2 &= [a_{12}, a_{22}, \dots, a_{P2}]^T & \mathbf{W}_{A2} &= [w_{a12}, w_{a22}, \dots, w_{aP2}]^T \\ \mathbf{D}_2 &= [d_{12}, d_{22}, \dots, d_{P2}]^T & \mathbf{W}_{D2} &= [w_{d12}, w_{d22}, \dots, w_{dP2}]^T \\ \mathbf{D}_1 &= [d_{11}, d_{21}, \dots, d_{P1}]^T & \mathbf{W}_{D1} &= [w_{d11}, w_{d21}, \dots, w_{dP1}]^T \end{aligned} \quad (6)$$

3.3. Adaptive vector quantization

Sub-band signal vectors \mathbf{A}_2 , \mathbf{D}_2 and \mathbf{D}_1 are quantized independently and adaptively according to the transformed WED distortion measure defined by weighting vectors \mathbf{W}_{A2} , \mathbf{W}_{D2} and \mathbf{W}_{D1} . Since the quantization procedure is identical for all $J+1$ sub-band vectors, it will be illustrated only for \mathbf{D}_1 and corresponding weighting vector \mathbf{W}_{D1} . The total LSF WED distortion induced by quantization of \mathbf{D}_1 is in first approximation proportional to its weighted variance. To keep this distortion constant, the number of bits allocated for coding of the vector \mathbf{D}_1 must be varied. One approach to such adaptive quantization is the Classified Vector Quantization (CVQ) [6], where the input vector is first classified into one of several classes and then vector quantized using the codebook for that particular class. These codebooks are of different sizes, so by choosing one of them the resolution (and bit rate) is adapted according to some statistical properties of the input vector. This is a forward adaptive scheme with the class index representing the forward side information. It was found that suitable vector classification is the one based on the square root weighted magnitude vector \mathbf{DW}_1 ,

$$\mathbf{DW}_1 = [dw_1, dw_2, \dots, dw_P]^T \quad (7a)$$

$$\text{where } dw_i = \sqrt{w_{d1i}} \cdot |d_{i1}| \quad (7b)$$

\mathbf{DW}_1 vectors are classified according to the nearest neighbor rule based on the Euclidean distance measure, using the same procedure as in vector quantization (VQ) [6]. Classification templates are found from the actual data using the Linde-Buzo-Gray (LBG) clustering algorithm [6]. With such classification the sub-band vector space is effectively divided into several sub-spaces, with vectors of similar 'complexity' occupying each of the classes. After classification, input vector \mathbf{D}_1 is vector quantized using the WED distortion measure, using the weights defined by \mathbf{W}_{D1} . Optimal VQ codebooks are designed independently for each of the classes, from the actual data using the LBG clustering algorithm with WED distortion measure [6]. Since the codebook sizes are different, each class requires different number of bits. For properly chosen classifier, the class with the highest probability requires the least number of bits. For classes requiring very accurate quantization, the sub-optimal split VQ was performed to reduce the complexity and the codebook size. The same quantization procedure was performed for remaining sub-band vectors ($\mathbf{A}_2, \mathbf{D}_2$), but with independent classifiers and independent codebooks for each of the classes.

4. EXPERIMENTAL RESULTS

The proposed variable bit-rate LSF Wavelet coder was evaluated on a database with the parameters given in Table 1. All results are presented for the biorthogonal Wavelet 'bior2.2' with two level decomposition, since it offers very good trade-off between the compression ratio and the introduced delay. The VQ resolution was chosen such that the average spectral distortion is equal to 1dB.

Table 1. Experimental conditions

Speakers :	2 male / 2 female
Sentences:	from newspapers
# of frames:	120000
Preemphasis	0.9375
Sampl. Frequency:	8000 Hz
Frame rate:	100Hz
Window:	25ms / Hamming
LPC	10 ord. / Autocorr.

LSF vectors from the input database were transformed into the Wavelet domain. This way, separate training databases were obtained for all three sub-band vectors \mathbf{A}_2 , \mathbf{D}_2 and \mathbf{D}_1 . The Inverse Harmonic Mean weights [3] were used during the study and these were also transformed to the Wavelet domain, using the procedure outlined in this paper. Classified VQ codebooks were designed next for each of the sub-band vectors and the number of classes was varied to determine the one offering highest bit-rate compression.

The average bit-rate for transparent quantization of each of the sub-band vectors depends on the rate of that particular vector and on the number of classes. More

precisely it depends on the probability of each class multiplied by the VQ resolution of that particular class, plus the additional bit-rate for coding of the chosen class index. For some classes the vector quantization was performed on the whole sub-band vector, while for the 'complex' classes split-VQ was used to reduce the complexity. The average required number of bits denoted with b_{d1} , b_{d2} and b_{a2} for adaptive vector quantization of the sub-band vectors \mathbf{D}_1 , \mathbf{D}_2 and \mathbf{A}_2 (including the class index) is given in Table 2 as a function of number of classes. Class index is Huffman coded for cases with more than 2 classes, while the VQ codebook index is product coded (in the case of split-VQ). Minimum and maximum code lengths are also given for cases with more than one class. Since the rate of the vector \mathbf{D}_1 is one half of the frame rate, while the rates of \mathbf{D}_2 and \mathbf{A}_2 are one fourth, the total average bit rate can be found by scaling b_{d1} , b_{d2} and b_{a2} with 0.5, 0.25 and 0.25 respectively.

Table 2. Bit-rates for adaptive vector quantization

#cl	\mathbf{D}_1 VQ			\mathbf{D}_2 VQ			\mathbf{A}_2 VQ		
	b_{d1}	min	max	b_{d2}	min	max	b_{a2}	min	max
1	12	12	12	22	22	22	38	38	38
2	9.80	7	18	20.38	19	27	37.22	36	40
4	9.55	3	21	19.06	15	31	-	-	-
8	9.38	1	23	18.89	13	33	-	-	-
16	9.60	2	25	-	-	-	-	-	-

Best results are obtained using CVQ with 8 classes for \mathbf{D}_1 and \mathbf{D}_2 and 2 classes for \mathbf{A}_2 , summing up to the total average bit rate of **18.72 bits/frame**. For non-adaptive case (#cl=1) the required bit rate is **20.66 bits/frame**. Benefit of the adaptive quantization is the most pronounced for \mathbf{D}_1 sub-band vector. For the case with eight classes, the whole vector \mathbf{D}_1 is coded using a single bit in 37% of cases, while the longest code of 23 bits is needed in only 3.7% of cases. To reduce the complexity, 16 split VQ codebooks are used for eight classes of \mathbf{D}_1 having all together only 3958 scalar entries (one class has full vector VQ, six classes have 2 vector split VQ, while the most complex one is coded using 3 vector split VQ). Since the vector \mathbf{A}_2 requires very high resolution, it was quantized with 5 vector split VQ resulting in rather sub-optimal quantization. This vector is the main contributor to the total average bit rate, so further reductions are possible with more sophisticated quantization schemes. For full adaptive coding the total coding/decoding delay is 13 frames, but if d_2 is coded with a fixed rate (one class) and only d_1 and a_2 are coded adaptively, the delay would still be only 9 frames. It must also be noted that much better results can be obtained if the simple autocorrelation LPC method is replaced by

more sophisticated LPC analysis techniques that result in much higher correlation of the successive LSF vectors.

5. CONCLUSION

A very effective method for reduction of the average bit-rate required for coding of the speech power spectrum envelope is presented in this paper. It is based on the inter-frame decorrelation of LSF vectors using the Discrete Wavelet Transform. By transforming the weighted Euclidean distance measure between LSF vectors into the same kind of distortion measure in the Wavelet domain, adaptive quantization of the sub-band signals can be done. The sub-band signal vectors are dynamically classified into several classes and each class is quantized using a specially designed vector quantizer. The average bit-rate with adaptive vector quantization can be reduced by approximately two bits per frame compared to the non-adaptive case.

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7. REFERENCES

- [1] D. Petrinovic, "Transform Coding of LSF Parameters Using Wavelets", *International Conference on Spoken Language Processing*, Sydney, November 1998, Vol.6, pp. 2603-2606
- [2] K.K. Paliwal, B.S. Atal, "Efficient Vector Quantization of LPC Parameters at 24 bits/frame", *IEEE International Conference on Acoustics, Speech and Signal Processing*, 1991, pp. 661-664
- [3] R. Laroia, N. Phamdo, N. Farvardin, "Robust and Efficient Quantization of Speech LSP Parameters Using Structured Vector Quantizers", *IEEE International Conference on Acoustics, Speech and Signal Processing*, 1991, pp. 641-644
- [4] D. Chang, S. Ann, C.W. Lee, "A Classified Vector Quantization of LSF Parameters", *Signal Processing*, Vol. 59, No. 3, June 1997, pp. 267-273
- [5] W.R. Gardner, B.D. Rao, "Theoretical Analysis of the High-Rate Vector Quantization of LPC Parameters", *IEEE Trans. Speech Audio Processing*, Vol. 3, No. 5, Sep. 1995, pp. 367-381
- [6] A.Gerso, R.M.Gray, "*Vector Quantization and Signal Compression*", Kluwer Academic Publishers, Boston / Dordrecht / London 1997