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Contribution of Transverse Bulkheads to Hull Stiffness of Large Container Ships

Preliminary communication

Ultra large container ships are rather flexible and exposed to significant wave deformations. Therefore, the hydroelastic strength analysis is required for these types of ships. The coupling of a beam structural model and a 3D hydrodynamic model is preferable for reasons of simplicity. In this paper, the contribution of large number of transverse bulkheads to general hull stiffness is analysed. The prismatic pontoon with the cross-section of a large container vessel is considered for this purpose. The 3D FEM torsional analysis is performed with transverse bulkheads included and excluded. The correlation analysis of the obtained deformations indicated the influence of transverse bulkheads on the ship hull stiffness. The analysis is done by employing the torsional theory of thin-walled girders.

Keywords: *container ship, finite element method, stiffness, thin-walled girder, torsion*

Doprinos poprečnih pregrada krutosti trupa velikih kontejnerskih brodova

Prethodno priopćenje

Vrlo veliki kontejnerski brodovi prilično su elastični i stoga podložni velikim valnim deformacijama. Zato se danas njihova čvrstoća istražuje metodama hidroelastičnosti. Pritom se radi jednostavnosti sprežu gredni strukturni model i 3D hidrodinamički model. U ovom članku istražen je doprinos velikoga broja poprečnih pregrada kontejnerskih brodova općoj krutosti trupa. Za te potrebe razmatran je prizmatični ponton s poprečnim presjekom velikoga kontejnerskog broda. Konstruiran je 3D model konačnih elemenata i provedena je analiza uvijanja s uključenim i isključenim poprečnim pregradama. Korelacijskom analizom deformacija za ova dva slučaja ustanovljen je utjecaj poprečnih pregrada na krutost brodskoga trupa. Pritom se koristila teorija uvijanja tankostjenih nosača.

Ključne riječi: *kontejnerski brod, metoda konačnih elemenata, krutost, tankostjeni nosač, uvijanje*

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1 Introduction

Nowadays sea transport is rapidly increasing and ultra large container ships are built [1]. Since they are rather flexible, their hydroelastic response becomes an imperative subject of investigation. In the early design stage, the coupling of a FEM beam structural model with a 3D hydrodynamic model based on the radiation-diffraction theory is reasonable [2], [3].

The 1D FEM structural model is quite sophisticated since it takes into account the bending and shear stiffness, as well as the torsional and warping stiffness [4], [5]. The general hull stiffness is increased due to the large number of transverse bulkheads in holds. There are two types of bulkheads, i.e. ordinary watertight bulkheads and grillage ones. The distance between them is determined by the container length.

Transverse bulkheads stretch within one web frame spacing and are quite stiff. They can be directly included in the 1D FEM model as a short beam element with a closed cross-section [6], [7]. However, due to the large number of transverse bulkheads and to the model discontinuity, it is more practical and reasonable to take into account their continuous contribution to the general hull stiffness.

Different attempts to take the influence of transverse bulkheads into account have been made. One of the first approaches was to increase the deck thickness based on the equivalence of the deformation energy of transverse bulkhead girders and the increased deck energy [8]. Today, the usual way is to model transverse bulkheads by axial elastic springs at their joints to the ship hull. The spring effect is condensed in lumped bimoments [9]. Furthermore, in the case of a large number of transverse bulkheads, the lumped bimoments might be distributed along the hull girder [10]. The distributed bimoments are manifested as additional torque load, which depends on the variation of the twist angle as pure torsional torque. Therefore, only the torsional stiffness of the ship hull is increased due to the bulkhead influence.

The effect of transverse structure on the deformation of thin-walled girders is a challenging subject of contemporary investigations [11], [12], [13]. Recent literature shows that the problem is rather complex and the complicated solutions offered there reduce the applicative advantages of the combined beam theory and the thin-walled girder theory (1D + 2D) with respect to the direct 3D FEM analysis. In any case, the reliability of the 1D + 2D theory has to be checked by the correlation analysis with 3D FEM solutions.

In the light of the above circumstances, especially of the needs of the ship hydroelasticity analyses, where results of dry natural vibrations of the ship hull are required (modes, frequencies, modal stiffness, modal mass), a simpler solution is preferable and more convenient. That was the motivation for the investigation of this challenging problem. Thus, the 3D FEM analysis of the prismatic hold structure with and without transverse bulkheads is performed. The equivalence of the maximum twist angle in the 1D and 3D models is used as a condition for determining the change of torsional beam stiffness. The reliability of approach is checked by the correlation for 1D and 3D warping functions and stress distribution.

2 Outline of the thin-walled girder theory

The thin-walled girder torsional theory is developed under assumptions that a considered structure is of membrane type (only in-plane deformation occurs) and that there is no distortion of the cross-section (twist angle is constant along the cross-section contour).

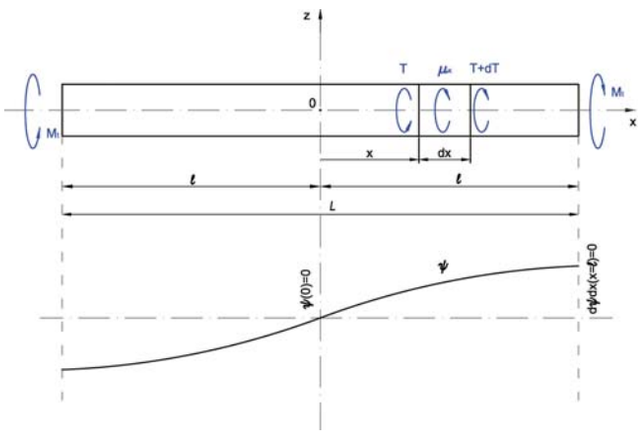


Figure 1 **Beam torsion**
Slika 1 **Uvijanje grede**

A prismatic girder exposed to torsion is shown in Figure 1. The equilibrium of sectional torque, T, and the distributed external torsional load, μ_x, yields

$$\mathrm{d}T = -\mu_x \mathrm{d}x. \tag{1}$$

According to the theory of thin-walled girders, the sectional torque consists of a pure torsional part and a warping contribution [14]

$$T = T_t + T_w = GI_t \frac{\mathrm{d}\psi}{\mathrm{d}x} - EI_w \frac{\mathrm{d}^3\psi}{\mathrm{d}x^3}, \tag{2}$$

where

E, G – Young's modulus and shear modulus
I_t, I_w – torsional and warping modulus
ψ – twist angle

Substitution of (2) into (1) leads to the ordinary differential equation of the fourth order

$$EI_w \frac{\mathrm{d}^4\psi}{\mathrm{d}x^4} - GI_t \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = \mu_x. \tag{3}$$

Its solution reads

$$\psi = A_0 + A_1 x + A_2 \operatorname{ch}\beta x + A_3 \operatorname{sh}\beta x + \psi_p, \tag{4}$$

where

$$\beta = \sqrt{\frac{GI_t}{EI_w}} \tag{5}$$

and A_i are integration constants, while ψ_p represents a particular solution which depends on μ_x.

Let us consider the twisting of the girder shown in Figure 1, which is loaded by torque M_t at the ends, while $\mu_x = 0$. The warping of the girder ends is suspended. In this case the twist angle ψ is an anti-symmetric function and therefore $A_0 = A_2 = 0$. The remaining constants A_1 and A_3 are determined by satisfying the boundary conditions

$$x = l: \quad T = M_t, \quad u = \frac{\mathrm{d}\psi}{\mathrm{d}x}\bar{u} = 0, \tag{6}$$

where u is the warping function (axial displacement) and $\bar{u}$ is the relative sectional warping due to the unit beam deformation, $\mathrm{d}\psi/\mathrm{d}x$, defined according to the theory of thin-walled girders [15], [16]. The final expressions for the twist angle reads

$$\psi = \frac{M_t l}{GI_t}\left[\frac{x}{l} - \frac{\operatorname{sh}\beta x}{\beta l \cdot \operatorname{ch}\beta l}\right]. \tag{7}$$

Now, it is possible to determine sectional forces, i.e. pure torsional and warping torques (2)

$$T_t = M_t\left(1 - \frac{\operatorname{ch}\beta x}{\operatorname{ch}\beta l}\right), \quad T_w = M_t \frac{\operatorname{ch}\beta x}{\operatorname{ch}\beta l} \tag{8}$$

and warping (sectorial) bimoment

$$B_w = EI_w \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = -M_t \frac{\operatorname{sh}\beta x}{\beta \operatorname{ch}\beta l}. \tag{9}$$

Furthermore, the warping function (6) takes the form of

$$u = \frac{M_t}{GI_t}\left(1 - \frac{\operatorname{ch}\beta x}{\operatorname{ch}\beta l}\right)\bar{u}. \tag{10}$$

Torques T_t and T_w are the result of shear stresses τ_t and τ_w due to pure torsion and suspended warping, respectively. The warping bimoment B_w represents the work of axial normal stress σ on the displacement $\bar{u}$ at a cross-section, i.e.

$$B_w = \int_s \sigma t \bar{u} \, \mathrm{d}s, \tag{11}$$

where

$$\sigma = E \frac{\mathrm{d}^2 \psi}{\mathrm{d}x^2} \bar{u}. \tag{12}$$

Thus, by substituting (12) into (11) one finds the expression for warping modulus in (9)

$$I_w = \int_s \bar{u}^2 t \, \mathrm{d}s. \tag{13}$$

3 Modelling of transverse bulkheads

The length of transverse bulkheads in large container ships is equal to one web frame spacing. They are of grillage type and therefore quite stiff. As a result, the bulkhead influence on the hull warping reduction is significant.

In the torsional thin-walled girder theory, bulkheads can be modelled by axial elastic foundation at their joint to the hull structure [9], [10]. In the case of a large number of bulkheads, the line foundation can be spread to the area foundation of the hull shell. The corresponding axial (tangential) surface load yields

$$q = \kappa u = \kappa \bar{u} \frac{\mathrm{d}\psi}{\mathrm{d}x}, \tag{14}$$

where κ is the spread bulkhead stiffness and u is the warping function (6).

Axial load q causes an additional bimoment per unit length on the relative sectional warping $\bar{u}$

$$b = \int_s q \bar{u} \, \mathrm{d}s. \tag{15}$$

By substituting (14) into (15) one writes

$$b = k \frac{\mathrm{d}\psi}{\mathrm{d}x}, \tag{16}$$

where

$$k = \kappa \int_s \bar{u}^2 \, \mathrm{d}s \tag{17}$$

is the sectional bulkhead stiffness.

According to the theory presented in [9] and [10], the bulkhead bimoment causes a distributed torque

$$\mu_b = \frac{\mathrm{d}b}{\mathrm{d}x} = k \frac{\mathrm{d}^2 \psi}{\mathrm{d}x^2}, \tag{18}$$

where relation (16) is used for b. The torque μ_b is the transformed bulkhead load and has to be equilibrated by sectional torques T_t and T_w (2). Thus, by substituting (18) into (3), the differential equation for girder torsion with bulkhead influence is obtained:

$$EI_w \frac{\mathrm{d}^4 \psi}{\mathrm{d}x^4} - \left(GI_t + k\right) \frac{\mathrm{d}^2 \psi}{\mathrm{d}x^2} = 0. \tag{19}$$

Generally, the value of k has to be calculated for a given bulkhead structure. It is a result of flexural bulkhead stiffness.

4 Effect of transverse bulkheads

In order to take the influence of bulkheads into account, another approach can also be applied. A ship hull consists of a large number of open cross-section segments (holds) and of closed ones (bulkheads). For the open section, the torsional modulus I_t is quite small, and therefore the warping modulus I_w plays the main role. In a short bulkhead area, the torsional modulus of closed section I_t^0 is one order of magnitude higher than I_t, while I_w^0 is of the same order as I_w. For the reason of simplicity we can consider a uniform girder with the equivalent torsional modulus I_t^*, where $I_t < I_t^* < I_t^0$, and the equivalent warping modulus I_w^* equal to I_w. In this case, the differential equation (3) takes the form of

$$EI_w \frac{\mathrm{d}^4 \psi}{\mathrm{d}x^4} - GI_t^* \frac{\mathrm{d}^2 \psi}{\mathrm{d}x^2} = 0, \tag{20}$$

where

$$GI_t^* = GI_t + GI_b. \tag{21}$$

GI_b is the additional hull torsional stiffness due to bulkheads as closed cross-section segments. Parameters k and GI_b, in (19) and (21) respectively, are equivalent quantities.

Instead of bulkhead modelling by equivalent axial elastic foundation, as it is usually done in literature, it is possible to determine the contribution of bulkheads by the 3D FEM analysis, as it is elaborated in Section 7. Let us assume, for the time being, that the end twist angles of a prismatic girder without and with transverse bulkheads are known, $\psi(l)$ and $\psi^*(l)$ respectively. Referring to (7), one writes

$$\psi(l) = \frac{M_t l}{GI_t}\left(1 - \frac{\mathrm{th}\, y}{y}\right) \tag{22}$$

$$\psi^*(l) = \frac{M_t l}{GI_t^*}\left(1 - \frac{\mathrm{th}\, y^*}{y^*}\right), \tag{23}$$

where

$$y = \beta l = l\sqrt{\frac{GI_t}{EI_w}} \tag{24}$$

$$y^* = \beta^* l = l\sqrt{\frac{GI_t^*}{EI_w}}. \tag{25}$$

Ratios of Eqs (23) and (22) lead to the transcendental equation for determining the unknown parameter y^*

$$\frac{1}{y^{*2}}\left(1 - \frac{\mathrm{th}\, y^*}{y^*}\right) = \frac{1}{y^2}\left(1 - \frac{\mathrm{th}\, y}{y}\right)\frac{\psi^*(l)}{\psi(l)}. \tag{26}$$

Now, the new value of torsional modulus can be determined by employing (24) and (25), i.e.

$$\frac{I_t^*}{I_t} = \left(\frac{y^*}{y}\right)^2. \tag{27}$$

According to (21), the contribution of bulkheads to torsional stiffness is

$$\frac{I_b}{I_t} = \left(\frac{y^*}{y}\right)^2 - 1. \tag{28}$$

The twist angle (7) and the warping function (10) in non-dimensional form read respectively:

$$\frac{GI_t^*}{M_t l}\psi^* = \frac{x}{l} - \frac{\mathrm{sh}\beta^* x}{y^* \mathrm{ch}\, y^*} \tag{29}$$

$$\frac{GI_t^*}{M_t}\frac{u^*}{\bar{u}} = 1 - \frac{\mathrm{ch}\beta^* x}{\mathrm{ch}\, y^*}. \tag{30}$$

Referring to (8), the twisting and warping torques take the following form:

$$\frac{T_t^*}{M_t} = 1 - \frac{\mathrm{ch}\beta^* x}{\mathrm{ch}\, y^*}, \quad \frac{T_w^*}{M_t} = \frac{\mathrm{ch}\beta^* x}{\mathrm{ch}\, y^*}. \tag{31}$$

Furthermore, the twisting torque can be split into the hull part and the bulkhead contribution

$$\frac{T_t^*}{M_t} = \frac{T_h^*}{M_t} + \frac{T_b^*}{M_t}, \tag{32}$$

where, proportionally to their torsional moduli (21),

$$\frac{T_h^*}{M_t} = \frac{I_t}{I_t^*}\frac{T_t^*}{M_t}, \quad \frac{T_b^*}{M_t} = \frac{I_b}{I_t^*}\frac{T_t^*}{M_t}. \tag{33}$$

Ratios I_t/I_t^* and I_b/I_t^* are defined by (27) and (28).

Finally, the warping bimoment (9) takes the following non-dimensional form:

$$\frac{B_w^*}{M_t l} = -\frac{\mathrm{sh}\,\beta^* x}{y^* \mathrm{ch}\, y^*}. \tag{34}$$

The presented approach is based on the known ratio of end values of the twist angle for a girder without and with transverse bulkheads. Its reliability can be checked by the known ratio of warping functions in the middle of the girder. According to (10), it follows that

$$\frac{u^*(0)}{u(0)} = \left(\frac{y}{y^*}\right)^2 \frac{1 - \dfrac{1}{\mathrm{ch}\, y^*}}{1 - \dfrac{1}{\mathrm{ch}\, y}}. \tag{35}$$

An additional way to check the obtained results is to compare the normal stress ratio at girder ends represented by the warping bimoments (9)

$$\frac{B_w^*(l)}{B_w(l)} = \frac{y\,\mathrm{th}\, y^*}{y^*\,\mathrm{th}\, y}. \tag{36}$$

Actually, ratios (35) and (36) are related to the first and second derivative of the twist angle, (6) and (9) respectively.

5 Ship particulars

A 7800 TEU container vessel of the following main particulars is considered, Figure 2.

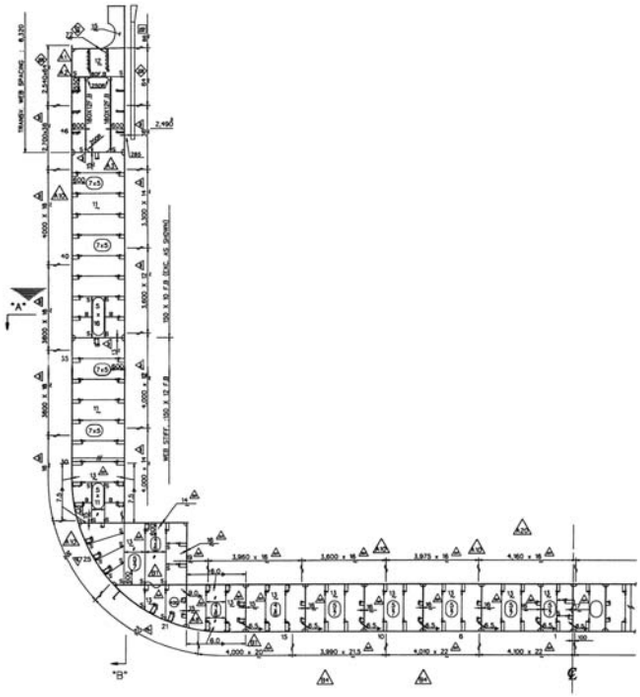
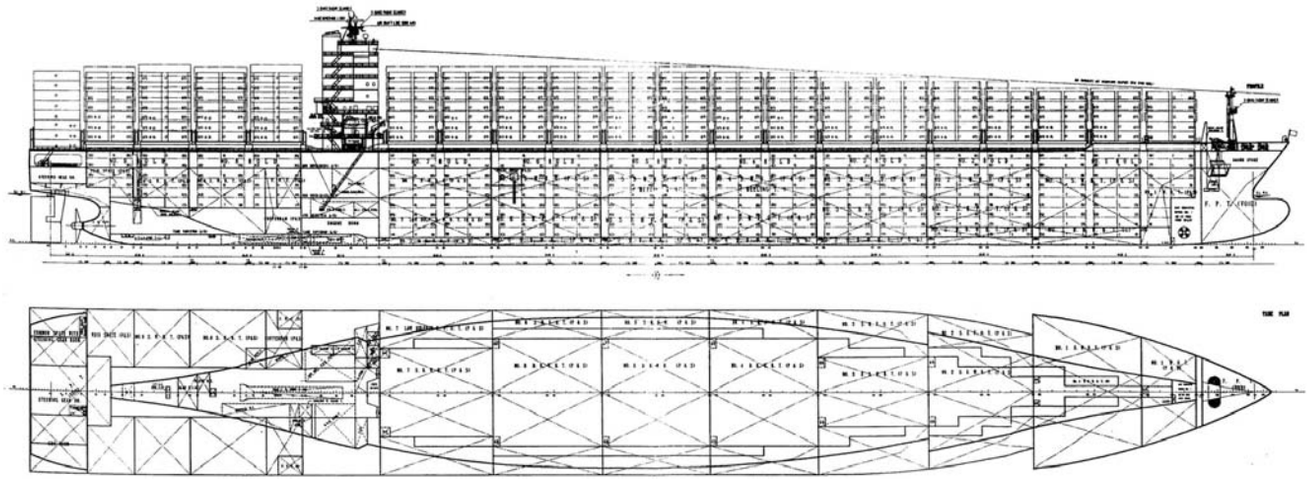


Figure 3 **Midship cross-section**
Slika 3 **Glavno rebro**

Figure 2 **A 7800 TEU container vessel**
Slika 2 **Kontejnerski brod nosivosti 7800 TEU**



Length overall	L_{oa} = 334 m
Length between perpendiculars	L_{pp} = 319 m
Breadth	B = 42.8 m
Depth	H = 24.6 m
Draught	T = 14.5 m
Displacement	Δ = 135530 t

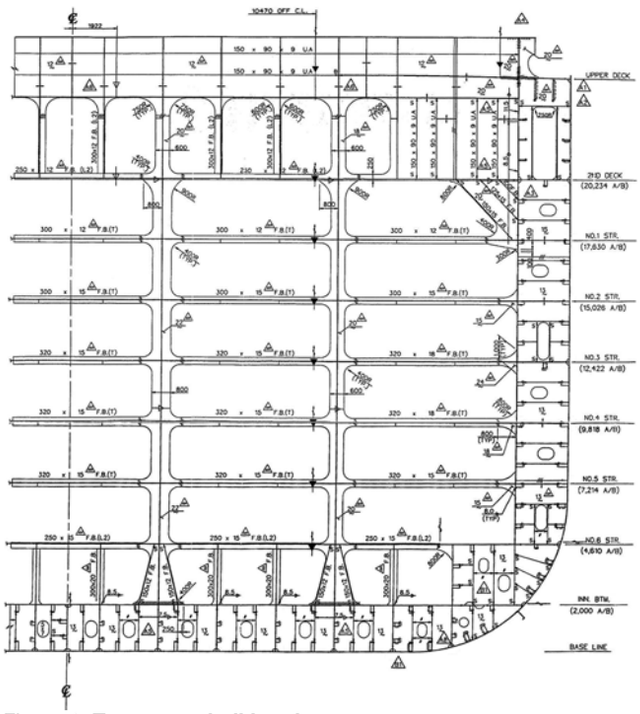
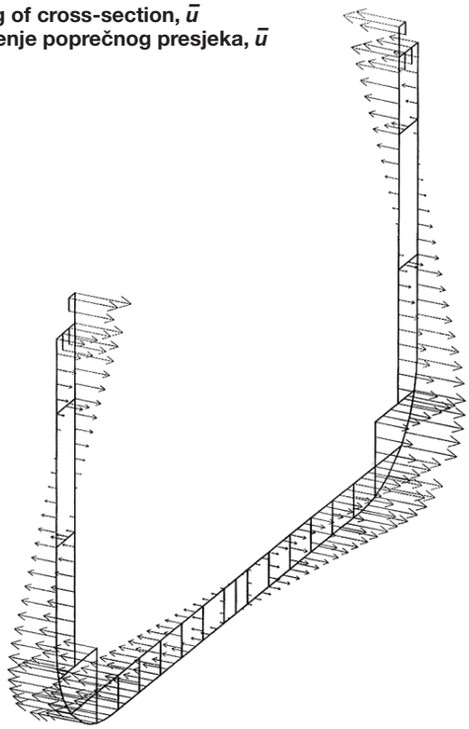


Figure 4 **Transverse bulkhead**
Slika 4 **Poprečna pregrada**

Figure 5 **Warping of cross-section, $\bar{u}$**
Slika 5 **Vitoperenje poprečnog presjeka, $\bar{u}$**



The midship cross-section is shown in Figure 3, while Figure 4 shows the transverse bulkhead. Properties of the open cross-section are determined by the STIFF program [17].

Cross-section area	A = 6.394 m^2
Horizontal shear area	A_{sh} = 1.015 m^2
Vertical shear area	A_{sv} = 1.314 m^2
Vertical position of neutral line	z_{NL} = 11.66 m
Vertical position of shear - - torsional centre	z_D = –13.50 m
Horizontal moment of inertia	I_{bh} = 1899 m^4
Vertical moment of inertia	I_{bv} = 676 m^4
Torsional modulus	I_t = 14.45 m^4
Warping modulus	I_w = 171400 m^6

Position of deformation centre, z_D, is rather low due to the open cross-section. The relative warping of cross-section, u, is illustrated in Figure 5. Young's modulus, shear modulus and Poisson's ratio are: $E = 2.06 \cdot 10^8$ kN/m^2, $G = 0.7923 \cdot 10^8$ kN/m^2, ν= 0.3, respectively.

6 FEM models of a hull segment

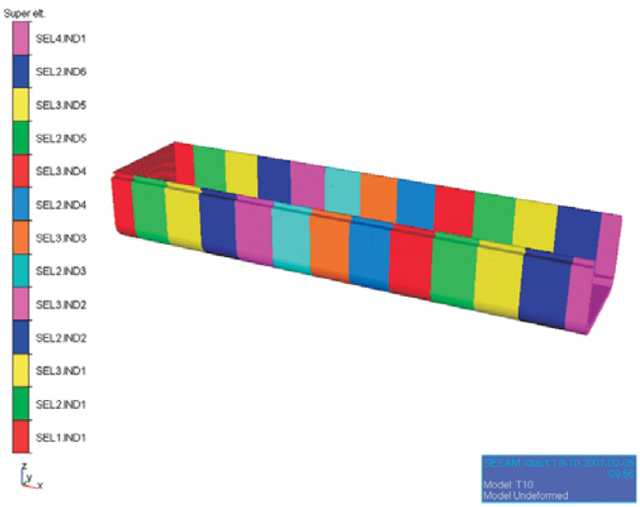


Figure 6 **Prismatic FEM model of a hull part**
Slika 6 **Prizmatični model konačnih elemenata dijela trupa**

The front holds of the ship as a prismatic thin-walled girder with the length of $L = 2l = 174$ m are considered. The FEM model is generated by the software [18]. It is constructed of four different types of superelements, and includes the total of 13 superelements, Figures 6 and 7. The shell finite elements are used. The model is clamped at the fore end and the only warping is suspended at the aft end. The vertical distributed load is imposed at the aft cross-section, generating the total torque M_t = 40570 kNm, Figure 8, [19].

There are two types of transverse bulkheads within the ship hold space, i.e. the ordinary watertight bulkheads and bulkheads of grillage construction. Both types stretch within one web frame spacing. The bulkhead top ends with the stool. Such a bulkhead design makes them quite strong and therefore the general hull stiffness is increased.

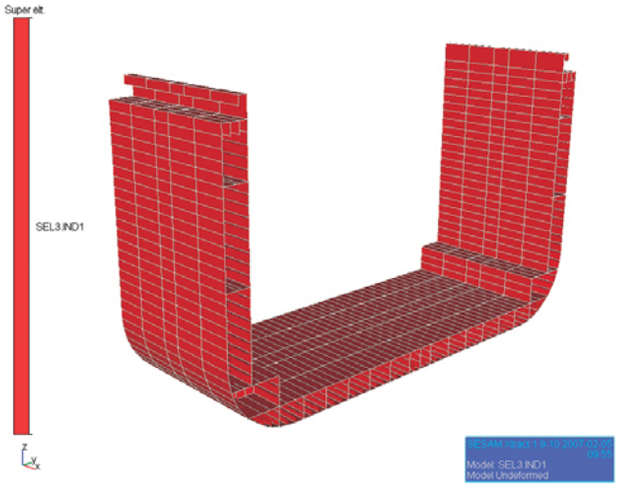


Figure 7 **Superelement No. 3**
Slika 7 **Superelement br. 3**

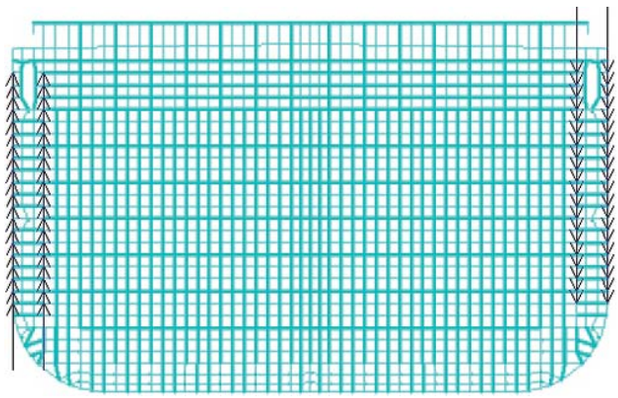


Figure 8 **Load at the aft end**
Slika 8 **Opterećenje kraja modela**

Figure 9 **Prismatic FEM model of a hull part with transverse bulkheads**
Slika 9 **Prizmatični model konačnih elemenata dijela trupa s poprečnim pregradama**

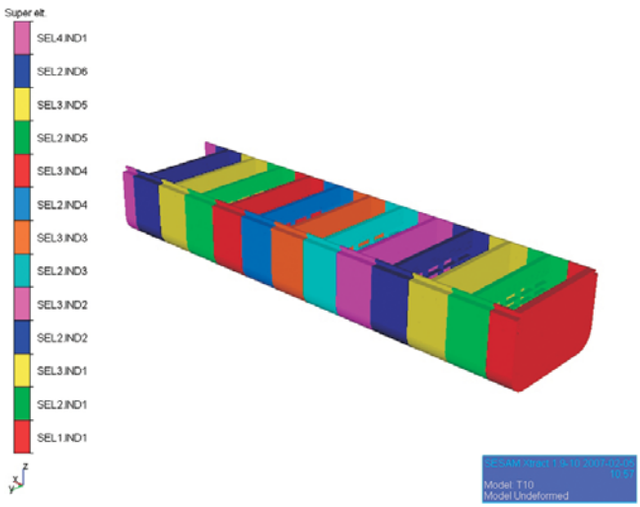


Figure 10 **Superelement No. 3 with a transverse bulkhead**
Slika 10 **Superelement br. 3 s poprečnom pregradom**

The FEM model of the ship segment with transverse bulkheads and a typical superelement with the watertight bulkhead are shown in Figures 9 and 10 respectively. The boundary conditions and imposed load are the same as in the case of prismatic model without transverse bulkheads.

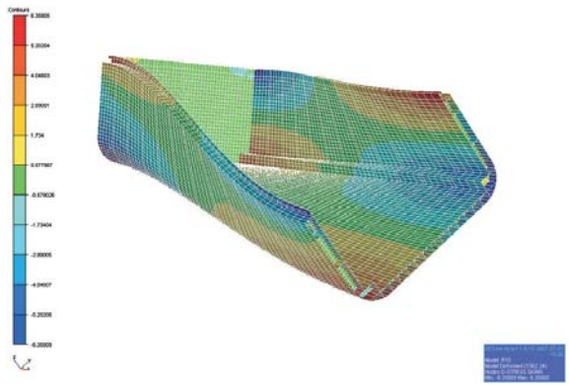
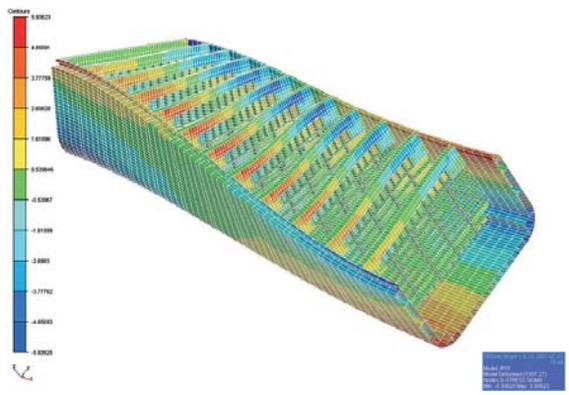


Figure 11 **Deformation and stresses of the model without transverse bulkheads, σ_x [N/mm²]**
Slika 11 **Deformacije i naprezanja modela bez poprečnih pregrada, σ_x [N/mm²]**

Figure 12 **Deformation and stresses of the model with transverse bulkheads, σ_x [N/mm²]**
Slika 12 **Deformacije i naprezanja modela s poprečnim pregradama, σ_x [N/mm²]**



Deformed models, without and with transverse bulkheads, are shown in Figures 11 and 12 respectively. Distortion of the cross-section is negligible as a result of a double skin cross-section with very strong web frames. Due to the same reason, the bending stresses are negligible in comparison to the membrane stresses; therefore, the structure behaves as a membrane one. Different colours in Figures 11 and 12 denote the levels of von Mises membrane stress. High stress concentration in the hatch coaming and the upper deck at the model ends confirms the well-known fact caused by the suspended warping of the cross-section, [20].

7 Influence of transverse bulkheads

Since the 3D FEM model behaves as a membrane structure without distortion of the cross-section, the obtained results are comparable to those of the 1D analysis.

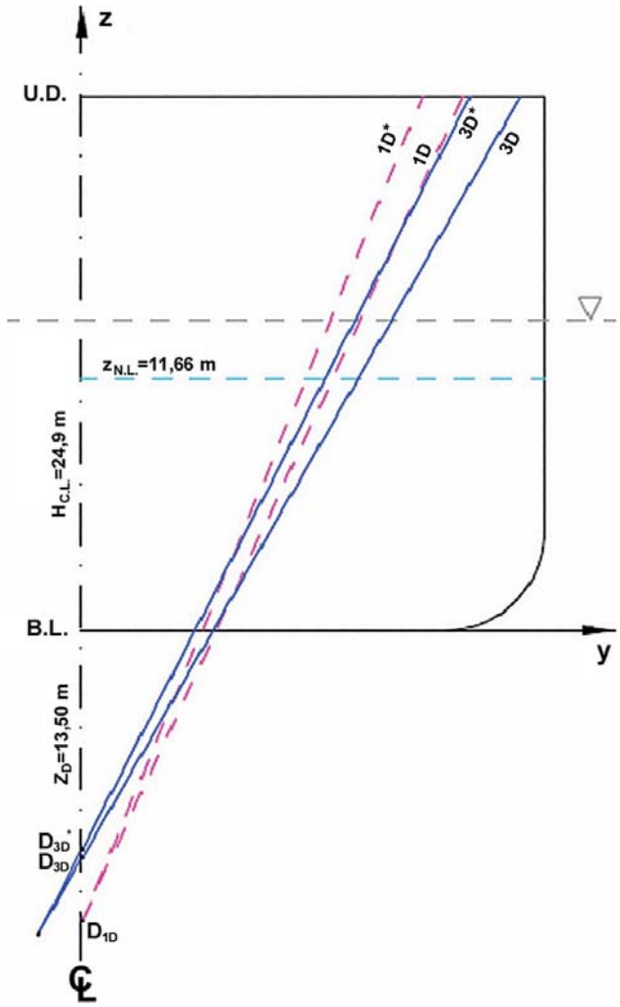


Figure 13 **Rotation of the cross-section**
Slika 13 **Zakret poprečnog presjeka**

Rotation of the model free cross-section, determined analytically by the beam theory (1D) and numerically by FEM (3D) for the pontoon with and without transverse bulkheads, is shown in Figure 13. The twist angle of the 3D analysis is somewhat higher than that of the 1D analysis.

$$\frac{\psi_{3D}}{\psi_{1D}} = \frac{0.27690 \cdot 10^{-3}}{0.22969 \cdot 10^{-3}} = 1.2055. \qquad \text{(a)}$$

Vertical position of the deformation centre in the 3D model is above that of the 1D model, points D_{3D} and D_{1D} in Figure 13, respectively. The influence of transverse bulkheads on the position of the deformation centre is quite weak, point D_{3D}. The warping of cross-section determined by the 3D analysis is rather close to that of the 1D analysis, Figure 5. Therefore, the warping correlation could be done only for one representative point of extreme displacement value. Let us chose the joint of the bilge and the inner bottom, Figure 5, where

$$\frac{u_{3D}}{u_{1D}} = \frac{1.02594\,\text{mm}}{1.05192\,\text{mm}} = 0.9753. \qquad \text{(b)}$$

Thus, the correlation of 3D and 1D analyses results is quite good concerning warping, while the 3D FEM model is more elastic than the 1D model from the twisting point of view. This could be caused by the shear influence on torsion which is not taken into account in the beam analysis, [10]. That fact might be the subject of further investigations. However, it does not have a significant influence on the relative bulkhead contribution to the hull stiffness.

The twist angle ratio of the model with and without transverse bulkheads reads

$$\frac{\psi^*_{3D}}{\psi_{3D}} = \frac{0.24876 \cdot 10^{-3}}{0.27690 \cdot 10^{-3}} = 0.89837. \qquad \text{(c)}$$

The warping ratio in the bilge point is

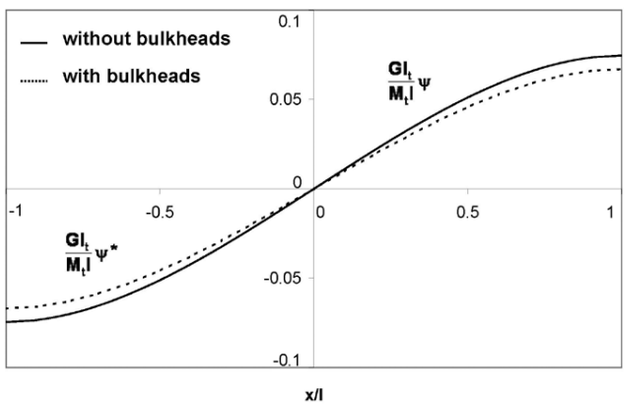
$$\frac{u^*_{3D}}{u_{3D}} = \frac{0.90013\,\text{mm}}{1.02594\,\text{mm}} = 0.87737. \qquad \text{(d)}$$

The axial normal stress ratio in the hatch coaming, Figures 11 and 12, yields

$$\frac{\sigma^*_{3D}}{\sigma_{3D}} = \frac{5.93623\,\text{N/mm}^2}{6.35805\,\text{N/mm}^2} = 0.93365. \qquad \text{(e)}$$

In the considered numerical example, according to (24), $y = 0.49541$, while the solution of Eq. (26) gives $y^* = 0.7464$. The variation of torsional stiffness, Eq. (27), is $I_t^*/I_t = 2.27$. It means that the bulkhead contribution is $I_b/I_t = 1.27$.

Figure 14 **Twist angle**
Slika 14 **Kut uvijanja**



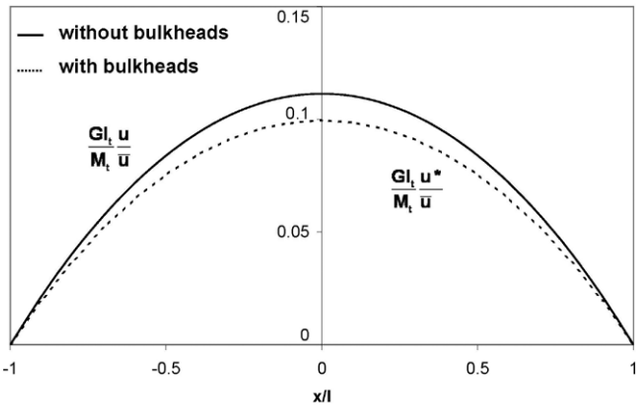


Figure 15 **Deck warping**
Slika 15 **Vitoperenje palube**

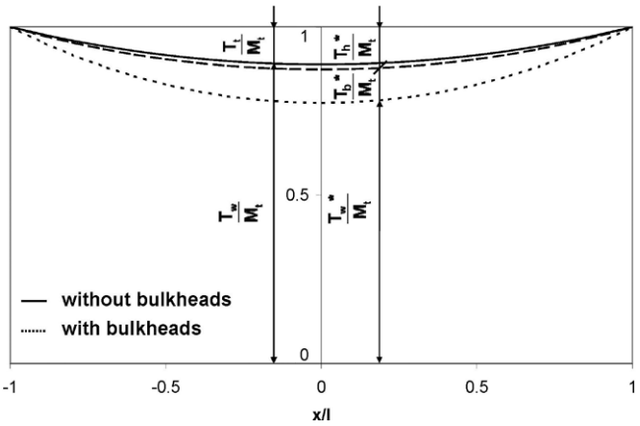


Figure 16 **Twisting and warping torques**
Slika 16 **Torzijski momenti uvijanja i vitoperenja**

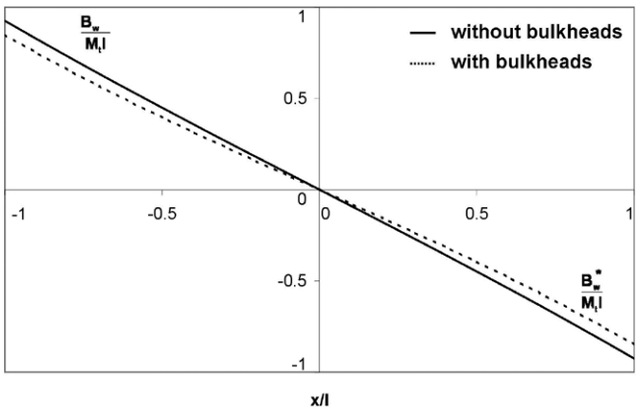


Figure 17 **Warping bimoment**
Slika 17 **Bimoment vitoperenja**

The girder displacements and sectional forces are determined for both cases, i.e. without and with transverse bulkheads, and are shown in Figures 14, 15, 16 and 17 in non-dimensional form. The corresponding formulae from Sections 2 and 4 are used. The twist angle ψ is reduced according to given values of the 3D FEM analysis, Figure 14. The warping of the cross-section u is also reduced, Figure 15. Its variation defined by the 1D analysis, Eq. (35) is

$$\frac{u^*(0)}{u(0)} = 0.89388. \tag{f}$$

Discrepancy between the 1D analysis and the 3D FEM analysis, value (d), is only 1.9%.

The warping bimoment shown in Figure 17 is also reduced due to bulkheads. The 1D ratio, Eq. (36), yields

$$\frac{B_w^*(l)}{B_w(l)} = 0.91633. \tag{g}$$

By comparing it to the 3D FEM stress ratio (e), a discrepancy of -1.9% is obtained. This fact confirms quite good simulation of the bulkhead effect in the thin-walled girder theory.

The torques distributions are shown in Figure 16. The hull twisting torque in the case with bulkheads, T_h, is very close to the pure twisting torque without bulkheads, T_t. The warping torque T_w is now reduced in comparison to T_w due to the bulkhead contribution, T_b.

The influence of the increased value of torsional stiffness on vibrations can, for instance, be analysed in the case of uncoupled natural vibration of a free thin-walled girder with suspended boundary cross-section warping. The corresponding formula for natural frequencies derived in Appendix reads, (A16)

$$\omega_n = \frac{n\pi}{L}\sqrt{\frac{GI_t}{J_t^0}}\sqrt{1+\left(\frac{n\pi}{L}\right)^2\frac{EI_w}{GI_t}}, \quad n = 0, 1, 2 \ldots \tag{37}$$

The following relation between natural frequencies of a hull segment with and without transverse bulkheads exists:

$$\frac{\omega_n^*}{\omega_n} = \frac{\sqrt{1+\left(\frac{2y^*}{n\pi}\right)^2}}{\sqrt{1+\left(\frac{2y}{n\pi}\right)^2}}. \tag{38}$$

For the first natural frequencies of elastic modes one finds $\omega_1^*/\omega_1 = 1.05594$. Thus, a 127% torsional stiffness increase due to bulkheads results in a 5.6% increase in the first frequency [M3] in the considered case. It is evident from (38) that the variation of higher mode natural frequencies is decreased.

8 Bending stiffness analysis

8.1 Horizontal bending

Horizontal bending is analysed by the FEM model adapted for this purpose. The model aft end is entirely free and loaded by distributed loads, as shown in Figure 18. The vertical load generates a torque of $M_t = 40570$ kNm, while the total horizontal force F_y, acting about the deformation centre, equilibrates it. In this way, the girder is only exposed to horizontal bending.

In the cases of the model without and with transverse bulkheads, the horizontal force of pure bending takes values of $F_y = 1500$ kN and $F_y^* = 1565$ kN, respectively. The corresponding maximum deflections yield $\delta_y = 19.8654$ mm and $\delta_y^* = 20.4069$ mm, Figures 19 and 20. Thus, the moment of inertia of the cross-section of the reinforced model can be expressed by that of the model without bulkheads:

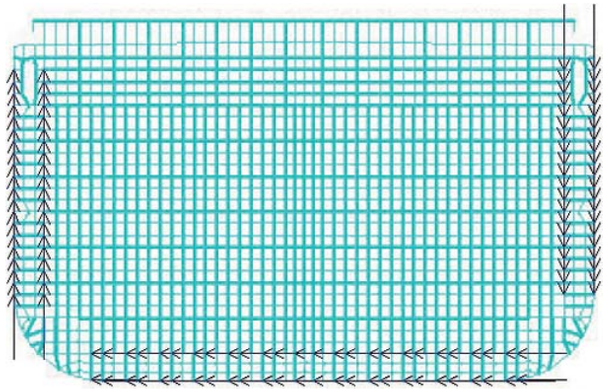


Figure 18 **Load at the model free aft end in the case of horizontal bending**
Slika 18 **Opterećenje horizontalnog savijanja na slobodnom stražnjem kraju modela**

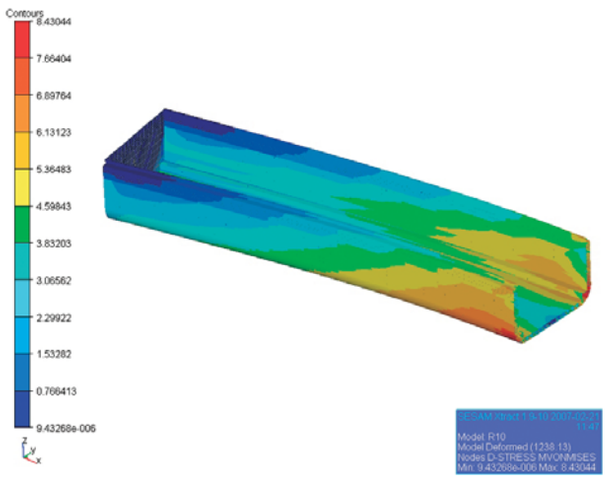
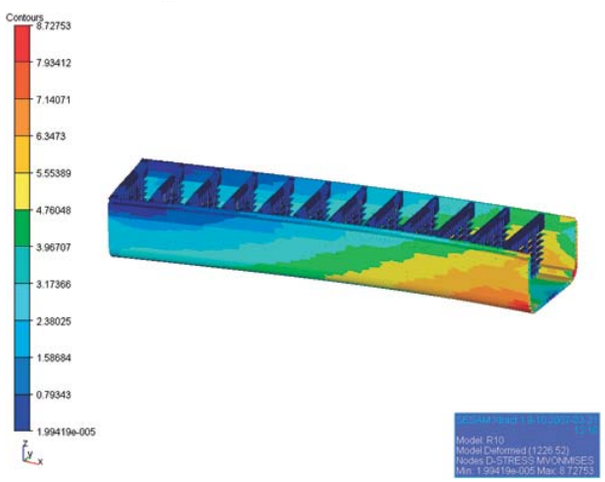


Figure 19 **Horizontal bending of the model without transverse bulkheads, σ_{vM} [N/mm²]**
Slika 19 **Horizontalno savijanje modela bez poprečnih pregrada, σ_{vM} [N/mm²]**

Figure 20 **Horizontal bending of the model with transverse bulkheads, σ_{vM} [N/mm²]**
Slika 20 **Horizontalno savijanje modela s poprečnim pregradama, σ_{vM} [N/mm²]**



$$I_z^* = \frac{\delta_y / F_y}{\delta_y^* / F_y^*} I_z = 1.01565\, I_z. \tag{h}$$

The correction is rather small, approximately 1.56% and its influence on vibration is almost negligible. Stress concentration in the bilge area at the fixed model end due to bending is evident, Figures 19 and 20.

Since pure torque M_t and horizontal forces F_y and F_y^* for the model without and with transverse bulkheads are known, it is possible to determine the vertical position of the deformation centre:

$$z_D = \frac{h}{2} - \frac{M_t}{F_y}, \quad z_D^* = \frac{h}{2} - \frac{M_t}{F_y^*}, \tag{i}$$

where h is the double bottom height. The obtained results are compared with those of pure torsion determined in Section 7, in Table 1. The 3D FEM analyses show that the torsional centre and the shear centre are not the same points. In the thin-walled girder theory these two centres are not distinguished, and the unique deformation centre is determined. Probably, the suspended warping in the 3D FEM torsional analysis has some influence on the vertical position of the torsional centre. We can see that the transverse bulkheads also influence the position of the torsional and shear centres.

Table 1 **Vertical position of the deformation centre, z_D [m]**
Tablica 1 **Vertikalni položaj središta deformacije, z_D [m]**

	Without bulkheads	With bulkheads
Torsional centre, 3D FEM twisting	-10.60	-10.25
Shear centre, 3D FEM bending	-12.52	-11.96
Deformation centre, 2D strip theory	-13.50	

8.2 Vertical bending

A similar FEM analysis is performed for the investigation of vertical bending stiffness. The total vertical force, imposed at the free model end, is F_v = 2000 kN. The corresponding deflection at the same place, in the case of the model without and with transverse bulkheads, yields δ_z = -28.3706 mm and δ_z^* = -28.1697 mm, respectively. Thus, for the corrected vertical moment of inertia of the cross-section one finds:

$$I_y^* = \frac{\delta_z}{\delta_z^*} I_y = 1.00713\, I_y. \tag{j}$$

It is obvious that the influence of transverse bulkheads on the vertical stiffness is even lower than on the horizontal stiffness.

9 Conclusion

Hydroelastic analysis of large container vessels becomes an actual problem. For the reason of simplicity, a beam model of hull girder is coupled with a 3D hydrodynamic model. Instead of calculating the transverse bulkhead stiffness, the contribution of bulkheads to the global stiffness of the ship hull is determined by the 3D FEM analysis of a prismatic ship-like pontoon. This

is a simple and reliable engineering approach. It was found that the increase in torsional stiffness is considerable in the illustrated numerical example. The influence of this fact on the resonant ship hull response to wave excitation is significant and therefore has to be taken into account. On the other hand, the influence of the transverse bulkheads on vertical and horizontal bending stiffness is rather small and may be neglected.

However, some discrepancies between the thin-walled girder theory and the 3D FEM still exist. In the analysed numerical example of a ship hull segment, the twist angle determined by the beam analysis is significantly smaller than that obtained by the 3D FEM analysis. Even the twist angle of the beam without bulkheads is still lower than that of the 3D FEM model reinforced by bulkheads. Also, there are some discrepancies of the shear centre position between the 1D and 3D models without bulkheads. On the other hand, agreement between the cross-section warping is excellent. This problem will be the subject of further investigation.

Most of present papers dealing with problems of thin-walled structures are concentrated on the investigation within the thin-walled girder theory. The validation of results should be based on the correlation analysis with 3D FEM models which simulate the structure behaviour in a more realistic way. Also, some model tests and full scale measurements are very valuable for this purpose.

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Appendix

Torsional beam vibrations

The differential equation of uncoupled torsional beam vibrations can be written as an extension of the static equation (3), [4], [21]

$$EI_w \frac{\partial^4 \psi}{\partial x^4} - GI_t \frac{\partial^2 \psi}{\partial x^2} + J_t^0 \frac{\partial^2 \psi}{\partial t^2} = \mu_x(t), \qquad \text{(A1)}$$

where the twist angle ψ and the distributed torque μ_x are time dependent quantities. The symbol J_t^0 denotes the polar mass moment of inertia. Natural vibrations are harmonic and Eq. (A1) is reduced to the homogeneous form

$$EI_w \frac{\mathrm{d}^4 \psi}{\mathrm{d}x^4} - GI_t \frac{\mathrm{d}^2 \psi}{\mathrm{d}x^2} - \omega^2 J_t^0 \psi = 0, \qquad \text{(A2)}$$

where ψ and ω are the natural mode and the natural frequency, respectively.

Solution of (A2) is assumed in exponential form

$$\psi = \mathrm{e}^{\alpha x}. \qquad \text{(A3)}$$

By substituting (A3) into (A2) one finds the following biquadratic characteristic equation:

$$\alpha^4 - \frac{GI_t}{EI_w}\alpha^2 - \frac{\omega^2 J_t^0}{EI_w} = 0. \qquad \text{(A4)}$$

Its four roots yield

$$\alpha_j = \pm\gamma, \quad \pm i\eta, \qquad \text{(A5)}$$

where

$$\gamma = \sqrt{\frac{GI_t}{2EI_w}}\sqrt{\sqrt{1 + 4\omega^2 \frac{J_t^0 EI_w}{(GI_t)^2}} + 1} \qquad \text{(A6)}$$

$$\eta = \sqrt{\frac{GI_t}{2EI_w}}\sqrt{\sqrt{1+4\omega^2\frac{J_t^0 EI_w}{(GI_t)^2}}-1}. \qquad \text{(A7)}$$

Thus, the solution of (A2) takes the following form:

$$\psi = A_1 \operatorname{sh}\gamma x + A_2 \operatorname{ch}\gamma x + A_3 \sin\eta x + A_4 \cos\eta x. \qquad \text{(A8)}$$

Let us consider vibrations of a free beam with suspended warping at its ends. The corresponding boundary conditions read

$$x = \pm l: \quad T = 0, \quad u = 0 \qquad \text{(A9)}$$

that leads to

$$x = \pm l: \quad \frac{\mathrm{d}\psi}{\mathrm{d}x} = 0, \quad \frac{\mathrm{d}^3\psi}{\mathrm{d}x^3} = 0. \qquad \text{(A10)}$$

In the case of symmetric modes, $A_1 = A_3 = 0$, while for anti-symmetric modes $A_2 = A_4 = 0$. The corresponding eigenvalue problems yield

$$\begin{bmatrix} \gamma \operatorname{sh}\gamma l & -\eta \sin\eta l \\ \gamma^3 \operatorname{sh}\gamma l & \eta^3 \sin\eta l \end{bmatrix} \begin{Bmatrix} A_2 \\ A_4 \end{Bmatrix} = \{0\} \qquad \text{(A11)}$$

$$\begin{bmatrix} \gamma \operatorname{ch}\gamma l & \eta \cos\eta l \\ \gamma^3 \operatorname{ch}\gamma l & -\eta^3 \cos\eta l \end{bmatrix} \begin{Bmatrix} A_1 \\ A_3 \end{Bmatrix} = \{0\}. \qquad \text{(A12)}$$

For a nontrivial solution, determinants of (A11) and (A12) have to be zero. That leads to the frequency equations

$$\gamma\eta(\gamma^2+\eta^2)\operatorname{sh}\gamma l \ \sin\eta l = 0 \qquad \text{(A13)}$$

$$\gamma\eta(\gamma^2+\eta^2)\operatorname{ch}\gamma l \ \cos\eta l = 0 \qquad \text{(A14)}$$

with the same eigenvalue formula for the symmetric ($n = 0, 2\ldots$) and anti-symmetric ($n = 1, 3\ldots$) modes

$$\eta l = \frac{n\pi}{2}, \quad n = 0, 1, 2\ldots \qquad \text{(A15)}$$

Taking (A15) into account, one finds the following expression for natural frequencies of torsional vibrations from (A7)

$$\omega_n = \frac{n\pi}{2l}\sqrt{\frac{GI_t}{J_t^0}}\sqrt{1+\left(\frac{n\pi}{2l}\right)^2\frac{EI_w}{GI_t}}, \quad n = 0,1,2\ldots \qquad \text{(A16)}$$

Integration constants A_2 and A_4, and A_1 and A_3 are determined from (A11) and (A12), respectively. Symmetric and anti-symmetric natural modes according to (A8) yield

$$\psi_n = \eta_n \sin\eta_n l \cdot \operatorname{ch}\gamma_n x + \gamma_n \operatorname{sh}\gamma_n l \cdot \cos\eta_n x, \quad n = 0,2\ldots \qquad \text{(A17)}$$

$$\psi_n = \eta_n \cos\eta_n l \cdot \operatorname{sh}\gamma_n x - \gamma_n \operatorname{ch}\gamma_n l \cdot \sin\eta_n x, \quad n = 1,3\ldots \qquad \text{(A18)}$$

In case $n = 0$, the natural frequency $\omega_0 = 0$, Eq. (A16), and the natural mode $\psi_0 = 1$, Eq. (A17), that is related to the rigid body rotation.