Visualizations of Rose-Surfaces

Sonja Gorjanc (sgorjanc@grad.hr)
Faculty of Civil Engineering, University of Zagreb

Roses

Roses or *rhodonea* curves $R(n,d)$, treated here, can be expressed by the polar equations $r = \cos\frac{n}{d}\varphi$ or $r = \sin\frac{n}{d}\varphi$, where $n,\ d \in \mathbb{N}$ and $GCD(n,d) = 1$.

$n\cdot d$	order of $R(n,d)$	point O	number of double points	period	number of petals
odd	$n+d$	n-ple	$\frac{1}{2}n(d-1)$	$d\cdot\pi$	n
even	$2(n+d)$	$2n$-ple	$2n(d-1)$	$2d\cdot\pi$	$2n$

Table: Some properties of $R(n,d)$ according to [3, pp. 358-369].

According to [1], the implicit equation of $R(n,d)$ is the following:

$$\Big(\sum_{k=0}^{\lfloor d/2\rfloor}\sum_{j=0}^{k}(-1)^{k+j}\binom{d}{2k}\binom{k}{j}(x^2+y^2)^{\frac{n+d}{2}-k+j}\Big)^s - \Big(\sum_{k=0}^{\lfloor n/2\rfloor}(-1)^k\binom{n}{2k}x^{n-2k}y^{2k}\Big)^s = 0.$$

where $s = 1$ for $n\cdot d$ odd, and $s = 2$ for $n\cdot d$ even.

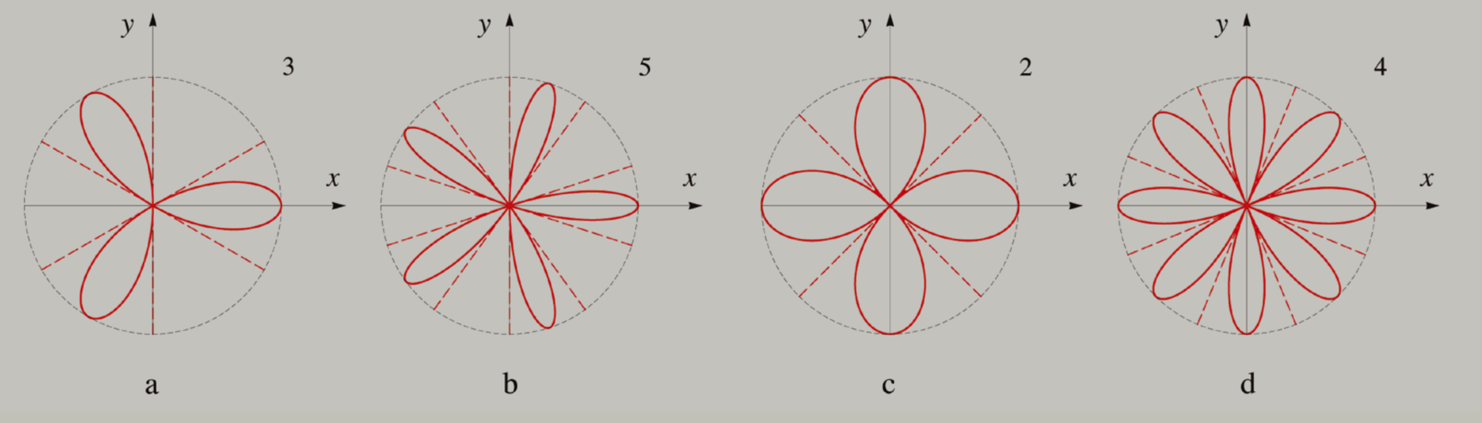


Figure: If n is odd, the rose $R(n,1)$ is $n-$petalled curve with n tangent lines at the origin (Fig. a and b). If n is even, the rose $R(n,1)$ is $2n-$petalled curve with n double tangent lines at the origin (Fig. c and d).

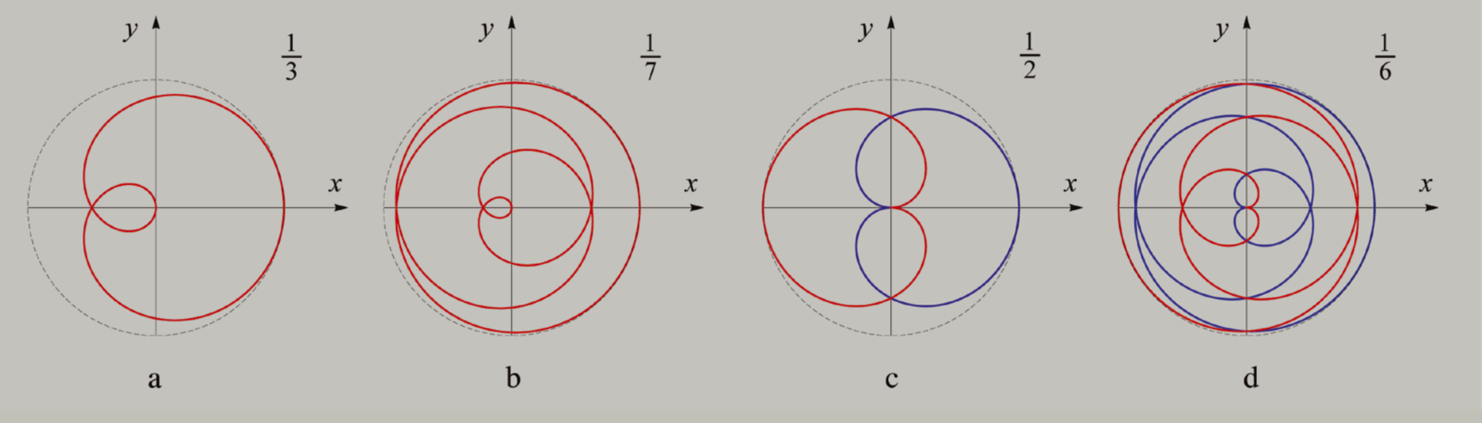


Figure: If d is odd, the rose $R(1,d)$ has only one petal (Fig. a and b). If d is even, the rose $R(1,d)$ has two petals (Fig. c and d).

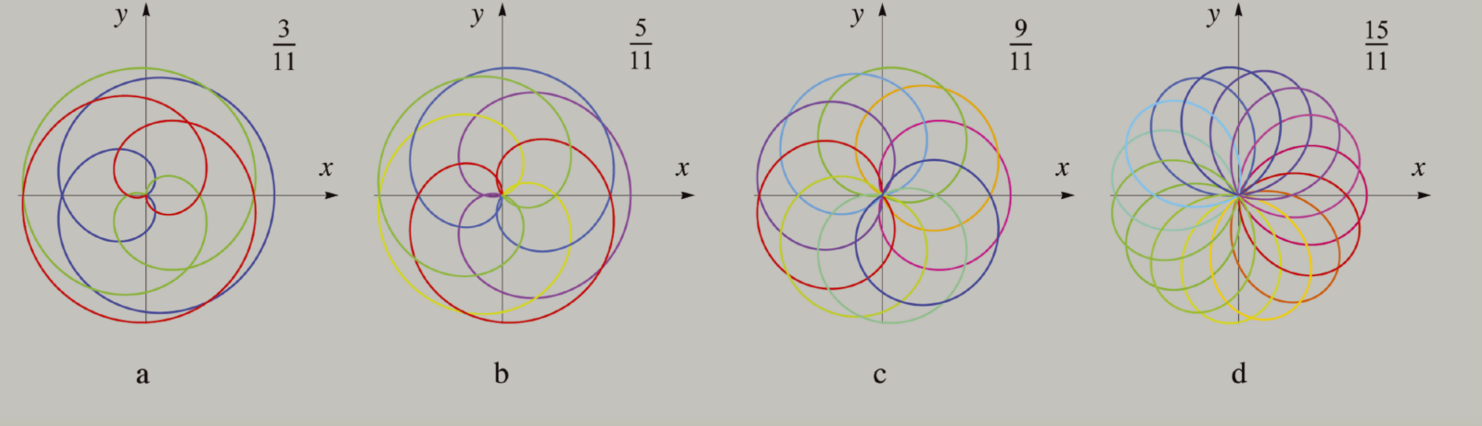
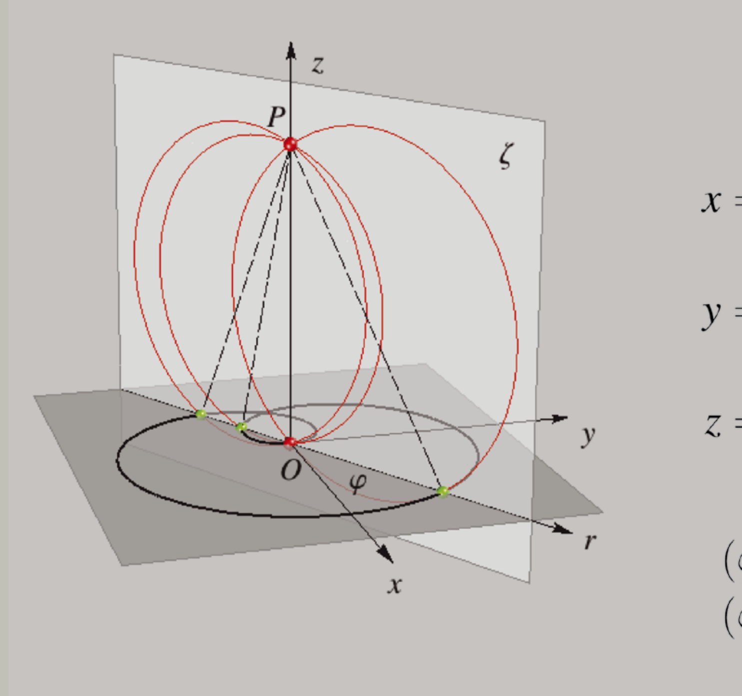


Figure: Four roses with petals coloured by different colors.

Rose-Surfaces

Definition

Let $P(0,0,p)$ be any point on the axis z and let $R(n,d)$ be a rose in the plane $z=0$. A *rose-surface* $\mathcal{R}(n,d,p)$ is the system of circles c_i which lie in the planes ζ through the axis z with diameters $\overline{PR_i}$, where $R_i \neq O$ are the intersection points of the rose $R(n,d)$ and the plane ζ.



Parametric equations of $\mathcal{R}(n,d,p)$

$$x = \frac{1}{2}\cos\varphi(\cos\frac{n}{d}\varphi + \sqrt{p^2+\cos^2\frac{n}{d}\varphi}\,\sin\theta)$$
$$y = \frac{1}{2}\sin\varphi(\cos\frac{n}{d}\varphi + \sqrt{p^2+\cos^2\frac{n}{d}\varphi}\,\sin\theta)$$
$$z = \frac{1}{2}(p + \sqrt{p^2+\cos^2\frac{n}{d}\varphi}\,\cos\theta),$$

$(\varphi,\theta) \in [0, d\cdot\pi] \times [0,2\pi]$ for $n\cdot d$ odd,
$(\varphi,\theta)) \in [0, 2d\cdot\pi] \times [0,2\pi]$ for $n\cdot d$ even.

According to [1], the implicit equation of $\mathcal{R}(n,d,p)$ can be written in the following form:

$$(x^2+y^2+z^2-p\cdot z)^{s\cdot d} = (x^2+y^2)^{\frac{s(d-n)}{2}}\Big(\sum_{k=0}^{\infty}(-1)^k\binom{\frac{n}{d}}{2k}x^{\frac{n}{d}-2k}y^{2k}\Big)^{s\cdot d}$$

where $s = 1$ for $n\cdot d$ odd, and $s = 2$ for $n\cdot d$ even.

$n\cdot d$		order	axis z	points O and P	double circles
odd	$d<n$	$n+d$	$(n-d)$-ple	n-ple	$\frac{1}{2}n(d-1)$
odd	$d>n$	$2d$	0-ple	d-ple	$\frac{1}{2}n(d-1)$
even	$d<n$	$2(n+d)$	$2(n-d)$-ple	$2n$-ple	$n(2d-1)$
even	$d>n$	$4d$	0-ple	$2d$-ple	$n(2d-1)$

Table: Some algebraic properties of $\mathcal{R}(n,d,p)$ according to [1] and [2, p251]

The tangent cones at O and P are proper for $P \neq O$, and split into planes for $P = O$.

Examples

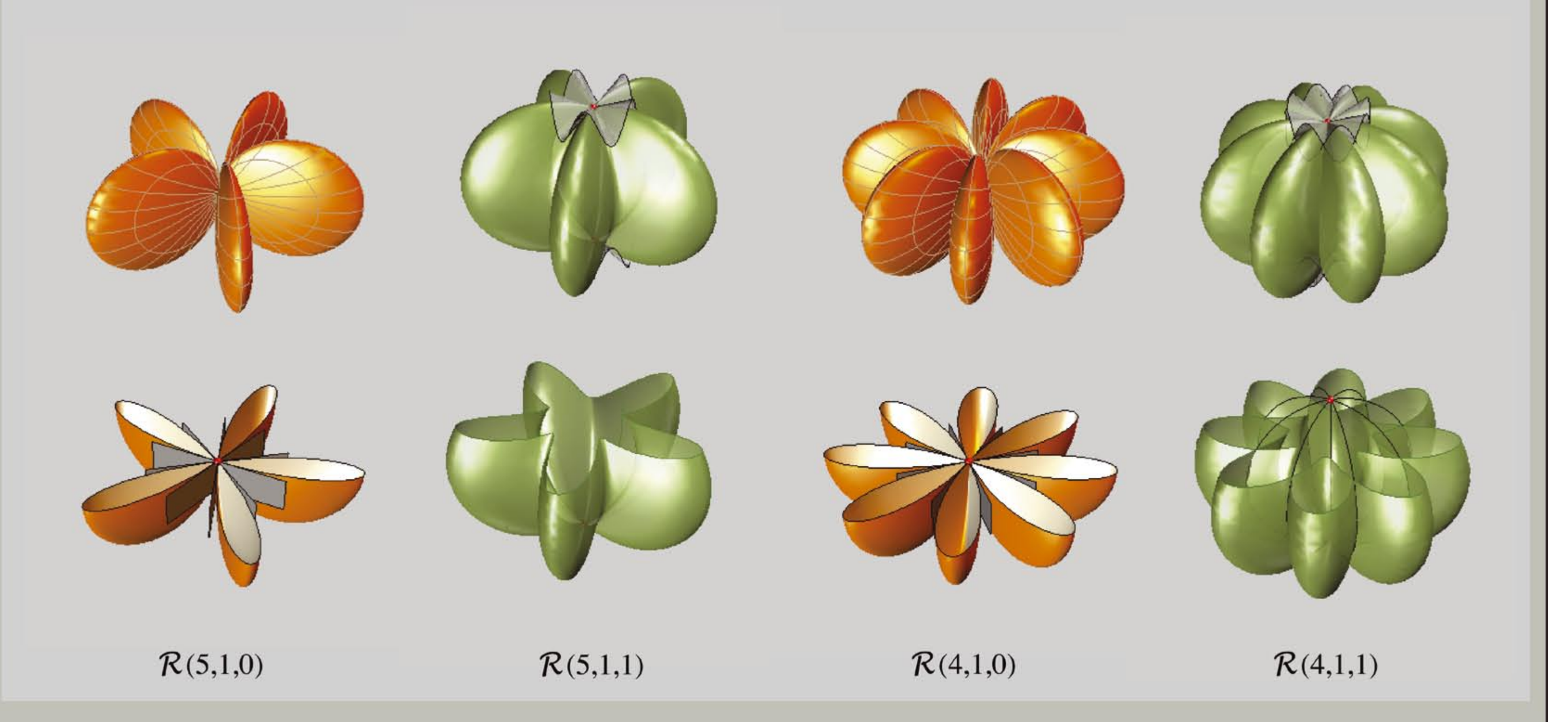


Figure: Four rose-surfaces with the tangent cones at the multiple points P and O

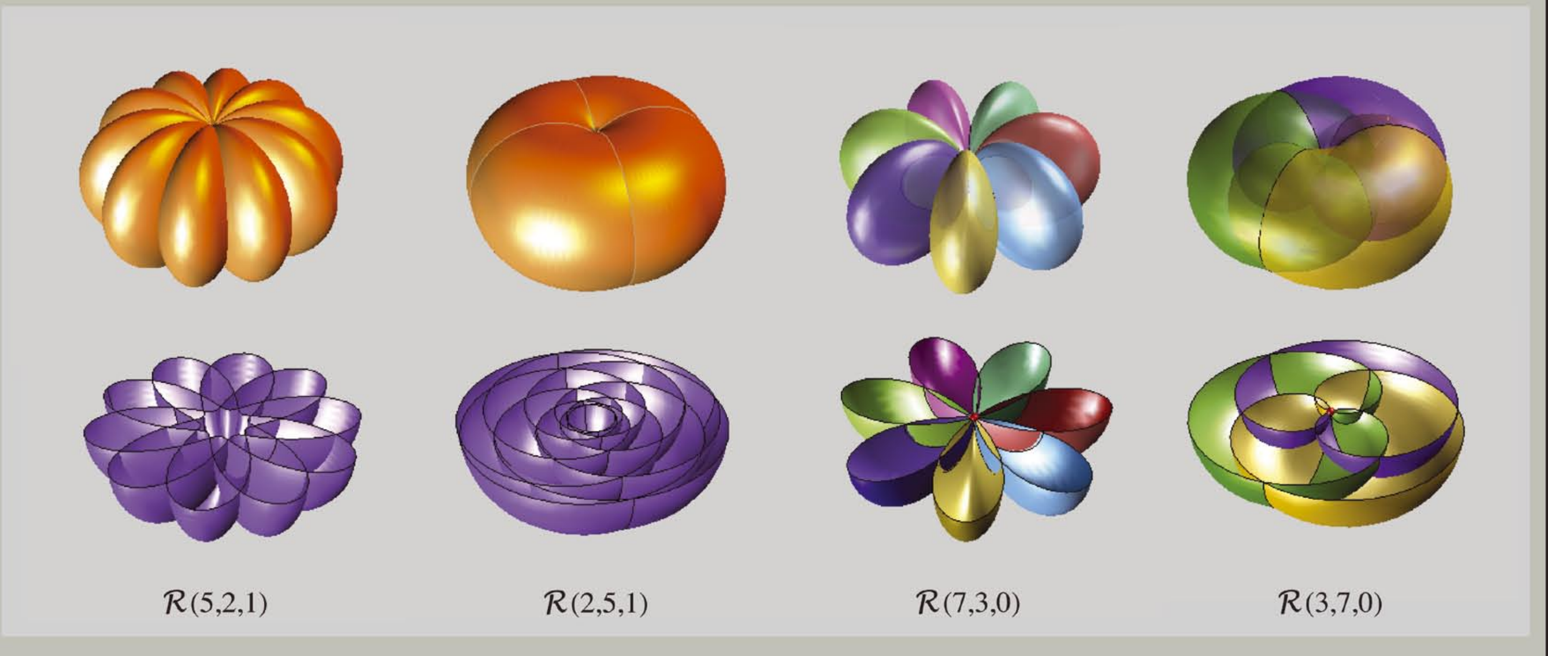


Figure: Four rose-surfaces.

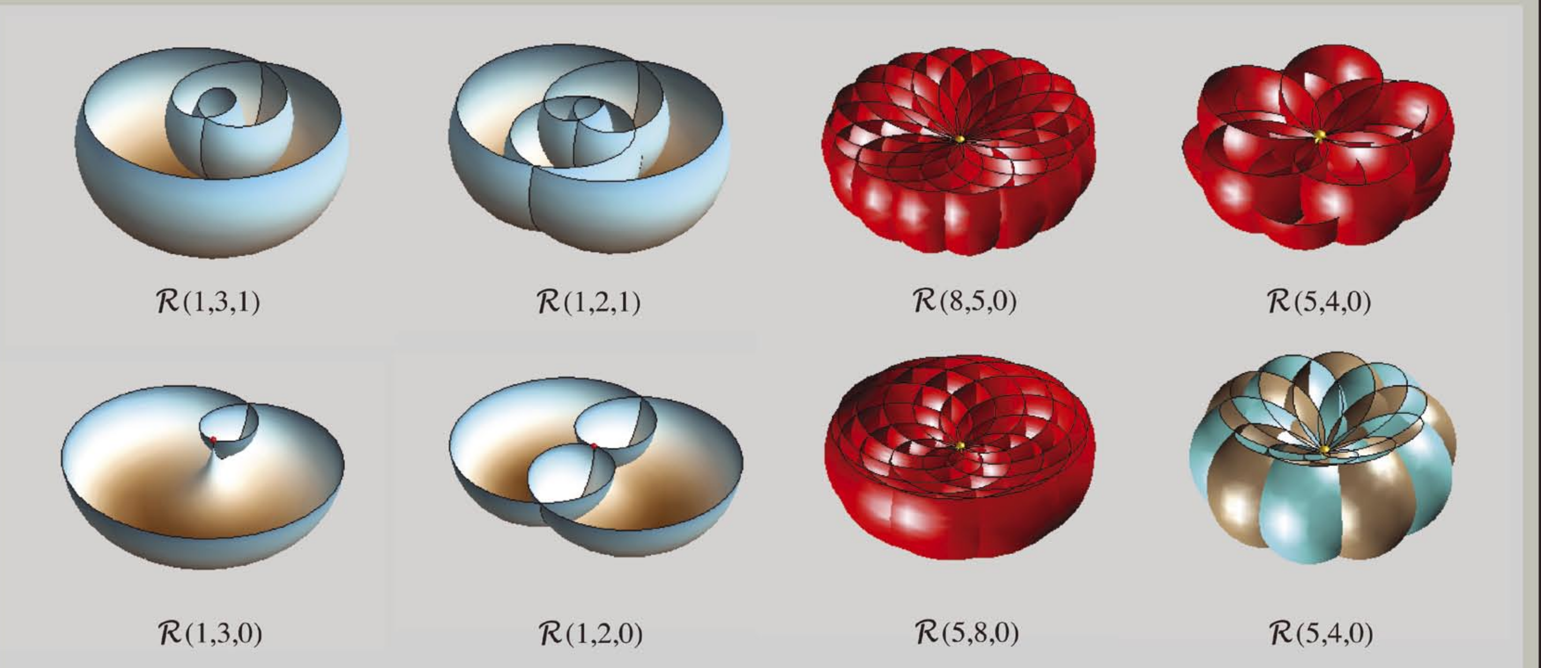


Figure: Seven rose-surfaces.

References

- Gorjanc, S.: *Rose-Surfaces*. (manuscript)
- Harris, J.: *Algebraic Geometry*. Springer, New York, 1995.
- Loria, G.: *Spezielle algebraische und transzendente ebene Kurven*. B. G. Teubner, Leipzig-Berlin, 1910.

Figures are made in the program Mathematica 7.