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# DAMAGE DETECTION FROM ANALYSIS OF DISPLACEMENT INFLUENCE LINES

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Abstract: Structural damage detection of a bridge structure using displacement influence lines and displacement influence surfaces is presented in this paper. Numerical testing is carried out for beam and plate-like bridge structures. Imperfections in a structure are related to changes in the static response of the structure. Static methods are used as they are more precise than modal testing, and therefore more attractive than the dynamic methods. Furthermore, displacement influence line can be obtained from measurements at only one point in the structure. In this work, the software "DARK" and "PLOCA" are used (author I. Kožar) for modelling intact and thevarious damaged cases. The displacement influence lines and influence surfaces are calculated for both of these cases and the central difference approximation is used to derive the curvatures from the displacement influence lines and influence surfaces. By plotting the difference in curvature of the displacement influence line or surfaces between the intact and the damaged case, a peak appears at the damaged elements.

The aim of this work is to find out a minimal number and the optimal location of the measurement points in the bridge structure that can enable locating the damages reliably.

## 1. INTRODUCTION

Nondestructive damage detection (NDD) is an important subproblem of damage assessment and should form the basis of any decision to repair, rehabilitate, or replace a structure. For critical structural systems such as aircrafts, bridges, and offshore platforms, an accurate and reliable NDD capability of the structural analysis is essential, since damage that is not detected and not repaired may lead to catastrophic structural failure.

In recent years, significant efforts have been devoted to developing nondestructive techniques for damage identification in structures.

In a typical load-bearing structure, degradation of structural properties because of damage manifests itself as a change in static and dynamic response.

Generally, existing damage identification methods can be classified into two major categories: the dynamic identification methods using dynamic test data, and the static identification methods using static test data. Both techniques are based on the correlation between two measured responses or comparison of the measured response to that obtained from an analytical model of the undamaged structure. The dynamic identification techniques have been developed more fully compared with the static methods.

There are several limitations with dynamic identification approach. First, the dynamic identification methods require the use of mass, stiffness and damping properties. On the other hand, the static methods only require the stiffness properties. Secondly, with dynamic methods, an adequate control of the excitation (including the elimination of spurious excitations) is essential for precise mode-shape measurement, and this can be difficult to achieve on site<sup>1</sup>. The influence of boundary conditions also has a significant effect on measured vibration frequencies and mode shapes<sup>2</sup>. Furthermore, higher modes are difficult to determine and measure and a large number of measurement points or measured frequencies are necessary to ensure reliability of the damage assessment<sup>3</sup>. The structural response measured in static tests is more precise than the structural response measured in modal testing<sup>2</sup>. In comparing the effect of measurement errors on results of damage detection, methods utilizing static test data are therefore expected to yield more reliable results than dynamic methods<sup>4</sup>. The main problem in static test is incomplete static displacement information due to a limited number of the measurement instruments.

### 2. DESCRIPTION OF THE DAMAGE DETECTION METHOD FROM DISPLACEMENT INFLUENCE LINES AND DISPLACEMENT INFLUENCE SURFACES

The displacement influence line and the displacement influence surface can be obtained from measurements or can be calculated, for which it is sufficient to take only one point in the structure.

Suppose we have two sets of the displacement influence lines or the displacement influence surfaces for two states of the structure; the first state is undamaged and the second is damaged state.

### Plate-like structures

 $\eta_w(x, y) = w(x, y)$  is the displacement influence surface for the undamaged state,

 $\overline{\eta_w}(x,y) = \overline{w}(x,y)$  is the displacement influence surface for the damaged state.

The difference between the displacement influence surfaces for two states is represented by:  $R(x, y) = \overline{w(x, y)} - w(x, y)$ (1)

Equation R(x, y) = 0 shows that the two states of the structure are identical.

When  $R(x, y) \neq 0$ , there is a difference in the displacement influence surfaces which points to changes in structural properties of the structure.

We assume that the system is geometrically and materially linear.

Then, the displacement influence surface curvatures can be written as

$$\rho_{xx}(x,y) = \frac{\partial^2 w(x,y)}{\partial x^2}, \quad \rho_{yy}(x,y) = \frac{\partial^2 w(x,y)}{\partial y^2}, \quad \rho_{xy}(x,y) = \frac{\partial^2 w(x,y)}{\partial x \partial y}$$
(2)

$$\overline{\rho}_{xx}(x,y) = \frac{\partial^2 \overline{w}(x,y)}{\partial x^2} , \quad \overline{\rho}_{yy}(x,y) = \frac{\partial^2 \overline{w}(x,y)}{\partial y^2} , \quad \overline{\rho}_{xy}(x,y) = \frac{\partial^2 \overline{w}(x,y)}{\partial x \partial y} .$$
(3)

The differences between the displacement influence surface curvatures for the damaged and the undamaged structure are:

$$R_{xx}(x,y) = \overline{\rho}_{xx}(x,y) - \rho_{xx}(x,y) \tag{4}$$

$$R_{yy}(x,y) = \overline{\rho}_{yy}(x,y) - \rho_{yy}(x,y)$$
(5)

$$R_{xy}(x,y) = \overline{\rho}_{xy}(x,y) - \rho_{xy}(x,y)$$
(6)

When the above differences in equation (4), (5) or (6) are not identical to zero there exist some changes in the displacement influence surface curvatures. By introducing equations

$$\rho_{xx} = \frac{m_{xx}}{D}, \rho_{yy} = \frac{m_{yy}}{D}, \rho_{xy} = \frac{m_{xy}}{D}$$
(7)

where  $m_{xx}, m_{yy}, m_{xy}$  are flexural moments and D is flexural stiffness, we can conclude that changes in the displacement influence surface curvatures are caused by the changes in flexural moments and the flexural stiffness.

Changes in the displacement influence surface curvatures can be caused:

- In static determined systems because of changes in flexural stiffness (flexural stiffness has no influence in flexural moments).
- In static undetermined systems, theoretically, change in flexural stiffness cause • change in flexural moment as well. Change in flexural stiffness is localized in a small field and it has much bigger intensity in curvature difference in comparison to changes in flexural moments.

Significantly a change in the displacement influence surface curvatures for the two structural states indicates the position of the flexural stiffness reduction.

#### **Beam structures**

The above procedure is similar for beam structures.

 $\eta_w(x) = w(x)$  is the displacement influence line for the undamaged state,

 $\overline{\eta_w}(x) = \overline{w}(x)$  is the displacement influence surface for the damaged state.

$$\rho(x) = \frac{d^2 w(x)}{dx^2}, \quad -\rho(x) = \frac{d^2 \overline{w}(x)}{dx^2} \quad .$$
(8)

The differences between the displacement influence line curvatures for the damaged and the undamaged structure are:

$$R_{xx}(x) = \overline{\rho}(x) - \rho(x) \tag{9}$$

When the previous difference in equation is not identical to zero there is some changes in the displacement influence surface curvatures. By introducing the next equation

$$\rho = \frac{M}{K} \tag{10}$$

where M is bending moment and K is bending stiffness, we can conclude that changes in the displacement influence line curvatures are caused by changes in the bending moment and the bending stiffness.

Significantly change in the displacement influence line curvatures for the two structure states indicates the position of the bending stiffness reduction.

# 3. DAMAGE DETECTION IN BEAM STRUCTURES USING DISPLACEMENT INFLUENCE LINES

The analysis has been carried out for simply supported and continuous beams with different combination of damages<sup>5</sup>. In this paper, one damage scenario of a four-span continuous beam is presented. The span length is L = 25 m. Numerical model of the continuous beam is divided in has n=200 finite elements. The length of each element is  $\Delta l = 0,50 m$ . The cross section area of the beam is  $A = 0,8567 m^2$ , second moment of area is  $I = 1,4 \cdot 10^{-1} m^4$ 

and Young's modulus is  $E = 3.5 \cdot 10^7 \text{ kN} / m^2$ . The applied force is F = 100 kN.

The displacement influence lines have been computed for different points in structure for both the undamaged and the damaged state. The damage has been simulated by reducing the bending stiffness of some finite elements by 20%. The curvature of the displacement influence lines has been calculated using central finite difference method.

In figures 2-5 the difference in curvatures of the two displacement influence lines and square of this result are shown.



Figure 1: Damaged model: reduction of bending stiffness in finite elements 40, 90, 126, 160



Figure 2: Measurement point 1 (in the middle of the first span)



Figure 3: Measurement point 2 (in the middle of the second span)



Figure 4: Measurement point 3 (in the middle of the third span)



Figure 5: Measurement point 4 (in the middle of the fourth span)

The conclusions from the conducted analyses on beams with different combination of damages are:

- a) The optimal location of the measurement point is in the middle of the span.
- b) One measurement point at every span in beam structures enables locating the damages reliably.
- c) The damages located in the range 0-0.1 L from the first or the last pinned support of the beam can not be detected by using the displacement influence line method.

# 4. DAMAGE DETECTION IN PLATE-LIKE STRUCTURES USING DISPLACEMENT INFLUENCE SURFACES

The analysis has been carried out for one-bay and two-bay plate-like structures with different combination of damages<sup>5</sup>. In this paper, one damage scenario of a two-bay plate like bridge structure is presented. (Figures 6-10)

The bridge structure has been modeled using software "PLOCA".

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Numerical model has 200 eight-node finite elements, of the size 2x1 m each, with 661 nodes as shown in Fig. 6. The thickness *d* of the finite elements is not the same for the whole model. It depends on the cross section of the bridge as follows: for elements  $41-160 d_1 = 0.8 m$ , for elements 1-40 and  $161-200 d_2 = 0.175 m$ . The Young's modulus is  $E = 3.158 \cdot 10^7 kN / m^2$  and the Poisson's ratio is v = 0.2. The applied force is 300 kN.

The influence surfaces for displacements a two points in the structure for both the undamaged and the damaged case have been computed. The damage has been simulated by reducing the thickness of some finite elements. The reduced thickness for elements 63, 77, 115, 150 is  $d_3 = 0.7 m$ .

Central difference approximation has been used to derive the curvatures from the displacement influence surfaces.

181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
81	82	83	84	85	1 86	87	88	89	90	91	92	93	94	95	2 96	97	98	99	100
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	70	80
41 ()	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	8
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	)ş
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 6: Damaged model

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	-	-				1,5.1	l 0 <sup>-9</sup> m	m			-2	,5·10 <sup>-9</sup>	mm			-	_		
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	<b>10</b> 0
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	70	80
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 7: Curvature difference in direction x for measurement point 1

						10	) <sup>-9</sup> mn	n			-2.	10 <sup>-9</sup> m	m						
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
81	82	83	84	85	86	87	88	89	90	91	92	93	- 94	95	96	97	98	99	100
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	70	80
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 8: Curvature difference in direction y for measurement point 1

						2,5.1	0 <sup>-9</sup> m	m			-2	,5·10 <sup>-9</sup>	<sup>9</sup> mm						
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	<b>10</b> 0
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	70	80
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 9: Curvature difference in direction x for measurement point 2

						2.1	0 <sup>-9</sup> mn	-1,5·10 <sup>-9</sup> mm											
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	70	80
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Figure 10: Curvature difference in direction y for measurement point 2

The conclusions from the conducted analyses on plate-like structures with different given damages are:

- a) The damages near supports (where flexural moment are minimal) can be detected and located only by using warping of deflection influence surface.
- b) In all the other cases using the combination of deflection influence surface curvature in both directions (x and y) is sufficient for locating the damages.
- c) The optimal location of the measuring point is in the middle of the bay.
- d) One measurement point in every bay in plate like structures enables locating the damages reliably.

## 5. CONCLUSIONS

Structural damage detection method through the analysis of deflection influence lines and influence surfaces is presented in this paper. This method can be used to detect and locate the reduction in the bending stiffness of beam or plate-like structures.

The method is based on the difference in curvature of the deflection influence lines or surfaces for the undamaged and the damaged case.

Comparison of the curvatures of the displacement influence lines makes a reliable method for locating the damaged section for simply-supported or continuous beams. Comparison of the curvatures of the displacement influence surfaces likewise ensures locating the damaged area for one-bay or continuous plate-like structures.

One measurement point in the middle of each span for beams as well as one measurement point in the middle of each bay for plate like structures enables locating the damages reliably. The presented method is not reliable if the damage is located near the first or the last pinned support of the beam structure analysed.

The small number of the required measurement points ensures simple on-site testing.

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