CRACK INITIATION LIFE AT COMBINED HCF/LCF LOADING

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Abstract: *Explicit expression is derived for components subjected to HCF loading, enabling initiation life assessment on the basis of the designed French curve and the Goodman plot in the Smith diagram. For components under combined HCF/LCF loading, the formula for determining the crack initiation life is derived and the closed form expression for estimating the crack propagation threshold is obtained.*

Keywords: *Crack propagation threshold; Initiation life; French curve; Goodman plot*

1. Introduction

The parts of high-speed engines, especially the turbine and compressor discs and blades, are subjected to the combined low cycle fatigue (LCF) and high cycle fatigue (HCF) loading. LCF stresses are actually the "steady" stresses, which result in one cycle for every start-up and shutdown operation [1], and HCF stresses are caused by in-service vibrations. Thus, the stress history consists of N_B stress blocks (one for each operation) with n_{HCF} HCF cycles and one LCF cycle (Fig. 1). Actually, such type of stress history is usual for all machine parts subjected to substantial load due to start-stop operations. Because of usually extremely high cyclic frequencies of in-service loading spectra, the fatigue life of e.g. 10^7 cycles can be reached in few hours. Consequently, the integrity of parts stressed in this way is critical, and a number of HCF failures had been detected till 1960's, and a number of LCF failures in 1970's, especially in US fighter engines [1]. Whereas in HCF region crack propagation life is very short compared to crack initiation life, i. e. almost the entire fatigue life consists of the crack initiation life, it is recommended that stresses, for the safety reasons, should not exceed the crack propagation threshold (CPT). For many other components, like high-pressure vessels, crack propagation is not allowed even in the LCF region [2], and it is necessary to determine CPT curve, the stress level of which should not be exceeded. It means that stresses at the level of CPT should represent a novel fatigue limit, and such approach to design might be called damage intolerant design.

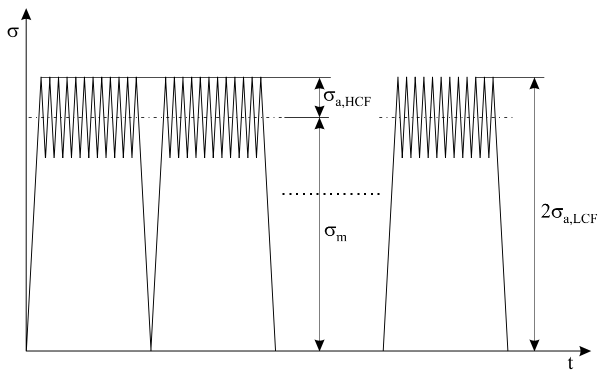

Fig. 1. Common stress history of the turbines and compressors discs and blades

2. Crack propagation threshold and crack initiation life for HCF loading

When French established his well-known curve [3], and suggested distribution of σ - N diagram field in three regions - the region of damage between Wöhler curve and French curve, the region of failure on the right of the Wöhler curve, and the region of "overload" on the left of the French curve - he didn't know that he actually plotted the CPT, a years before the Fracture Mechanics was established. The original French procedure of testing consists of cyclic loading that is stopped after a previously determined number of cycles, and after continued at the endurance limit level, or slightly bellow it. If the specimen is fractured after sufficiently long number of cycles, it means that the specimen had been damaged (i.e. cracked) in previous loading. Thus, the unfractered specimens had not been damaged. All the tests resulting in initial crack and all the tests resulting in uncracked specimen, represented by corresponding points, are separated by French curve. In strain approach to fatigue design, more suitable to LCF loading, those points are distributed by corresponding CPT curve in log N - log ε diagram. Recently, the French procedure is simplified, because the crack initiation is perceived by modern devices, but the name of French is not more in use [4,1]. In the region of the finite fatigue life, clasping the fatigue lives between the boundary of quasi-static failure N_q and the boundary of the infinite fatigue life region, this curve is well described by the Wöhler type equation [4,1]

$$N_i \sigma^{m_i} = C_i , \tag{1}$$

where N_i is the crack initiation life for the certain stress level σ, and m_i and C_i are material constants.

At steady loading (N = 1/4), the CPT equals the ultimate strength σ_U, and for the sufficiently long fatigue life, which can be taken e.g. N_{gr}, it equals the endurance limit σ_0, meaning the entire fatigue life consists of the crack initiation life.

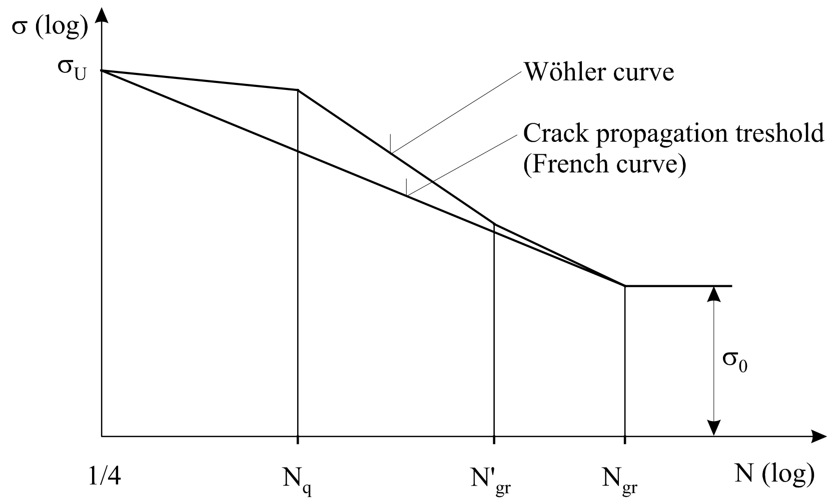

Fig. 2. Approximate design of the French curve

It is reasonable to assume that there is a unique CPT curve between these two points, because the crack initiation mechanism is similar for both HCF and LCF loading. Therefore, the curve of the CPT could be represented in the log σ - log N diagram (Fig.

2.), with the straight line passing the points (1/4; σ_U) and (N_{gr}; σ_0). It is easy now to calculate the slope of this curve:

$$m_i = \frac{\log(4N_{gr})}{\log(\sigma_U / \sigma_0)} \tag{2}$$

This expresion is found to be in good correllation with experimentaly obtained values. For example, the fatigue strength exponent b of steel 42 Cr Mo 4V (after DIN) for initiation life at r = -1 loading, was found to equal 0,0692 [5], meaning $m_i = 1/b$ = 14,5. Exactly the same value was obtained after Eqn. (2) for $N_{gr} = 3\cdot10^7$.

In the same way as the Wöhler curves for various stress ratios are used for designing the fatigue strength plots in Smith diagram, the French curve is used in order to obtain the CPT curve for any stress ratio r, in the same diagram. So, the Goodman plot of the fatigue strength is N_f = const plot in the same time, and Goodman CPT plot is in the same time N_i = const plot, and exhibits also the boundary of crack initiation for any r. Just like the most frequently used fatigue strength plot is Goodman straight line, the best approximation of the CPT is the Goodman plot again, which is also the straight line (Fig.3), passing the points ($\sigma_{0N,i}/2$; $\sigma_{0N,i}$) and (σ_U; σ_U). Indeed, Nicholas and Zuiker [1] declare that this plot is the straight line by definition. Thus, any straight line of the certain slope in Smith diagram passing the point (σ_U; σ_U) is the constant initiation life plot and in the same time the constant fatigue life plot. Of course, these

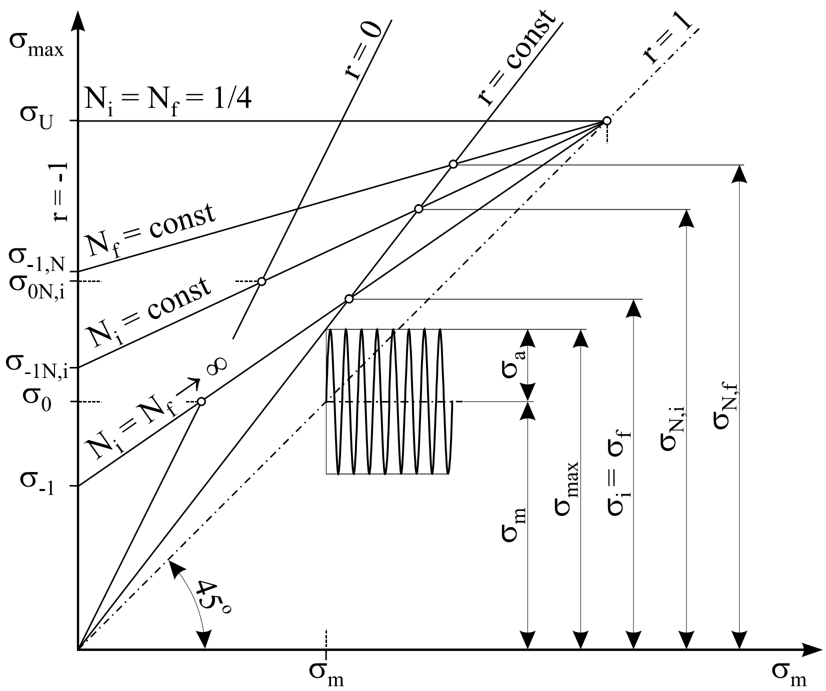

Fig. 3. Crack propagation thresholds in Smith diagram

lives are different. Therefore, for the same lives $N_i = N_f$, these plots are different (Fig. 3.). Whereas the N_i = const curve is the straight line passing the point (σ_U; σ_U), it is enough to know only the one point more to determine it. For any $N_q < N_i \leq N_{gr}$, this point is obtained from the (only one) French curve (1), usually for the stress ratio r = -1 or r = 0. For the purpose of this paper, the French curve at r = 0 is used, which enables determining the level of the pulsating stress at the CPT for certain N_i, by knowing the crack initiation life N_{gr} at the endurance limit level:

$$\sigma_{0N,i} = \sigma_0 \left(N_{gr} / N_i\right)^{1/m_i} \tag{3}$$

Whereas the maximum stresses in Smith diagram change along the load line r = const, the fatigue limit for a certain stress ratio is determined as the ordinate of the intersection point between the fatigue limit plot N_f = const and load line [6, 7, 8], the maximum value of the stress at the CPT, for the same stress ratio, is analogously obtained as the intersection point between the crack initiation plot N_i = const and the load line. At common circumstances, when no pre-load stress is applied, the equation of the load line is

$$\sigma_{\max} = k\sigma_m , \tag{4}$$

where $k = 2/(1+r)$ is the slope of the load line. The equation of the Goodman plot for the crack initiation life N_i = const is

$$\sigma_{N,i} = \sigma_{0N,i} + k_\sigma (\sigma_m - \sigma_{0N,i} / 2), \tag{5}$$

where $k_\sigma = (\sigma_U - \sigma_{0N,,i})/(\sigma_U - \sigma_{0N,i}/2)$ is its slope. The maximum stress at the CPT for the arbitrary stress ratio r is now obtained

$$\sigma_{N,i} = \frac{2 - k_\sigma}{2 - k_\sigma (1 + r)} \sigma_{0N,i} , \tag{6}$$

which might represent a novel fatigue limit for the high frequency HCF loading at arbitrary stress ratio. The safety margin against crack propagation is determined as

$$s = \sigma_{N,i} / \sigma_{\max} . \tag{7}$$

By means of the French curve equation, it is easy now to obtain the crack initiation life for the maximum stress that reaches the $\sigma_{N,,i}$ stress:

$$N_i = N_{gr} \left(\frac{2 - k_\sigma}{2 - k_\sigma (1 + r)} \frac{\sigma_0}{\sigma_{N,i}} \right)^{m_i} . \tag{8}$$

For the maximum stress σ_{max} (Fig. 3.), it is obtained the crack initiation life

$$N_i^{'} = N_i \cdot s^{m_i} , \tag{8a}$$

meaning the safety margin against the crack initiation life is

$$s_N = N_i^{'} / N_i = s^{m_i} . \tag{9}$$

In the region of LCF, the fatigue strength and fatigue life are determined by cyclic strains, not by stresses. Therefore, the relations obtained in that region, are only the approximations. For more precise calculations of fatigue life in such a case, it is necessary:

- Instead the conventional values of stresses and strength, to calculate with real one

$$\sigma^* = \sigma(1 + \varepsilon) , \tag{10}$$

and also with real deformations

$$\varepsilon_r = \ln(1+\varepsilon)\,. \tag{11}$$

- Instead with ultimate strength σ_U, to calculate with fatigue strength coefficcient σ'_f, which has to be additionaly decreased by the factor of mean stress sensitivity (equals 1,1…1,5), or to increse the mean stress for the same amount [9].

It is clear that N_i represents the mean value if all other random variables taking part in it are represented with their mean values. The standard deviation of N_i can be approximated by the widely known Gaussian approximation formula.

3. Crack propagation threshold and crack initiation life for combined HCF/LCF loading

For the stress history described in Fig. 1., the crack initiation life expressed in number of stress blocks N_B is derived on the basis of Palmgren - Miner hypothesis of linear damage accumulation, where the level of damage is defined as

$$D = \sum_{j=1}^{n_B} \frac{n_j}{N_j} = \sum_{j=1}^{n_B} \left(\frac{n_{HCF,j}}{N_{HCF,j}} + \frac{1}{N_{LCF,j}} \right). \tag{12}$$

The CPT is reached for $D = 1$, when number of blocks n_B becomes $N_{B,i}$:

$$\sum_{j=1}^{N_{B,i}} \left(\frac{n_{HCF,j}}{N_{HCF,j}} + \frac{1}{N_{LCF,j}} \right) = N_B \left(\frac{n_{HCF}}{N_{HCF,i}} + \frac{1}{N_{LCF,i}} \right) = 1\,, \tag{13}$$

where from follows the crack initiation life expressed in stress blocks [1]:

$$N_{B,i} = \frac{1}{\dfrac{n_{HCF}}{N_{HCF,i}} + \dfrac{1}{N_{LCF,i}}}\,. \tag{14}$$

It is easy now to obtain the total initiation life:

$$N_i = N_{B,i}(1+n_{HCF}) \cong N_{B,i} \cdot n_{HCF} = \frac{1}{\dfrac{1}{N_{HCF,i}} + \dfrac{1}{N_{LCF,i} \cdot n_{HCF}}}\,. \tag{15}$$

The initiation life $N_{LCF,i}$ is obtained after the French curve (3) at $r = 0$:

$$N_{LCF} = N_{gr} \left(\sigma_0 / \sigma_{\max} \right)^{m_i} \tag{16}$$

This equation is also used for the calculation of the HCF initiation life by substituting in it an equivalent stress amplitude obtained by reducing a HCF stress amplitude (with stress ratio $r_{HCF} > 0$) to an equivalent HCF stress amplitude at $r = 0$ (Fig. 4). It is obtained after Fig. 4 the value of the reduced stress amplitude:

$$\sigma_{a,eq} = \frac{1}{\dfrac{1}{\sigma_U} + \dfrac{1}{\sigma_a} - \dfrac{\sigma_m}{\sigma_a \sigma_U}}\,. \tag{17}$$

By substituting Eqn. 16. in Eqn. 15 twice (for a LCF stress σ_{max} and for a reduced HCF stress after Eqn. 17.), the explicit formula is obtained for determining the crack initiation life at combined HCF/LCF loading:

$$N_i = \frac{N_{gr}\sigma_0^{m_i}}{\dfrac{1}{\left(\dfrac{1}{\sigma_U}+\dfrac{1}{\sigma_a}-\dfrac{\sigma_m}{\sigma_a\sigma_U}\right)^{m_i}}+\dfrac{\sigma_{\max}^{m_i}}{n_{HCF}}} \tag{18}$$

It is interesting that the same expression is derived by the constant amplitude equivalent stress approach, established on the basis of experimental investigations of Yamada and Albrecht [10].

It is not possible to get an explicit solution for the CPT curve $\sigma_{max} = \sigma_{N,i} = f(\sigma_m)$ for a certain value of initiation life, from Eqn.(15). Therefore, a simple computer program is made for determining those constant initiation life curves for a number of materials, including the titanium alloy Ti-6Al-4V, frequently used for the aircraft components. For input data as the ultimate strength σ_U, endurance limit σ_0, and material constants m_i and N_{gr}, by means of equations presented above, the points of the CPT curves are obtained, indicating the stress states in Smith diagram, that produce crack initiation after $N_i = C$ cycles. The calculations were carried out for the various values of C, and for total number of stress blocks to initiation $N_{B,i} = 10^2 \ldots 10^5$.

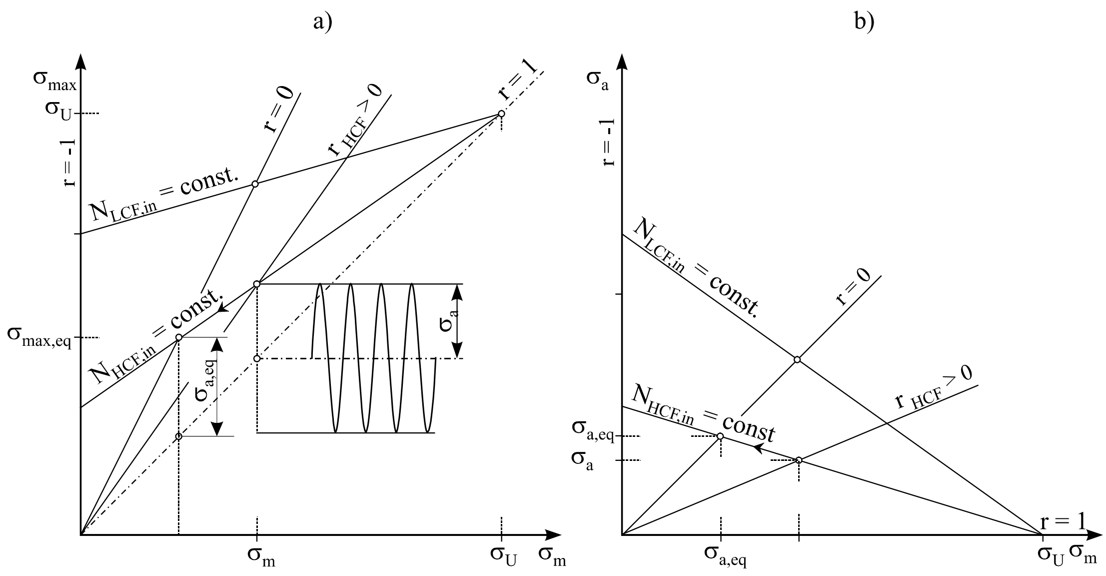

Fig. 4 The plots of the CPT for HCF and LCF loading in a) Smith diagram, and b) Haigh diagram, and reducing the HCF stress amplitude σ_a to an equivalent stress amplitude $\sigma_{a,eq}$ at $r = 0$.

The resulting CPT curves for titanium alloy Ti-6Al-4V, $C = 10^7$ and for $N_{B,i} = 10^2$, 10^3, 10^4 and 10^5, are exhibited in Smith diagram, Fig. 5. In view of these curves for any metallic material, it was observed that the presence of the LCF component restricts the safe design space compared to that under pure HCF, as more as the share of LCF component is greater. They can be well approximated with parabolas

$$\sigma_{N,i} = b - a(\sigma_m - b)^2, \tag{19}$$

whose point of maximum is determined with

$$b = 1{,}2\sigma_{0N,i} + (\sigma_U - 1{,}2\sigma_{0N,i})(1 - \log N_{B,i}/\log N_{gr})^{1,4}. \tag{20}$$

The constant a is derived on the basis of condition that Goodman line is a tangent on parabola:

$$a = \frac{1}{4}\frac{k_\sigma^2}{(1 - 0{,}5k_\sigma)\sigma_{0N,i} + b(4k_\sigma - 1)}. \tag{21}$$

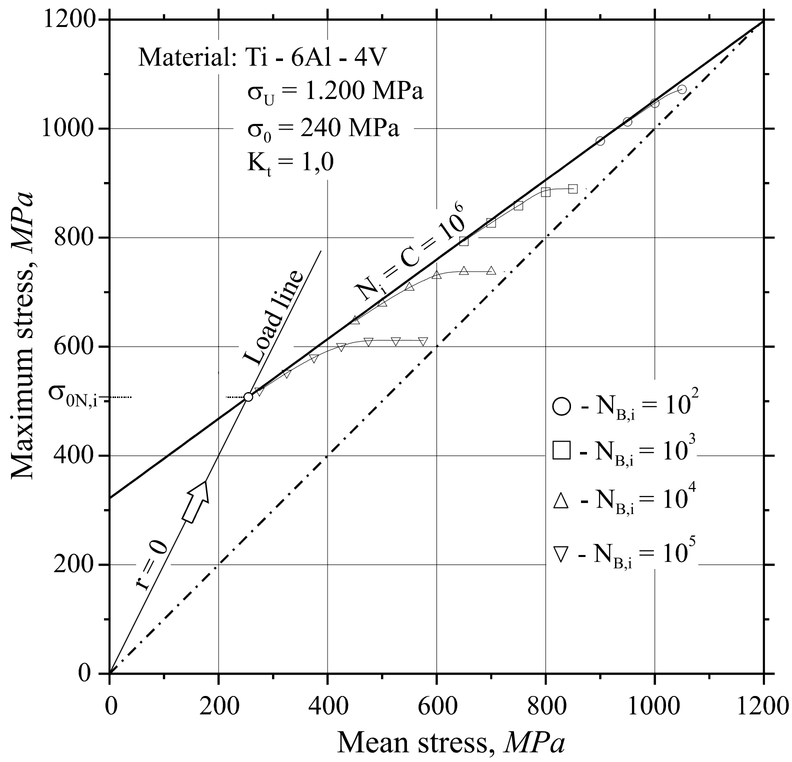

Fig. 5. Crack propagation thresholds obtained in Smith diagram for a combined HCF/LCF loading of a titanium alloy Ti-6Al-4V

For the mean stress values equal the boundary value

$$\sigma_{m,b} = b - k_\sigma/(2a) \tag{22}$$

the CPT parabolas become one with corresponding Goodman plot without any discontinuity, and for mean stresses less than $\sigma_{m,b}$, the CPT curve $N_i = C$ becomes a Goodman plot. Because the load line of HCF/LCF loading is that of LCF loading, its belonging stress ratio is $r = r_{LCF} = 0$. Its intersection point with Goodman plot is the ($\sigma_{0N,i}/2$; $\sigma_{0N,i}$) point. It means that for any $\sigma_{m,b} < \sigma_{0N,i}/2$, the CPT is the mentioned parabola, and for $\sigma_{m,b} \geq \sigma_{0N,i}/2$, the CPT is a Goodman plot. In the first case the maximum stress at the level of the CPT is the intersection point of the parabola Eqn. (18) and load line, obtained as

$$\sigma_{N,i} = \frac{2}{a}\left(ab - 1 + \sqrt{1 - ab}\right), \tag{23}$$

which is always greater then $\sigma_{0N,i}$, and represents a fatigue limit when no crack propagation is allowed. It is obviously the function depending on $\sigma_{0N,i}$ only. When CPT is a Goodman plot, the boundary value of maximum stress is $\sigma_{N,i} = \sigma_{0N,i}$.

4. Conclusion

Smith's diagram as a tool for estimating both the stress states at the level of the crack propagation treshold and crack initiation life, is presented in this paper. For a HCF loading at arbitrary stress ratio, the unique and explicit relations are derived, which enable estimation of the initiation life, by means of the French curve only, which one can be approximated by means of the Wöhler curve.

For the combined LCF/HCF loading, the procedure is explained, the curves of the crack propagation treshold in Smith diagram are obtained, and the formulae are derived for estimating the stresses at the crack propagation treshold level. For the components where the crack propagation is not allowed, the crack propagation treshold curves represent a fatigue limit, enabling the prediction of the crack initiation life of the part.

For more reliable design against crack propagation, presented herein, it should be necessary to pay the special attention to the way of clasping all the differences in geometry and service conditions between the specimens used to obtain the French curve and the real part.

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