Gain Scheduling-based Friction Compensation
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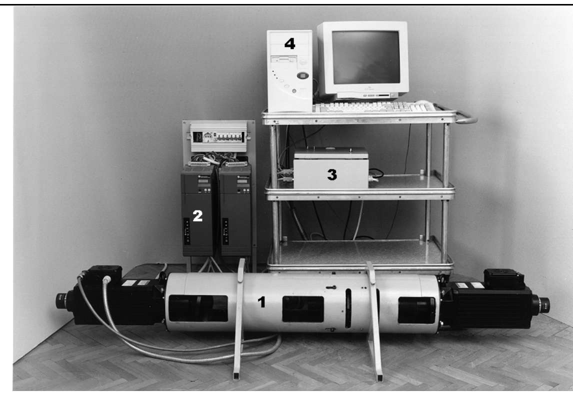
Abstract: In this contribution it is experimentally proved that friction can be regarded as damping in case of low speed motion. According to that, friction can be seen like part of the process and not like disturbances. Utilizing the root-locus method for the purpose of analysis, it is established that friction effect (stick-slip, hunting) can be rejected at low speed by means of large speed controller integral gain, with the stabilizing damping effect of the friction. The integral gain of the speed controller is realized by gain-scheduling utilizing the look-up table for the whole speed range. Remarkably good servosystem behavior is experimentally verified on a laboratory model by utilization of the proposed control concept.

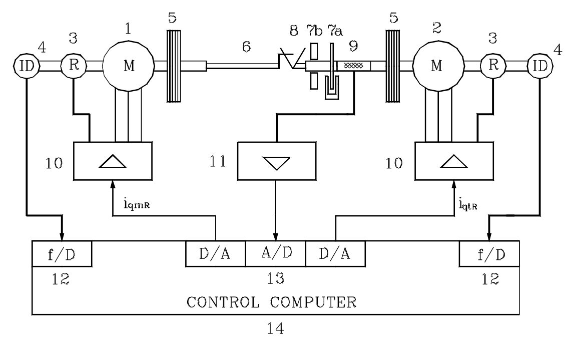
I INTRODUCTION

The purpose of the investigation performed is the development of an simple algorithm for the compensation of the non-linear friction characteristic in the low speed region, with the aim of improving the dynamic behavior of servo systems. Friction compensation at low speeds is needed in various machines and drives such as robots, CNC machine tools, radar antennae etc. [3] ,[4], [5], [9], [15]. The experimental verification of the proposed controller structure has shown a significant reduction in the friction effect in servodrives. Therefore, it is reasonable to expect that by application of the described controller structure, the dynamic behavior of various electromechanical systems can be significantly improved in the low speed region: for the direction change in the circular interpolation algorithm in CNC machine tools, thus improving the production quality, providing more precise robot manipulator positioning, etc.

Experiments described in this paper were conducted on a laboratory model of the servosystem designed for the purpose of investigating elasticity, backlash and friction effects in various mechanisms and devices [1], [7], [9], [10]. The photograph and the functional scheme are shown in Fig.1. The servosystem consists of two brushless servodrives (driving and loading motor), suitably designed mechanical components (shafts, devices for the generation of friction and backlash), measuring terms (angular displacement, speed and coupling torque) and a digital control system. For the purpose of investigation of friction effects, sliding pair steel-car brake friction coating is utilized. The driving torque is generated by the brushless servodrives with rated motor torque M_n = 14.8 Nm and rated speed n_n = 2000 rpm, while the angular displacement



1. Electromechanical plant   2. Servo unit
3. Interconnected circuits   4. Control computer   **a)**



1. 2.5 kW permanent-magnet synchronous machine (driving motor)
2. 2.5 kW permanent-magnet synchronous machine (load machine)
3. Resolver
4. Incremental encoder with sinusoidal output signals
5. Fly wheel with replaceable inertial disks
6. Elastic shaft
7. Braking devices for generation of friction in radial (7b) and axial (7a) rolling or sliding bearing
8. Backlash
9. Torque sensor (strain gauges glued on elastic shaft)
10. Servo unit (current-controlled frequency converter)
11. Torque measurement amplifier
12. Two-channel PC counter card for angle measurement
13. PC card with analog inputs and outputs
14. Control computer   b)

Fig.1. Laboratory model: a) photo of the model, b) functional scheme.

is measured by a precise incremental encoder with a resolution of 120000 pulses per revolution.

Owing to such high-quality angular displacement measurement, it is possible to measure micromotions (motions in the stick regime of friction). Since this investigation is conducted for rotational motion, the effect of friction is taken into account by the friction torque M_{fr}.

This paper is organized as follows: section 2 describes the physical background of method. A stability analysis is presented in section 3, followed by the description of simulation in section 4, and presentation of the experimental investigation in 5 section. A review of the results and applications of the proposed friction model is given in the conclusion (section 6).

II PHYSICAL BACKGROUND OF THE PROPOSED METHOD

This paper addresses the problem of friction effect on the dynamic behavior of servosystems in terms of an analogy between a servosystem with friction effect and a servosystem with friction effect replaced by an ideal damping element. [2].

The physical principles of the proposed friction effect compensation strategy are illustrated by the analogy between energy dissipation due to friction and damping by the ideal damping element, respectively.

The schematic representation of three different designs of a stiff transmission consisting of an actuator and a mass to be activated is given in Fig. 2.

The actuator control is designed utilizing the quasicountinuous-time approach shown in Fig. 3, where the discrete-time control system is replaced by an equivalent continuous-time control system. The controller is based on the linear P position controller, with the inner PI speed controller tuned according to the damping optimum [11],[16]. Time delays caused by D/A conversion, motor torque realization in the inner current (torque) control loop and digital speed measurement can be approximated by first-order lag terms [11], [15]. In order to simplify the control system design, the three lag terms $G_{SE}(s)$, $G_{ei}(s)$ and $G_{m\omega}(s)$ with nondominant time constants are in further discussion replaced by one mutual first-order lag term with the time constant: $T_\Sigma = T/2 + T_{ei} + T/2 = T_{ei} + T$.

In that case, according to Fig. 3 (assuming K_d=0), the following transfer function of the position control system is obtained:

$$G(s)=\frac{\alpha(s)}{\alpha_R(s)}=\frac{K_\omega K_\alpha}{T_I T_M T_\Sigma T_B s^4 + T_M T_I T_B s^3 + K_\omega T_I T_B s^2 + K_\omega T_B s + K_\alpha K_\omega} \qquad (1)$$

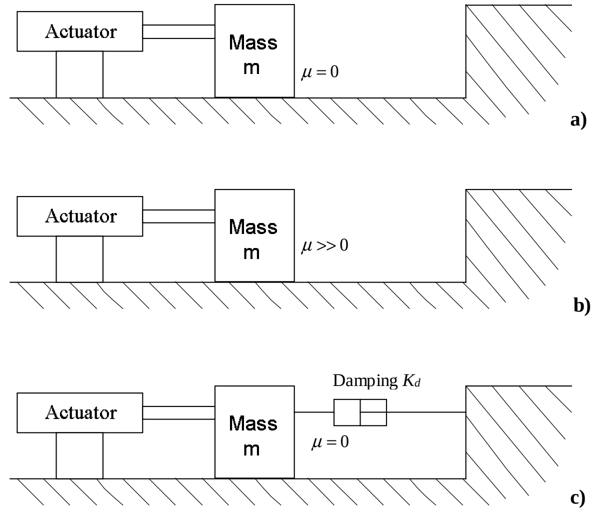


Fig. 2. Schematic representation of stiff transmission: : a) without friction and external damping , b) with friction but no external damping without friction but with external damping.

where the meaning of the individual coefficients is evident from Fig. 3.

Transfer function (1) provides a full description of the system shown in Fig. 2(a), whose behavior is not affected by either friction or an ideal damping element ($K_d = 0$).The second case shown in Fig. 2(b) refers to a real-time system, the operation of which is affected by friction. As a result of the markedly non-linear nature of the friction force, it is impossible to derive a simple transfer function for this case.

Fig. 2(c) describes a system, where the effect of friction is replaced by an ideal linear damping element $(K_d > 0)$. Considering the similar nature of the stabilizing damping effect of friction and the ideal damping element, this is reasonable to assume when discussing qualitatively the effect of friction on the stability of the controlled drive. Namely, both friction and the ideal damping element act by opposing motion, their dissipation power equaling the product of speed and resistance to motion. The difference between the damping effect of friction and that of the ideal damping element is expressed by the function which correlates speed and resistance to motion: this function is linear for the ideal damping element and non-linear for friction. The transfer function for this case is given by equ. (2).

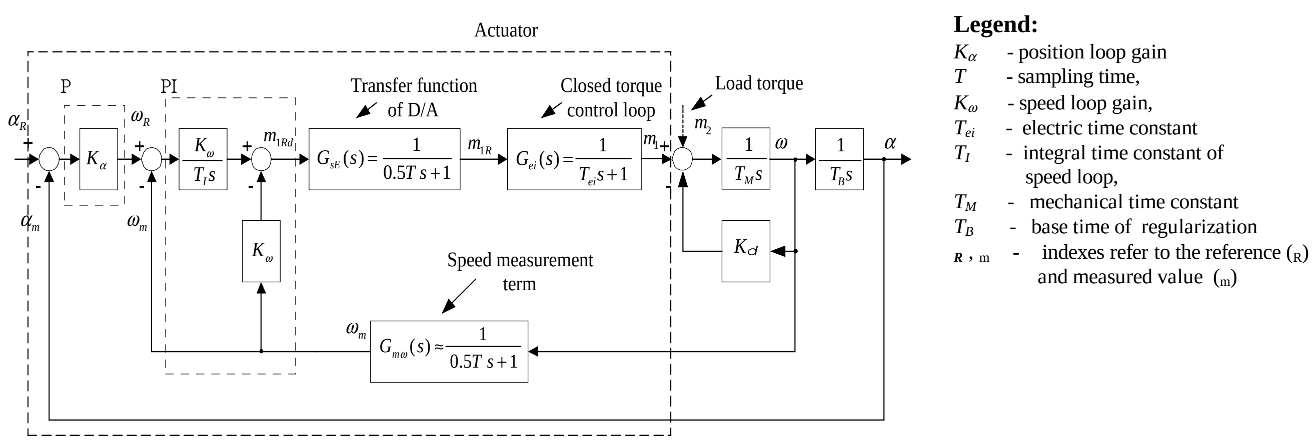


Fig. 3. **Structural block diagram of the equivalent continuous-time position control system of an servodrive with stiff transmission.**

$$G(s)=\frac{\alpha(s)}{\alpha_R(s)}=\frac{\frac{K_\omega^2}{K_d}K_a}{\frac{K_I}{K_\omega}T_M T_\Sigma T_B s^4+(K_d T_\Sigma+T_M)\frac{K_I}{K_\omega}T_B s^3+(K_d+K_\omega)\frac{K_I}{K_\omega}T_B s^2+K_\omega T_B s+K_a K\ \frac{K_\omega^2}{K_d}} \tag{2}$$

III STABILITY ANALYSIS

For the purpose of general analysis of the damping effect caused by friction, the friction effect is analyzed as a linear damping term with the gain K_d (Fig. 2 and 3). By applying the root-locus method [8], it is possible to investigate separately the influence of the integral gain $K_I=1/T_I$ increase (with respect to the integral gain K_L obtained by means of the damping optimum) and the increase of the gain K_d on the position of the closed position control loop poles in *s* - plain, i.e. on its stability (Figs. 4 and 5).

Fig. 4 shows the influence of integral gain increase K_I on the position of servosystem poles for K_d=0.00001. The integral gain increase produces a more rapid response to disturbance with a concomitant loss of stability. If $K_I=20K_L$ is assumed, where K_L denotes the integral gain obtained by the damping optimum [15], the system is markedly unstable in terms of oscillations.

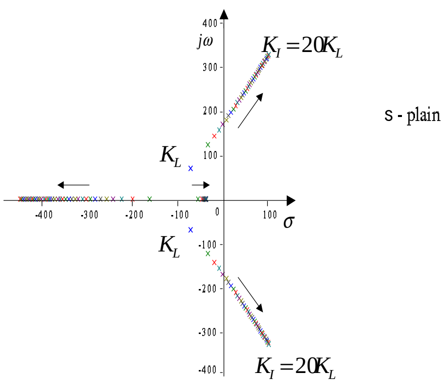


Fig. 4. The influence of the integral gain K_I on the position of the closed position control loop poles (with K_d = 0.00001).

With $K_I=20K_L$ and concomitant damping gain increase K_d in the ideal damping element, the unstable poles shift back to the left half-plain, thereby restoring system stability (see Fig. 5).

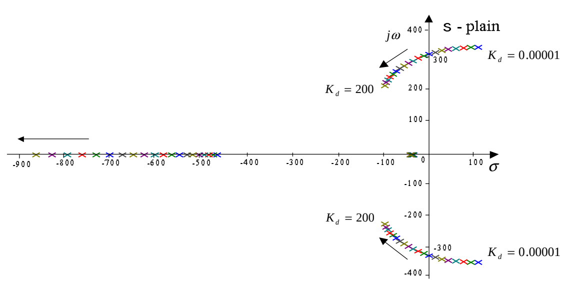


Fig. 5. The influence of the K_d gain increase of the ideal *damping term on the position of the closed position control loop poles with* K_I = 20K_L.

IV SIMULATION INVESTIGATION OF THE PROPOSED APPROACH

The feasibility of the proposed approach was tested by simulation. The parameters of the linear controller described by the transfer function (1) were calculated according to the damping optimum [15], [13].

The parameters of the controller tuned to the damping optimum are obtained by equalizing the coefficients of the polynomial $A(s)$ in the denominator of the closed-loop transfer function (1):

$$\begin{aligned}A(s)&=a_0+a_1 s+a_2 s^2+\ldots+a_n s^n=\\&=K_\alpha K_\omega+K_\omega T_B s+_\omega T_I T_B s^2+T_M T_I T_B s^3+T_I T_M T_\Sigma T_B s^4\end{aligned}, \tag{3}$$

with the coefficients of the characteristic polynomial of the damping optimum $N_{DO}(s)$ [14]:

$$\begin{aligned}N_{DO}(s)=a_o(1&+T_e s+D_2 T_e^2 s^2+D_3 D_2^2 T_e^3 s^3+\ldots\\&\ldots+D_n D_{n-1}^2\ldots D_3^{n-2} D_2^{n-1} T_e^n s^n)\end{aligned}. \tag{4}$$

The characteristic relationships are denoted by coefficients D_i, the equivalent time constant of the closed control loop being T_e. The controller parameters obtained by coefficient equalization are shown in Table 1 [14].

Due to physical constraints of the actuators, the integral performance of the PI controller must be restricted [11]. This project utilizes the “antireset windup” approach, where the integrator is adjusted to a value that provides the maximum permitted controller output.
In the simulations performed a fixed fifteen-fold increase of the internal gain $K_I=15/T_I$ was employed. The effect of friction was simulated by using a reset integrator model [6]

TABLE I

. Speed and Position Controller Perameters Tuned to The Damping Optimum

T_e	K_a	T_I	K_ω
$\frac{T_\Sigma}{D_2 D_3 D_4}$	$\frac{T_B}{T_e}$	$D_2 T_e$	$\frac{T_M}{D_3 D_2 T_e}$

shown in Fig. 6. This model was obtained by simplifying the bristle model described in [6].

In this model the friction force in the static friction region is dependent on position and proportional to bristle displacement *p*.

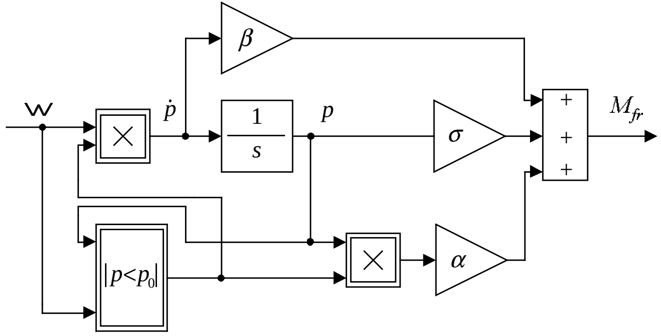


Fig. 6. Simulation reset integrator model.

The relative speed between contact surfaces ω represents the reset integrator model input. Provided that the condition:

$$|p| < p_0 \Rightarrow \dot{p} = \omega , \qquad (4)$$

is fulfilled, the logic block output $|p| < p_0$ assumes the value of 1, in which case the relative speed ω equals the bristle bending speed $\dot{p}$, with p_0 denoting maximum bristle displacement. . The bristle displacement p ids obtained by integrating the bristle displacement speed $\dot{p}$. When the bristle displacement p is multiplied by σ, the Coulomb friction M_C is obtained, while multiplication of bristle displacement p by a yields the difference between static and Coulomb friction ($Ms - M_C$).When these values ($p\sigma + pa$) are added to the product of the damping coefficient β and the bristle bending speed $\dot{p}$, the friction moment M_{fr} in the static friction region is obtained.

When bristle displacement assumes its maximum value p_0, i.e. on fulfillment of the condition :

$$\dot{p} = 0 \wedge p = p_0 \ \text{ for } \left\{ \begin{matrix} \omega > 0 \wedge p \geq p_0 \\ \vee \\ \omega < 0 \wedge p \leq -p_0 \end{matrix} \right\}, \qquad (5)$$

asperite bonds break and motion occurs. The logic block output $|p < p_0|$ assumes the value of 0, integration stops, the correction factor a no longer applies and the effect of damping β ceases to act. The friction moment in this region becomes $M_{fr} = p_0\sigma$, which corresponds to Coulomb friction. By introducing damping β oscillations in the static friction region, typically associated with the bristle model, can be avoided.

Step responses of the closed position control loop are shown in Fig. 7. The "dissipative torque" of the ideal damping term (K_d=70) is equal to the product of the gain K_d and the speed ω, while with friction it is approximately constant. When the speed, and thus the torque of the linear damping term, reaches a certain value, the damping effect of the friction is no longer capable of stabilization, and limit-cycle oscillations occur. In the case when the reference value is 0.64° (Fig. 8(a)) , the stabilizing effect of the friction is sufficient, and the system settles at the new value after 0.25 s. In Fig. 8(b) the step response for the reference value α_R = 0.73° is shown. The stabilizing friction effect is not sufficient and the system enters limit-cycle oscillations.

V. Experimental results

Fig. 8. shows the simplified structural block diagram of the position cascade control of an servodrive with friction effect compensation based on gain scheduling [12] of the speed controller integral gain. In distinction from the common PI speed controller, this controller utilizes gain scheduling for integral gain K_I tuning. With the increase of the integral gain K_I , fast load torque compensation is obtained, i.e. the load torque effect caused by variation of the friction torque at low speeds can be efficiently compensated. Utilizing the proper parameter tuning logic (Fig. 8), the integral gain K_I is held at the appropriate value, which guarantees stable system behavior ($K_I = f(\omega_R , K_L)$).

$$K_I = \begin{cases} 15K_L, & \omega_R < 0.0005 \\ K_L, & \omega_R > 0.0005 \end{cases} \qquad (6)$$

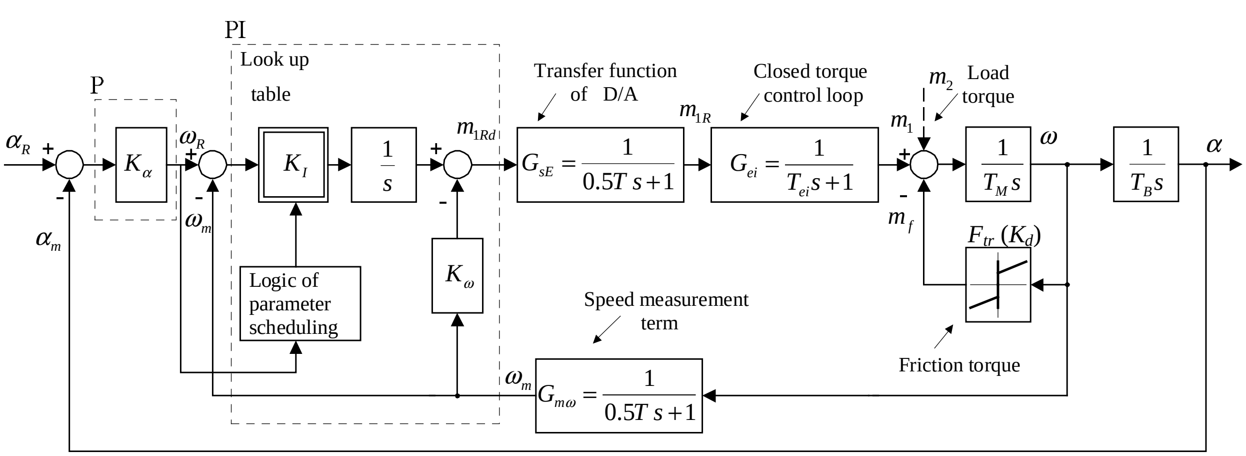


Fig. 8. Structural block diagram of the position cascade control system of the electrical drive with friction effect compensation utilizing speed controller gain scheduling

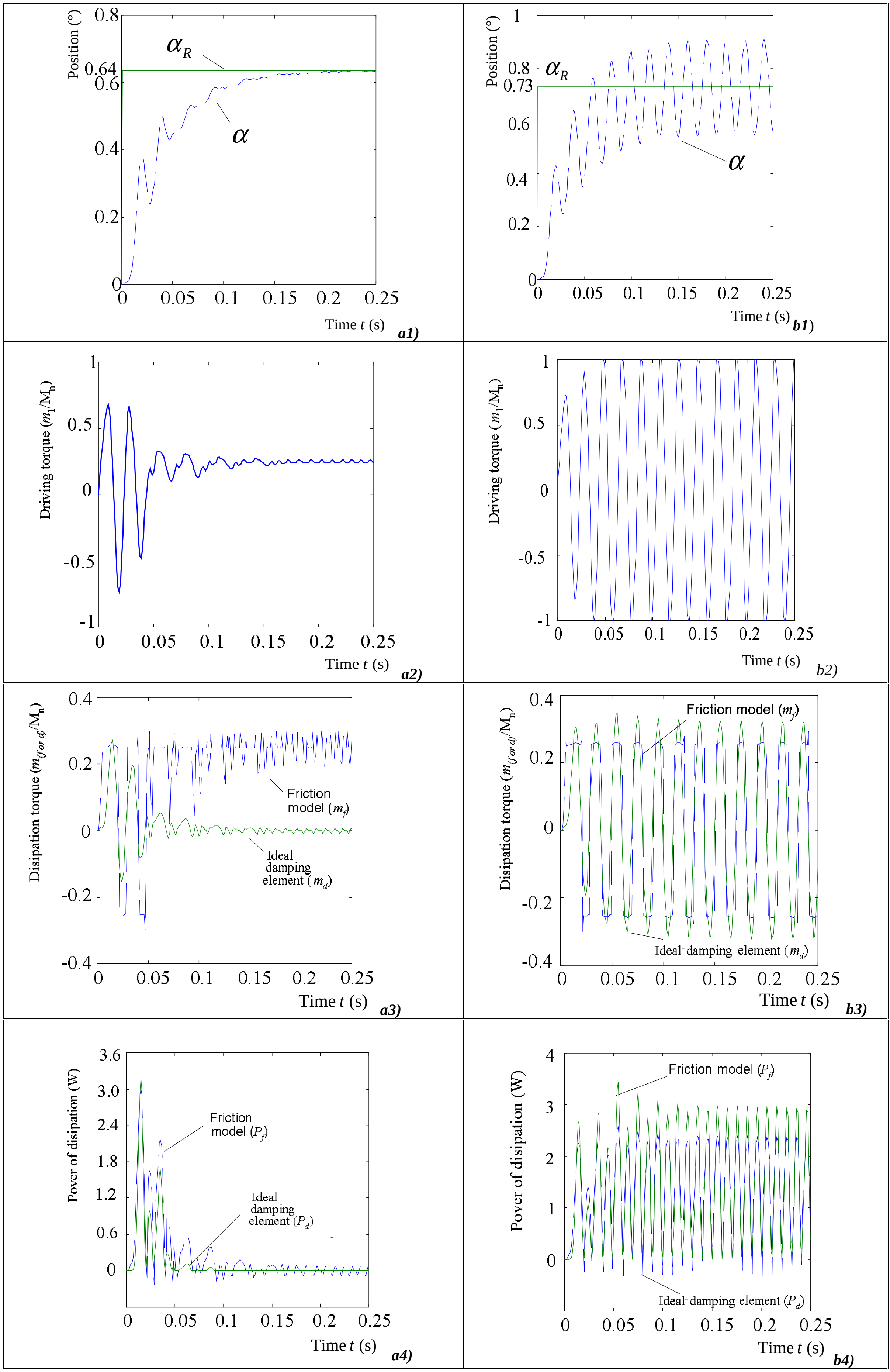


Fig. 7. Step responses of a simulated position control system with friction of::

a) sufficiently stabilizing effect b)insufficiently stabilizing effect..

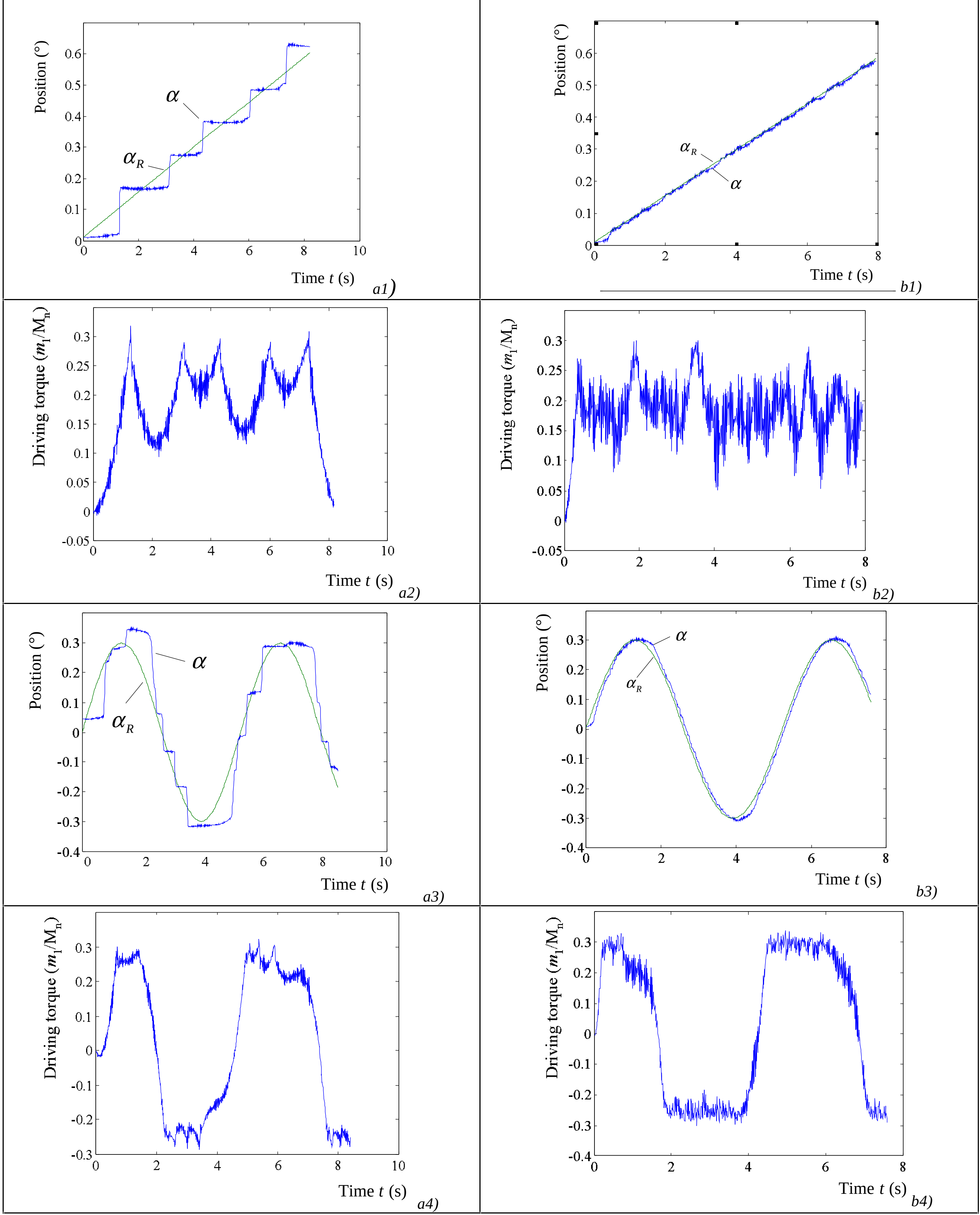


Fig. 9. Corresponding responses of the control system with: a) linear speed controller tuned according to the damping optimum and b) speed controller with compensation of the friction effect.

The effect of the proposed friction compensation method on the servodrive is shown in Fig. 9. The results are shown for the discrete-time implementation of the control system with linear speed controller tuned according to the damping optimum (Fig. 5(a1-a4)), and the speed controller with compensation of the friction effect by means of gain-scheduling (Fig. 5(b1-b4)). Experiments have been conducted on a laboratory model [9], [15]. By comparison of the corresponding responses, it is easy to notice the significant improvement in friction effect compensation obtained by utilizing gain-scheduling . It should be noted that this improvement is achieved without significant control effort (i.e. motor torque m_1), which is especially important with respect to transmission mechanism durability in controlled servodrives.

VI Conclusion

This paper deals with a novel approach to friction effect compensation in servodrives. The recognition of friction as part of actuator operation is its major contribution. Due to the damping effect of friction in the low speed range, the speed controller integral gain may be increased without any risk of stability loss. . By employing large speed controller integral gains, it is possible to effectively reject the influence of abrupt friction torque changes (stick-slip, hunting), thus significantly reducing the positioning and tracking error of servosystems with marked friction effect.

Apendix

Simulations are performed with normalized parameters (Table 2). Base values for normalization are; a rated motor torque M_n = 14.8 (Nm) and rated speed n_n = 2000 (rpm) with T_B=1 (s). Servodrive total inertia is 0.0336 (kgm^2). Simulation step size is $T_s = T/10$.

TABLE II.
Normalized Parameters

T_M	0.4773 (s)
T	0.001 (s)
T_{ei}	0.0025 (s)
T_Σ	0.0035 (s)
p_o	0.6e-6
σ	416667
a	43333
β	7
D_2	0.37
D_3= D_4	0.5

Acknowlegment

The authors would like to thank the Ministry of Science and Technology and the Ministry of Defense of the Republic of Croatia for the financial support of this project.

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