# STRESS AND STRAIN DISTRIBUTION IN CONCRETE BEAMS REINFORCED WITH FRP BARS

Tomislav KIŠIČEK [1] Zorislav SORIĆ [1] Josip GALIĆ [1]

[1] Civil Engineering Faculty, University of Zagreb, Kačićeva 26, 10000 Zagreb, Croatia

**Keywords**: bond/slip behaviour, concrete, curvature, deflection, FRP bars, numerical procedure.

## 1 INTRODUCTION

The most common material in the construction of buildings is a concrete reinforced with steel reinforcement. If reinforced concrete element was placed in aggressive environment, its deterioration due to corrosion of steel reinforcement will start soon. In the last few decades a lot of attention has been devoted to fiber reinforced polymer (FRP) as a replacement for steel reinforcement. FRP materials could be produced from glass (G), aramid (A) or carbon (C) fibers which are connected with epoxy resin, polyester or vinyl-ester, and therefore they can be distinguished as GFRP, AFRP and CFRP products. Beside the fact that FRP products do not corrode, they have other good characteristics such as: high tensile strength, good behavior under dynamic loadings (CFRP), little relaxation, moisture resistance, and low weight. They are magnetic resistant and electric isolators. FRP products also have deficiencies such as elastic behavior until failure and big differences between longitudinal and transversal mechanical characteristics and between tensile and compressive stresses. Main deficiencies of FRP when compared to steel reinforcement are their non-ductile behavior and creep rupture. In general, FRP materials have lower modulus of elasticity than steel reinforcement, and lower longitudinal coefficient of thermal expansion. FRP reinforcement is sensitive to ultraviolet exposure. GFRP is also sensitive to alkaline environment.

The iterative numerical procedure for calculation of stresses and strains of concrete and FRP reinforcement in concrete beam reinforced with FRP bars has been developed. The focus of this procedure was distribution of stresses and strains, between two adjacent cracks. This is important because strains of reinforcement and concrete are main parameters for calculation of curvature of cross section and therefore, for calculation of deflections of reinforced concrete beams.

The procedure included bond/slip behavior of reinforcement and formation of new crack(s) between initial ones. In the analysis the real bond/slip curve could be used, as well as idealized one. In this paper, the impact of bond/slip curves on speed and accuracy of iterative numerical procedure would be analyzed.

## 2 BOND-SLIP DISTRIBUTION BETWEEN TWO ADJACENT CRACKS

The model of bond-slip distribution of reinforcement and concrete between two adjacent cracks, described by Manfredi and Pecce [10], for concrete beams reinforced with steel reinforcement, is an iterative process. In this research this model has been rearranged for beams with FRP reinforcement. The procedure for calculation of bond-slip distribution is based on three equations that connect all variables:

1) Equilibrium of horizontal forces in the cross section (Fig. 1):

$$F_{ct} + F_{f1} = F_c + F_{s2}\,, \tag{1}$$

where: $F_{ct}$ is tensile force in concrete, $F_{f1} = \sigma_{f1} \cdot \sum \frac{d_b^2 \cdot \pi}{4}$ is tensile force in FRP reinforcement, $F_c$ is compressive force in concrete and $F_{s2}$ is compressive force in steel reinforcement (if exists). Concrete stress-strain relation in compression is taken as parabolic-horizontal line diagram. Concrete behaviour in tension is shown in figure 2., where: $\varepsilon_{ct,m}$ is concrete tensile strain corresponding to $f_{ct,m}$ which is concrete tensile strength, $f_{ct,u}$ is maximal concrete tensile stress corresponding to $\varepsilon_{ct,u}$ which is maximal concrete tensile strain between two adjacent cracks.

2) Equilibrium of forces due to stresses in the FRP bar, and their surface:

$$\frac{d\sigma_{f1}}{dx}-\tau_b\cdot\frac{\sum d_b\cdot\pi}{\sum\frac{d_b^2\cdot\pi}{4}}=\frac{d\sigma_{f1}}{dx}-\frac{4\cdot\tau_b}{d_b}=0 \tag{2}$$

where: $\sigma_{f1}$ is tensile stress of FRP reinforcement, $d_b$ is diameter of tensile reinforcement and $\tau_b$ is bond stress between reinforcement and concrete, and x is the distance in longitudinal direction of the FRP bars. Bond stress distribution between two adjacent cracks can be calculated from equation (2) which can be rewritten as:

$$\tau_b(x)=\frac{d\sigma_{f1}}{dx}\cdot\frac{d_b}{4} \tag{3}$$

3) Definition of slip of FRP bar in concrete:

$$\delta=\int_0^x(\varepsilon_{f1}-\varepsilon_c)\cdot dx \tag{4}$$

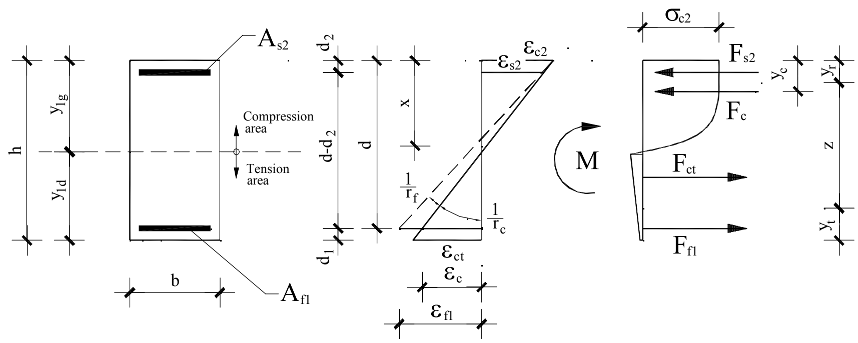


**Fig. 1** Internal forces in uncracked cross sections between two adjacent cracks

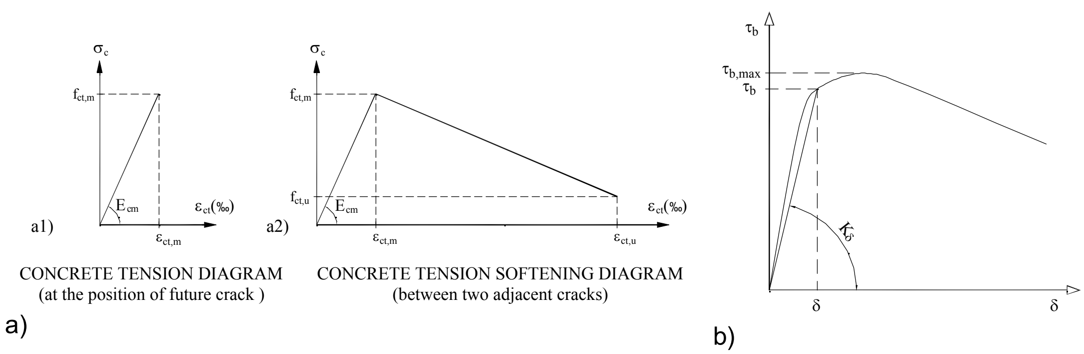


**Fig. 2** a) Concrete tension stress-strain diagrams, b) Bond stress-slip diagram

Differentiation of equation (2) with respect to x leads to:

$$\frac{d^2\sigma_{f1}}{dx^2}-\frac{d\tau_b}{dx}\cdot\frac{4}{d_b}=0 \tag{5}$$

Relationship between tensile stresses in FRP reinforcement in (i) and (i-1) cross section is given by equation:

$$\sigma_{f1,(i)}=\sigma_{f1,(i-1)}+\tau_{b,(i-1)}\cdot\frac{4}{d_b}\cdot\Delta x \tag{6}$$

where $\Delta x$ is distance between (i-1) and (i) cross sections.

Before the iterative process started, the initial bond stress, $\tau_b$, had to be assumed. The recommended initial value of $\tau_b$ is $\tau_{b,max}/2$ (Fig. 2b)). At the beginning of calculation, the initial cross-section of the beam element is named (i-1), and the tensile stress in FRP reinforcement in this section, $\sigma_{f1,(i-1)}$, could be easily obtained from equilibrium conditions (Fig. 3).

Tensile stress in FRP reinforcement, $\sigma_{f1,(i)}$ in next section (i), could then be calculated by using expression (6). Strains of concrete and FRP reinforcement for this cross section could be calculated from equilibrium conditions for uncracked cross section (Fig. 1). Slip, $\delta_{(i)}$, of FRP reinforcement in (i) cross section is given by equation (7):

$$\delta_{(i)} = \delta_{(i-1)} + \left( \frac{\varepsilon_{f1,(i)} - \varepsilon_{c,(i)}}{2} + \frac{\varepsilon_{f1,(i-1)} - \varepsilon_{c,(i-1)}}{2} \right) \cdot \Delta x \qquad (7)$$

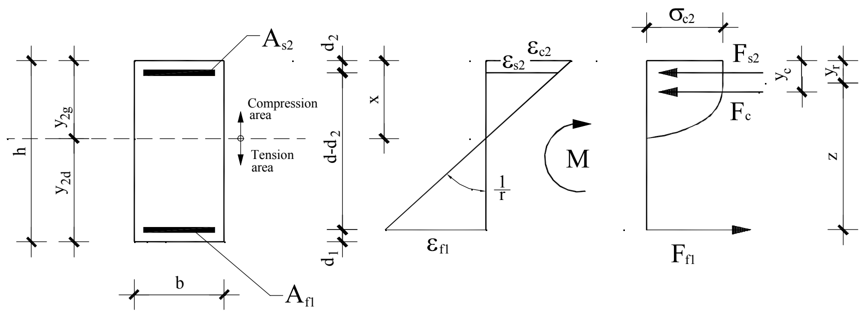


**Fig. 3** Internal forces in cracked cross section

Knowing the slip, $\delta_{(i)}$, corresponding bond stress $\tau_{b,(i)}$ can be determined from experimental bond-slip diagram (Fig. 2b)). This procedure should be repeated for every next cross section at distance $\Delta x$ from previous section, between two adjacent cracks. Finally this procedure reaches the next cracked cross section, where tensile stress in FRP reinforcement, $\sigma_{f1}$, is known from equilibrium conditions (Fig. 3) Then, tensile stresses in FRP should be compared, one obtained by equation (6), and one from equilibrium conditions. If these two values differ substantially the procedure should be repeated with new assumption of initial bond stress $\tau_b$ until difference between two values of $\sigma_{f1}$ became negligibly small.

## 3 EXAMPLE

Theoretical and experimental investigations (four point bending) of concrete beams reinforced with GFRP bars were made. Cross section of beams was rectangular (20/28 cm), beams were cast of concrete class C35/45 and tensile reinforcement was 3$\phi$12.7 mm GFRP bars. Beam of span was 300 cm, were subjected with two concentrated loading force at distance of 100 cm from supports. Modulus of elasticity of GFRP bars were 41600 N/mm$^2$, and their ultimate tensile strength was 724 N/mm$^2$. The bond/slip diagrams were obtained from experiments on standard specimen according to RILEM [8]. Theoretical calculations of load/deflections diagrams for such beams were made using the method which has incorporated calculations of stresses and strains between two adjacent cracks. The experimentally obtained bond/slip curves are shown on figure 4a). The results of the deflection calculations are shown on figure 4b) together with experimental results.

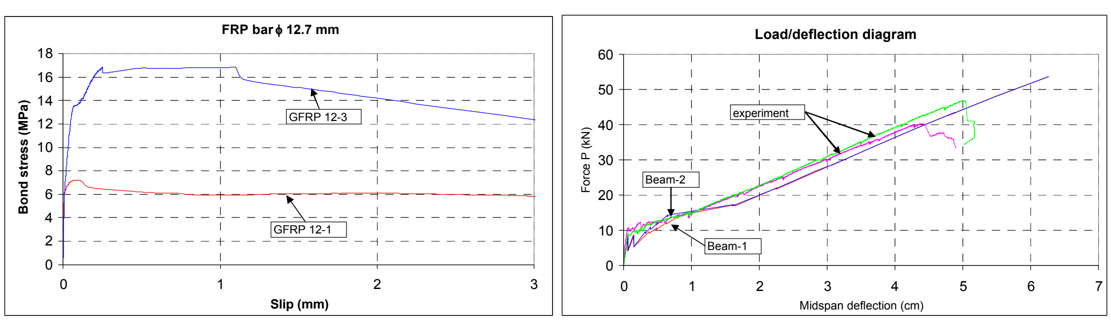


**Fig. 4** a) Bond/slip diagrams of GFRP bars $\phi$12.7 mm, b) Load/deflection diagrams

## 4 CONCLUSIONS

The following conclusions are deduced from the experimental results:

- Bond/slip characteristics of reinforcement have great influence on deflections during cracking of beam. Higher bond stress between reinforcement and concrete results in smaller deflections. After cracking formation is stabilized this influence is negligible. Bond/slip curves have great impact on calculation model, his speed and accuracy.
- The force-deflection curves, that were calculated with the procedure that was proposed here show good agreement with experimental curves. With this analytical procedure, at every point along the beam and for every step of loading, stresses and strains of reinforcement and concrete, bond stress and slip of reinforcement, as well as deflection and appearance of new cracks could be determined.

## ACKNOWLEDGEMENTS

The investigations described herein have been done in the frame of the research project No. 0082203 “Application of non-metal materials in concrete structures” which has been supported by a grant from the Ministry of Science, Education and Sport of the Republic of Croatia. All supports have been gratefully acknowledged.

## REFERENCES

[1] ACI Committee 440, "Guide for the Design and Construction of Concrete Reinforced with FRP Bars", 440.1R-01, American Concrete Institute, Farmington Hills, MI, 2001.

[2] Benmokrane, B., Chaallal, O., Masmoudi, R., "Flexural Response of Concrete Beams Reinforced With FRP Reinforcing Bars", ACI Structural Journal, V. 93, No. 1, January-February 1996., pp. 46-55

[3] Masmoudi, R., Theriault, M., Benmokrane, B., "Flexural Behaviour of Concrete Beams Reinforced With Deformed Fiber Reinforced Plastic Reinforcing Rods", ACI Structural Journal, V. 95, No. 6, November-December 1998., pp. 665-675

[4] Alsayed, S.H., Al-Salloum, Y.A., Almusallam, T.H., "Performance of Glass Fiber Reinforced Plastic Bars as a Reinforcing Material for Concrete Structures", Composites: Part B 31, 2000., pp. 555-567

[5] Abdalla, H.A., "Evaluation of Deflection in Concrete Members Reinforced With Fibre Reinforced Polymer (FRP) Bars", Composite Structures 56, 2002., pp. 63-71

[6] Branson, D.E., “Design procedures for computing deflections”, ACI Journal, September 1968., pp. 730-742

[7] Benmokrane, B., Zhang, B., Laoubi, K., Tighiouart, B., Lord, I., "Mechanical and Bond Properties of New Generation of Isorod CFRP Reinforcing Bars for Concrete Structures", Technical Progress Report, NSERC Research Chair in FRP Reinforcement for Concrete Structures, Canada, April 2001.

[8] Fib bulletin 10, "Bond of reinforcement in concrete", August, 2000

[9] Damjanic, F. B., "Reinforced concrete failure prediction under both static and transient conditions", Ph.D. Thesis, Department of Civil Engineering, University College of Swansea, United Kingdom, 1983.

[10] Manfredi, G., Pecce, M., "A refined R.C. beam element including bond–slip relationship for the analysis of continuous beams", Computers & Structures, Volume 69, Issue 1, October 1998, pp. 53-62