

EFFECT OF ZEROS ON IIR FILTERS WITH SYMMETRIC IMPULSE RESPONSE

Mladen Vucic and Hrvoje Babic
Faculty of Electrical Engineering and Computing
Unska 3, Zagreb, HR10000, Croatia

ABSTRACT

The properties of the symmetric impulse response filters with real and complex zeros are considered. The pole-zero locations of the rational transfer functions are found, by minimization of the symmetry error. The contribution of zeros to the symmetry error is shown and compared to the systems with all zeros at the origin. The design procedure of such filters based on time or frequency domain requirements is outlined.

1 INTRODUCTION

In discrete time systems the finite impulse response (FIR) can be obtained, what enables the design of causal systems with a symmetric impulse response and an ideal linear phase. The realization of FIR filters typically requires quite a large number of memory locations as well as multiplications. In practical applications, however, an ideal linear phase system is not always necessary. This means that IIR systems with approximately symmetric impulse response or approximate linear phase will also have application, since they are computationally more efficient than FIR filters.

The finite order systems based on a design requirement in the time domain are, generally, different from those based on the frequency domain requirements. We have recently used the impulse response symmetry as a design criterion for IIR filters [1]. In this paper we will extend the optimization of the IIR filters by adding real and complex zeros to the transfer functions and investigate their influence to the system properties.

2 IMPULSE RESPONSE SYMMETRY

The discrete form of the impulse response symmetry error is given by

$$E_S = \sum_{n=S}^{\infty} [h(n) - h(2S-1-n)]^2, \quad (1)$$

where $h(n)$ is the filter impulse response. Symmetry line is placed between samples $S-1$ and S . The error E_S is normalized to the energy of the impulse response

$$E = \frac{E_S}{E_0}; \quad E_0 = \sum_{n=0}^{\infty} h^2(n). \quad (2)$$

The value E expressed by transfer function poles and zeros, is used as the optimization criterion [1].

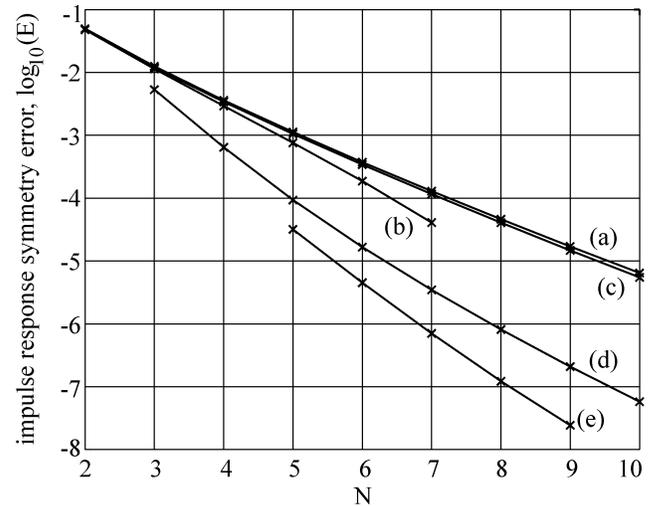


Figure 1. Symmetry error of optimized systems, $S=10$.

The real and imaginary part of poles, d_i , and zeros, c_k , were used as the goal function variables. Finally, the optimum poles and zeros were found from

$$\min_{c_k, d_i} E[c_k, d_i]. \quad (3)$$

A Quasi-Newton method with BFGS formula for Hessian matrix update was used for searching for the minimum [2].

The optimization procedure was performed from the third up to the tenth order transfer functions, $N=3$ to $N=10$, and the number of complex zeros, $M=2$ and $M=4$. The delay parameter was varied from $S=3$ to $S=30$ samples.

3 OPTIMUM SYSTEMS WITH COMPLEX ZEROS

The symmetry error for various numbers of zeros is shown in Figure 1. The curve (a) for all zeros at the origin and (b) $N-1$ real zeros are computed earlier and are presented in [1]. In this paper, the diagram is extended for the systems with (c) one real zero and (d) two and (e) four complex zeros.

The obtained symmetry errors are generally smaller for higher system orders and larger numbers of complex zeros, as shown in Figure 1. The largest increase of symmetry is obtained by (d) one complex pair, $M=2$. The addition of the second complex pair (e) is less efficient, but still useful. Adding more complex zeros would contribute little to the symmetry, so the increased realization complexity would not pay off. Therefore, we will consider only cases with $M=2$ and $M=4$.

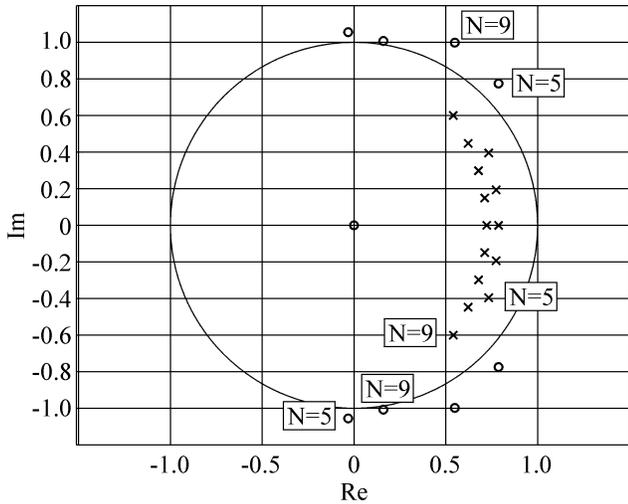


Figure 2. Pole and zero locations of the optimum system with four complex zeros, $S=10$.

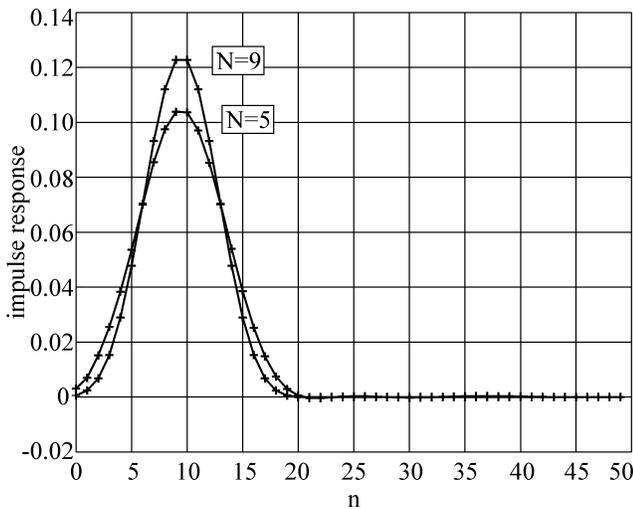


Figure 3. Impulse response of the optimum system with four complex zeros, $S=10$.

The symmetry of the impulse response is more improved by an additional pair of poles than by an additional pair of zeros for $N < 5$. It is more improved with an additional pair of zeros for $N \geq 5$, as one can conclude from Figure 1.

Contribution of one optimum real zero, (c), to the symmetry error is negligible, as well as the contribution of (b) the $N-1$ real zero.

For example, pole and zero positions for order $N=5$ and $N=9$ and four complex zeros are given in Figure 2. It is interesting to note that poles are very nearly equidistant in the frequency, what is typical for linear phase systems. Zeros are outside the unit circle.

The impulse responses of the optimum systems are quasi gaussian, Figure 3, with better symmetry and smaller ringing for larger system orders N , and number of complex zeros M . Delay of the response is $S-1/2$, while the length is practically $L=2S-1$.

The amplitude response in dB versus frequency in linear scale is given in Figure 4. The parabolic character of the

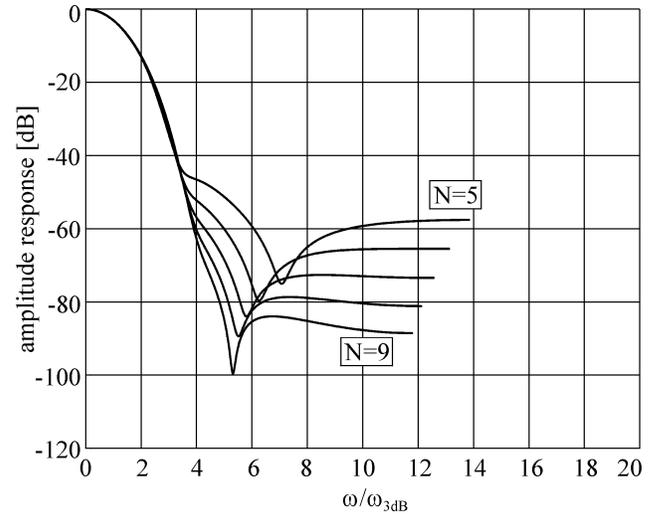


Figure 4. Amplitude responses of discrete time systems with four complex zeros, $S=10$.

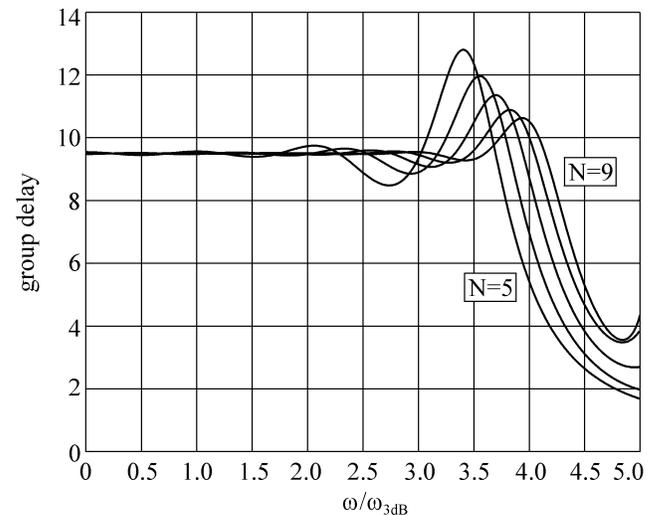


Figure 5. Group delay of discrete time systems with four complex zeros, $S=10$.

attenuation is obvious throughout the band up to $4\omega_{3dB}$. This means that the dominant part of the frequency response approximates gaussian response. In fact, the parabolic part is given by a simple equation

$$\alpha = -3 \left(\frac{\omega}{\omega_{3dB}} \right)^{2.2} \text{ dB}, \quad (4)$$

which is valid for systems with two and four complex zeros. The amplitude responses follow parabola in a wider band for larger system orders, as shown in Figure 4.

The group delay curves approximate a constant with a ripple, as shown in Figure 5. However, the ripple is not equal for a given N , but is increasing with frequency. Apparently, larger group delay error is tolerable in the frequency region where amplitude attenuation is high.

Properties of the obtained systems are generally similar to the continuous time systems with symmetric impulse response, presented in [3].

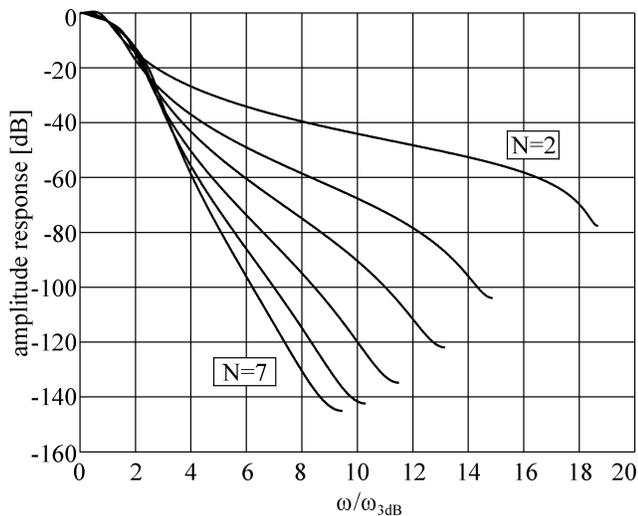


Figure 6. Amplitude responses of the optimum systems with $N-1$ real zeros, $S=10$.

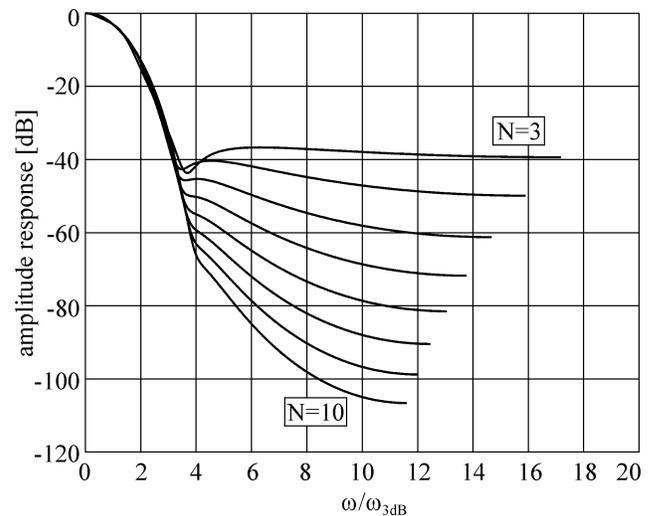


Figure 8. Amplitude responses of the optimum systems with two complex zeros, $S=10$.

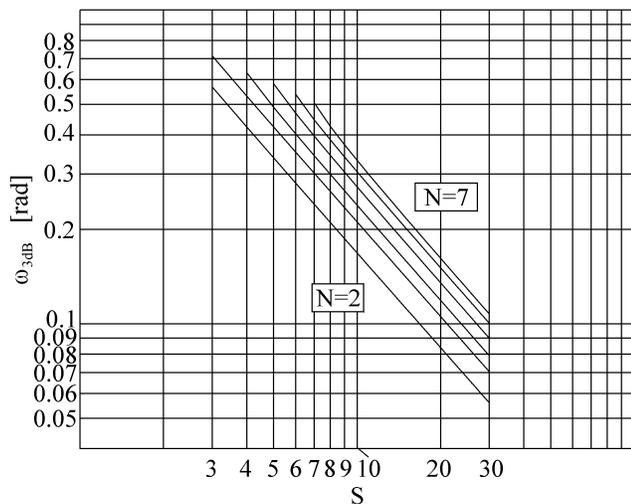


Figure 7. Cutoff frequencies ω_{3dB} of the optimum systems with $N-1$ real zeros.

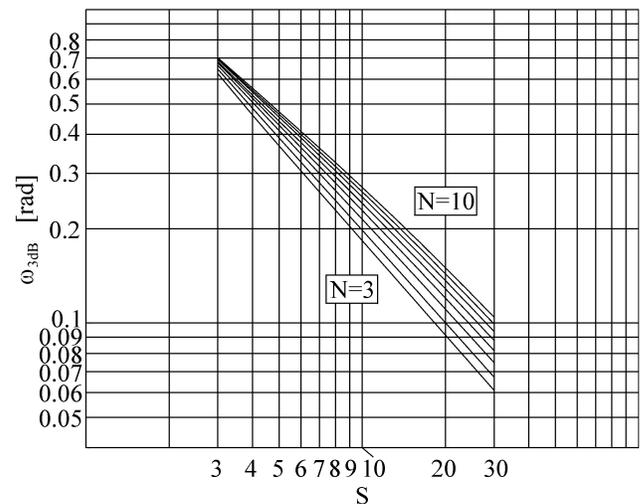


Figure 9. Cutoff frequencies ω_{3dB} of the optimum systems with two complex zeros.

4 FILTER DESIGN

Filter design is similar to the design of filters with all zeros at the origin, described in [1]. It can be based on either (i) tolerable symmetry error in the time domain, or (ii) attenuation in the frequency domain.

(i) System order, N , can be selected from the required symmetry error using appropriate curves in Figure 1. Symmetry line, S , can be chosen from the desired impulse response length $L=2S-1$. The resulting IIR filter is approximately equivalent to a gaussian FIR filter of the same length.

(ii) Filter design in the frequency domain can be based on attenuations α_p and α_s at passband ω_p and stopband ω_s . Required frequency ω_{3dB} can be calculated from the passband ω_p and attenuation α_p using (4).

The necessary system order N for the filter with $M=N-1$ real zeros can be found from Figure 6 and the required

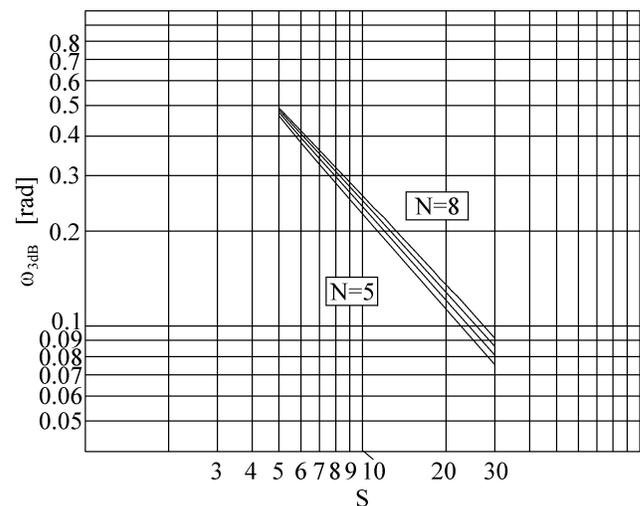


Figure 10. Cutoff frequencies ω_{3dB} of the optimum systems with four complex zeros.

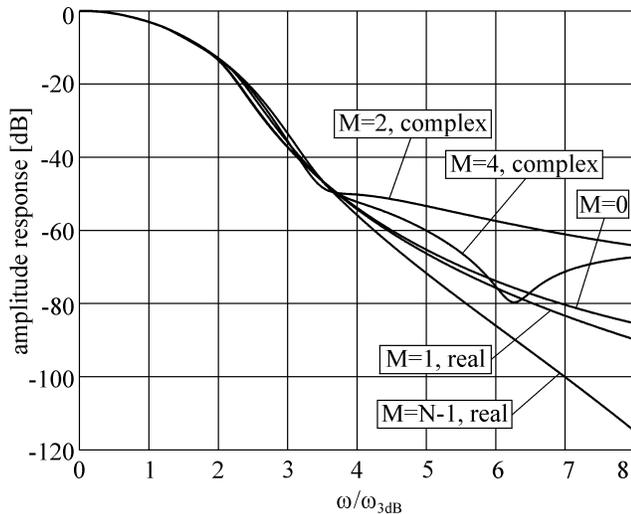


Figure 11. Amplitude response of optimum systems, with various zeros, $N=6$, $S=10$.

attenuation at the stop band frequency α_s . Although the figure shows a particular case where $S=10$, it can be used for a rough estimation of necessary system order N in the range $S=3$ to $S=30$.

The necessary S can be read out from the diagram in the Figure 7 using estimated N and calculated ω_{3dB} . The diagram was obtained by computing examples of the filters for various N and S . When parameters N and S are known, only one run of the optimization process (3) is usually sufficient to obtain the numerical values of poles and zeros.

Amplitude responses of filters with one and two pairs of complex zeros are shown in Figure 8 and Figure 4, respectively. Corresponding diagrams $\omega_{3dB}(N, S)$ are shown in Figure 9 and Figure 10. The procedure for estimation N and S is the same as described above.

It should be noted that the parabolic parts of the amplitude responses in Figure 4, Figure 6 and Figure 8 are not depended on the system order. In fact, the "tails" of the responses diverge from the parabolic part at various attenuations determined by N . Thus, the high stop band attenuations for these filters can be obtained by increasing N only when point (α_s, ω_s) is outside the parabolic part, i.e. outside the curve expressed by (4).

5 THE EFFECT OF ZEROS

The presence of zeros in the transfer function is generally useful. Being complex, they increase impulse response symmetry and reduce its ringing. In the frequency domain, they reduce the slope of the amplitude response and widen constant group delay bandwidth. For the illustration of their influence, the amplitude and group delay response of the filter with $N=6$ are shown in Figure 11 and Figure 12.

Optimum location of a single real zero is approximately at the point $-1.1+0j$. It is interesting that optimization of $N-1$ real zeros results with one zero with multiplicity $N-1$, located left of the point $-1+0j$, as shown in [1]. The real zeros show much lower influence to the impulse response symmetry as shown by curves (b) and (c) in Figure 1, than

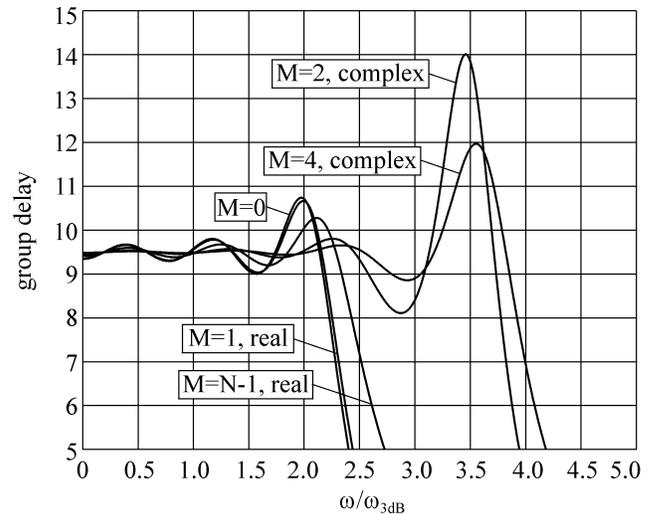


Figure 12. Group delay of optimum systems, with various zeros, $N=6$, $S=10$.

complex zeros shown by curves (d) and (e). The real zeros increase the amplitude response slope and the attenuation around Nyquist frequency, Figure 11.

Generally, the presence of zeros increases the realization complexity, compared to the system with all zeros at the origin. The optimum position for real zeros suggests the use of a zero, located at $z_r=-1+0j$, with multiplicity $M_r=N-1-M_z$, where M_z is the number of complex zeros. The realization of the factor

$$K = (z + 1)^{M_r} \quad (5)$$

of the transfer function will not require multipliers.

6 CONCLUSION

A method for optimization of IIR filters with symmetric impulse response was developed. The proposed error criterion produces the best symmetry of samples about the symmetry line, for a given system order. The properties of the transfer functions with real and complex zeros are analyzed. Two and four complex zeros are very useful for the increase of the impulse response symmetry, and improvement of the group delay response. The optimum real zeros influence the slope of the amplitude curves outside gaussian part and increase the attenuation near the Nyquist frequency.

7 REFERENCES

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