SPECTRAL CALCULATION OF THE OUTPUT VOLTAGE OF AN INVERTER WITH BIPOLAR PULSE WIDTH MODULATION

Ivan Mrčela[1], Viktor Šunde[2], Zvonko Benčić[3]
University of Zagreb, Faculty of Electrical Engineering and Computing
Zagreb, Croatia
ivan.mrcela@fer.hr[1], viktor.sunde@fer.hr[2], zvonko.bencic@fer.hr[3]

Abstract. Converters with pulse width modulation are used for connections between the DC and AC networks, e.g. in uninterrupted power supply systems, AC electromotor drives, for powering induction furnaces, in audio technique. Spectrum of signals sampled by pulse amplitude modulation and output voltage spectrum of the converter with pulse width modulation have similar properties. Spectrum of signals sampled by pulse amplitude modulation contains a harmonic of frequency equal to the frequency of the modulating signal and the harmonics of frequencies equal to the sum of frequency of the modulating signal and multiples of the sampling frequency. The output voltage spectrum of the converter with bipolar pulse width modulation contains harmonic of frequency equal to the frequency of the modulating signal and harmonics of frequencies equal to sum of the frequency of the modulating signal and multiples of the frequency of the carrier signal. It also contains harmonics of frequencies equal to the sum of the multiples of the frequency of the modulating signal and the multiples of the carrier signal. The comparison analysis was carried out for the harmonics of the output voltage of the converter with bipolar pulse width modulation in time domain. The dependency of the amplitudes and frequency spectre on the wave forms of the carrier signal and modulating signal was shown. Similarity of the output voltage spectrum of the converter and signal spectrum sampled by the pulse width modulation was also shown.

Keywords. output voltage converter with bipolar pulse width modulation, spectral analysis, Fourier series, carrier signal, reference signal.

INTRODUCTION

In [1] voltage spectrum calculation was carried out for the pulse width modulation converter (PWM--converter) using the Fourier transformation for a two variable function (double variable controlled waveform Fourier series).

The calculation was performed by splitting of the voltage waveform into a Fourier series and by calculating the series coefficients, assuming a periodical voltage. The periodical voltage is essential for the foregoing applications.

A periodical signal with period T_0 can be represented by Fourier series in trigonometric or in complex form. Trigonometric form of Fourier series is:

$$g(t) = a_0 + \sum_{k=1}^{\infty}\left[a_k \cdot \cos\left(\frac{2 \cdot k \cdot \pi \cdot t}{T_0}\right) + b_k \cdot \sin\left(\frac{2 \cdot k \cdot \pi \cdot t}{T_0}\right)\right] \tag{1}$$

Complex form of Fourier series is:

$$g(t) = \sum_{m=-\infty}^{\infty} c_m \cdot e^{\mathrm{j}\cdot\frac{2\cdot k\cdot\pi\cdot t}{T_0}} \tag{2}$$

If it can be assumed that the function $g(t)$ is square integrable over the interval $\left[-\frac{T_0}{2},\frac{T_0}{2}\right]$, coefficients a_0, a_k, b_k and c_k are:

$$a_0 = \frac{1}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) \cdot \mathrm{d}t \tag{3}$$

$$a_k = \frac{2}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) \cdot \cos\left(\frac{2\cdot k\cdot\pi\cdot t}{T_0}\right) \cdot \mathrm{d}t \tag{4}$$

$$b_k = \frac{2}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) \cdot \sin\left(\frac{2\cdot k\cdot\pi\cdot t}{T_0}\right) \cdot \mathrm{d}t \tag{5}$$

$$c_k = \frac{1}{2} \cdot \left(a_k - \mathrm{j}\cdot b_k\right) = \frac{1}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) \cdot e^{-\mathrm{j}\cdot\frac{2\cdot k\cdot\pi\cdot t}{T_0}} \cdot \mathrm{d}t \tag{6}$$

As an example a spectral calculation was carried out for the reference signal of the sinusoidal waveform sampled by pulse amplitude modulation.

Sampling of a signal by a series of the Dirac delta-functions gives the signal value (amplitude) for the moments of sampling. Sampling of a signal with pulse amplitude modulation can be mathematically modelled as a product of the modulating signal and carrier signal.

Sampling of a signal with pulse width modulation gives the median value signal for the sampling period by the ratio of durations of individual positive and negative pulses. Sampling of a signal with pulse width modulation can be mathematically modelled as a sign function of the difference between the modulating signal and the carrier signal.

Fig. 1. a) shows a block diagram of the sampling model for the sinusoidal signal by pulse amplitude modulation, and Fig. 1. b) shows a block diagram of the sampling model of the sinusoidal signal by pulse width modulation.

Modulation of the reference signal is, according to [2], carried out because the form of the modulated signal is suitable for transmission through the individual medium.

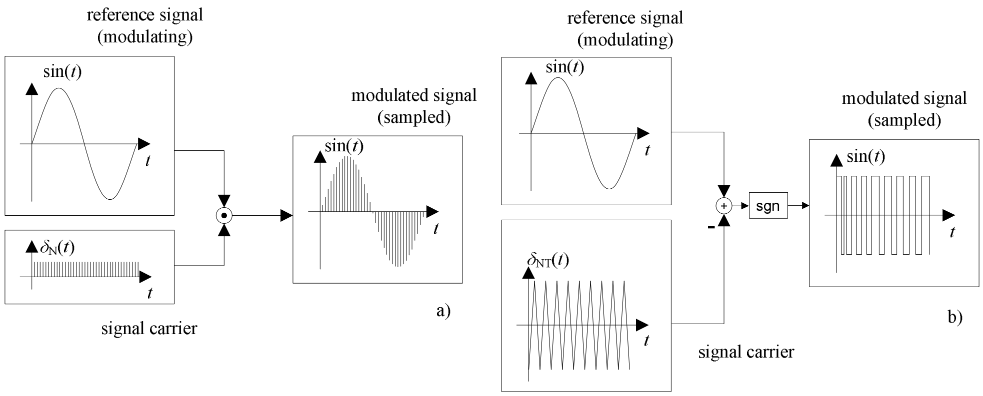

Fig. 1. Reference signal (modulating), carrier signal (transmitting signal) and modulated signal (sampled)

The Fourier series for the signal modulated by pulse amplitude modulation is calculated as a product of the modulating signal Fourier series and the carrier signal Fourier series. The first step in the modulated signal spectrum calculation is calculation of the coefficients separation for the carrier signal into a Fourier series. The Fourier series in the complex form of Dirac delta-function $\delta_S(t)$ is:

$$\delta_S(t)=\sum_{k=-\infty}^{\infty} c_k\cdot e^{\mathrm{j}\cdot\frac{2\cdot k\cdot\pi\cdot t}{T_s}}=\frac{1}{T_S}\cdot\sum_{k=-\infty}^{\infty} e^{\mathrm{j}\cdot\frac{2\cdot k\cdot\pi\cdot t}{T_s}} \tag{7}$$

Fourier series in complex form of the reference signal $f_r(t) = \sin(\omega_r \cdot t)$ is:

$$f_r(t)=\frac{1}{2\cdot\mathrm{j}}\cdot\left(e^{\mathrm{j}\cdot\omega_r\cdot t}-e^{-\mathrm{j}\cdot\omega_r\cdot t}\right) \tag{8}$$

The coefficients of the complex form Fourier series (8) are 1 and –1.
The complex form Fourier series of the sinusoidal waveform signal sampled by pulse amplitude modulation is:

$$\begin{aligned}
f_M(t)&=f_r(t)\cdot\delta_S(t)=\frac{1}{2\cdot\mathrm{j}}\cdot\left(e^{\mathrm{j}\cdot\omega_r\cdot t}-e^{-\mathrm{j}\cdot\omega_r\cdot t}\right)\cdot\frac{1}{T_S}\cdot\sum_{k=-\infty}^{\infty} e^{\mathrm{j}\cdot\frac{2\cdot k\cdot\pi\cdot t}{T_s}}=\\
&=\frac{1}{2\cdot\mathrm{j}\cdot T_S}\cdot\sum_{k=-\infty}^{\infty}\left[e^{\mathrm{j}\cdot\left(\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}-e^{\mathrm{j}\cdot\left(-\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}\right]=\\
&=\frac{1}{2\cdot\mathrm{j}\cdot T_S}\cdot\left[\sum_{k=-\infty}^{\infty} e^{\mathrm{j}\cdot\left(\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}-\sum_{k=-\infty}^{\infty} e^{\mathrm{j}\cdot\left(-\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}\right]=\\
&=\frac{1}{2\cdot\mathrm{j}\cdot T_S}\cdot\left[\sum_{k=-\infty}^{\infty} e^{\mathrm{j}\cdot\left(\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}-\sum_{k=-\infty}^{\infty} e^{-\mathrm{j}\cdot\left(\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}\right]=\\
&=\frac{1}{2\cdot T_S}\cdot\left[\sum_{k=-\infty}^{\infty}(-\mathrm{j})\cdot e^{\mathrm{j}\cdot\left(\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}+\sum_{k=-\infty}^{\infty}\mathrm{j}\cdot e^{-\mathrm{j}\cdot\left(\omega_r\cdot t+\frac{2\cdot k\cdot\pi\cdot t}{T_s}\right)}\right]
\end{aligned} \tag{9}$$

For the purpose of the expression (9) in trigonometric form, the real and imaginary parts of the coefficients in (9) must be separated. Real parts in both sums are zero, and the imaginary parts are –1 and 1. According to (6) it follows that $c_k = \frac{1}{2}\cdot\left(a_k - \mathrm{j}\cdot b_k\right)$ and according to (1), (9) in the trigonometric form:

$$f_{\mathrm{M}}(t) = \frac{1}{2\cdot T_{\mathrm{S}}}\cdot\left[\sum_{k=1}^{\infty}\sin\left(\omega_{\mathrm{r}}\cdot t + \frac{2\cdot k\cdot\pi\cdot t}{T_{\mathrm{S}}}\right) - \sum_{k=1}^{\infty}\sin\left(-\omega_{\mathrm{r}}\cdot t - \frac{2\cdot k\cdot\pi\cdot t}{T_{\mathrm{S}}}\right)\right] = \\ = \frac{1}{T_{\mathrm{S}}}\cdot\sum_{k=1}^{\infty}\sin\left(\omega_{\mathrm{r}}\cdot t + \frac{2\cdot k\cdot\pi\cdot t}{T_{\mathrm{S}}}\right) \tag{10}$$

The spectrum of the pulse amplitude modulation sampled signal is represented as a periodical spectrum of the sampled signal, with the period equal to the sampling period.

1. BIPOLAR PULSE WIDTH MODULATION FOR THE TRIANGULAR CARRIER SIGNAL

Spectral calculation was carried out for the harmonics of the modulated signal for the reference signal of a cosine waveform for the bipolar method of pulse width modulation. The calculation was carried out for the cosine waveform of the reference signal, since that form is the most often used for the applications listed in the Summary. The results of the spectral calculations for the modulated signal PWM-inverter were verified against the results of the calculations in [1].

The bipolar pulse width modulation is a two-level pulse width modulation, since the output voltage of the converter takes positive and negative values of the input voltage. For the interval where the reference signal is higher than the carrier signal the converter output voltage is positive, and in the interval where the reference signal is lower than the carrier signal the converter output voltage is negative.

Fig. 2. a) shows the comparison of the reference signal $f_{\mathrm{r}}(t)$ and the carrier signal $f_{\mathrm{N}}(t)$, and Fig. 2. b) shows the modulated signal $f_{\mathrm{rm}}(t)$.

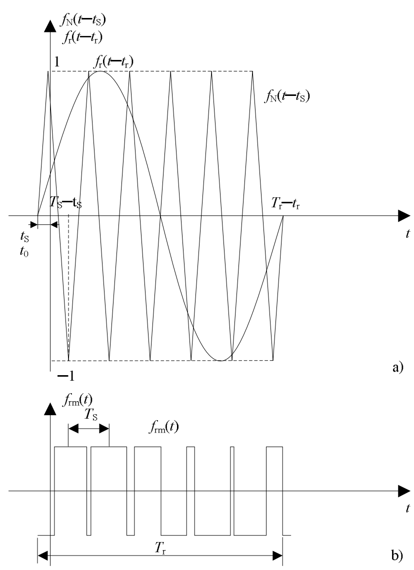

Fig. 2. Comparison of the carrier signal f_{n}(t) and the reference signal f_{r}(t) and the modulated signal f_{rm}(t)

The voltage waveform of the converter is equal to the waveform of the modulated signal. Time t_S is the time shift of the carrier signal, and time t_r is the time shift of the reference signal. T_S is the period of the carrier signal, T_r is the period of the reference signal. It is assumed that the phase shifts of the reference signal and the carrier signal are equal and that the ratio of T_S and T_r is an integer number. If the ratio of T_S and T_r is not an integer number, the modulated signal is not periodical and the Fourier transformation must be used instead of the Fourier series.

The amplitudes of the reference signal and the carrier signal are taken to have value of 1, for the sake of simpler calculations of the Fourier series coefficients. The reference signal amplitude can take any value in the interval of $\left[0,\, A_N\right]$, where A_N is the amplitude of the carrier signal.

Waveform of the modulated signal, as shown in Fig. 3. b), is periodical with the period of the reference signal T_r. Signal $f_{rm}(t)$ in Fig. 3. a) can be represented as the sum of l signals $f_{rm}(t)(1)$, ..., $f_{rm}(t)(l)$, shown in Fig. 3. b) – 3. d), where $l = \dfrac{T_r}{T_S}$. Fourier series of the signal in Fig. 3. a) can be represented as the sum of l Fourier series of the signals $f_{rm}(t)(1)$, ..., $\mathrm{f}_{rm}(t)(l)$ in Fig. 3. b) – 3. d).

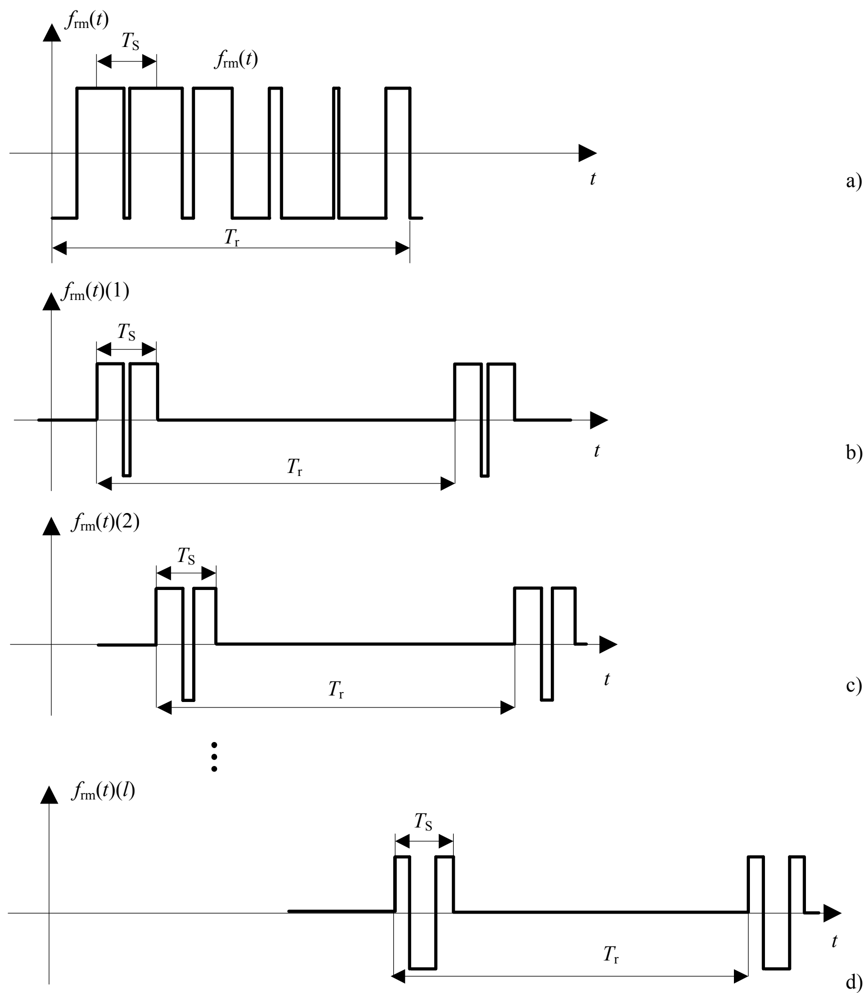

Fig. 3. Parts of a modulated reference signal

The coefficients c_k of the signal $f_{\mathrm{rm}}(t)(1)$ of the complex form Fourier series is calculated according to (6). The integration boundaries are the crossing points of the reference signal and the carrier signal, i.e. solutions for the system of two equations: equations for the carrier signal and the reference signal. Periods T_0 from (6) do not have to be equal, since the waveform signal in Fig. 3. a) consists of two periods: T_{S} and T_{r}. If the integration limits are the solutions for the carrier signal equation, the exponent periods from (6) and (2) are T_{S}. If the integration limits are the solutions of the reference signal equations, the exponent periods from (6) and (2) are T_{r}. Period T_0 from (6) is the period of the signal in the Fig. 3. b) T_{r} for any form of solution of the equation system.

$$\begin{aligned}
c_k(1) &= \frac{1}{T_{\mathrm{r}}}\cdot\left(\int\limits_{-\frac{T_{\mathrm{S}}}{2}+t_0}^{\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_{\mathrm{s}}\cdot t}\cdot \mathrm{d}t + \int\limits_{-\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}}^{\frac{T_{\mathrm{S}}}{2}+t_{\mathrm{S}}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_{\mathrm{s}}\cdot t}\cdot \mathrm{d}t - \int\limits_{\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}}^{-\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_{\mathrm{s}}\cdot t}\cdot \mathrm{d}t\right) = \\
&= \frac{\mathrm{j}\cdot T_{\mathrm{S}}}{2\pi\cdot k\cdot T_{\mathrm{r}}}\cdot\left(e^{-\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}} - e^{\mathrm{j}\cdot\pi\cdot k-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}} + e^{-\mathrm{j}\cdot\pi\cdot k-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}} - \right. \\
&\quad - e^{\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}} - e^{\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}} + \\
&\quad \left. + e^{-\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}}\right) = \frac{2\cdot T_{\mathrm{S}}}{\pi\cdot k\cdot T_{\mathrm{r}}}\cdot e^{-\mathrm{j}\cdot k\cdot\Theta_{\mathrm{s}}}\cdot\sin\left[\frac{\pi}{2}\cdot k\cdot\left(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1\right)\right]
\end{aligned} \tag{11}$$

The signal time shift $f_{\mathrm{rm}}(t)(i)$ and $f_{\mathrm{rm}}(t)(i+1)$ is T_{S}. The coefficient $c_k(i)$ of the signal $f_{\mathrm{rm}}(t)(\mathrm{i})$ is:

$$\begin{aligned}
c_k(i) &= \frac{1}{T_{\mathrm{r}}}\cdot\left(\int\limits_{-\frac{T_{\mathrm{S}}}{2}+t_{\mathrm{S}}+(i-1)\cdot T_{\mathrm{S}}}^{\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}+(i-1)\cdot T_{\mathrm{S}}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_{\mathrm{s}}\cdot t}\cdot \mathrm{d}t + \int\limits_{-\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}+(i-1)\cdot T_{\mathrm{S}}}^{\frac{T_{\mathrm{S}}}{2}+t_{\mathrm{S}}+(i-1)\cdot T_{\mathrm{S}}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_{\mathrm{s}}\cdot t}\cdot \mathrm{d}t - \int\limits_{\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}+(i-1)\cdot T_{\mathrm{S}}}^{-\frac{T_{\mathrm{S}}}{4}\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)+t_{\mathrm{S}}+(i-1)\cdot T_{\mathrm{S}}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_{\mathrm{s}}\cdot t}\cdot \mathrm{d}t\right) = \\
&= \frac{\mathrm{j}\cdot T_{\mathrm{S}}}{2\pi\cdot k\cdot T_{\mathrm{r}}}\cdot\left(e^{-\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}+(i-1)\cdot T_{\mathrm{S}}} - e^{\mathrm{j}\cdot\pi\cdot k-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}+(i-1)\cdot T_{\mathrm{S}}} + e^{-\mathrm{j}\cdot\pi\cdot k-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}+(i-1)\cdot T_{\mathrm{S}}} - \right. \\
&\quad - e^{\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}+(i-1)\cdot T_{\mathrm{S}}} - e^{\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}+(i-1)\cdot T_{\mathrm{S}}} + \\
&\quad \left. + e^{-\mathrm{j}\cdot\frac{\pi}{2}\cdot k\cdot(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1)-\mathrm{j}\cdot 2\cdot\pi\cdot k\cdot f_{\mathrm{s}}\cdot t_{\mathrm{s}}+(i-1)\cdot T_{\mathrm{S}}}\right) = \frac{2\cdot T_{\mathrm{S}}}{\pi\cdot k\cdot T_{\mathrm{r}}}\cdot e^{-\mathrm{j}\cdot k\cdot\Theta_{\mathrm{s}}}\cdot e^{\mathrm{j}\cdot 2\pi\cdot(i-1)\cdot k\cdot t}\cdot\sin\left[\frac{\pi}{2}\cdot k\cdot\left(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1\right)\right] = \\
&= \frac{2\cdot T_{\mathrm{S}}}{\pi\cdot k\cdot T_{\mathrm{r}}}\cdot e^{-\mathrm{j}\cdot k\cdot\Theta_{\mathrm{s}}}\cdot\sin\left[\frac{\pi}{2}\cdot k\cdot\left(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1\right)\right] = c_k(1)
\end{aligned} \tag{12}$$

$c_k(\mathrm{i})$ in (12) are functions depending on the waveform of the reference signal $f_{\mathrm{r}}(t)$. The coefficients c_k of the modulated reference signal are the sum of l coefficients $c_k(\mathrm{i})$:

$$\begin{aligned}
c_k &= \sum_{i=1}^{l} c_k(i) = l\cdot c_k(1) = l\cdot\frac{2}{\pi\cdot k\cdot l}\cdot e^{-\mathrm{j}\cdot k\cdot\Theta_{\mathrm{s}}}\cdot\sin\left[\frac{\pi}{2}\cdot k\cdot\left(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1\right)\right] = \\
&= \frac{2}{\pi\cdot k}\cdot e^{-\mathrm{j}\cdot k\cdot\Theta_{\mathrm{s}}}\cdot\sin\left[\frac{\pi}{2}\cdot k\cdot\left(f_{\mathrm{r}}(t-t_{\mathrm{r}})-1\right)\right]
\end{aligned} \tag{13}$$

If it is assumed that the reference signal is $f_{\mathrm{r}}(t-t_{\mathrm{r}}) = \cos(\omega_{\mathrm{r}}\cdot(t-t_{\mathrm{r}}))$, for comparison of the results with the results in u [1], the coefficients c_k are:

$$c_k = \frac{2}{\pi \cdot k} \cdot e^{-\mathrm{j} \cdot k \cdot \Theta_\mathrm{s}} \cdot \sum_{n=-\infty}^{\infty} J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \sin\left(\left(n-k\right) \cdot \frac{\pi}{2} + n \cdot \left(2 \cdot \pi \cdot f_\mathrm{r} \cdot t - \Theta_\mathrm{r}\right)\right) =$$

$$= \frac{1}{\mathrm{j} \cdot \pi \cdot k} \cdot e^{-\mathrm{j} \cdot k \cdot \Theta_\mathrm{s}} \cdot \sum_{n=-\infty}^{\infty} J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \left(e^{\mathrm{j} \cdot \left(\left(n-k\right) \cdot \frac{\pi}{2} + n \cdot \left(2 \cdot \pi \cdot f_\mathrm{r} \cdot t - \Theta_\mathrm{r}\right)\right)} - e^{-\mathrm{j} \cdot \left(\left(n-k\right) \cdot \frac{\pi}{2} + n \cdot \left(2 \cdot \pi \cdot f_\mathrm{r} \cdot t - \Theta_\mathrm{r}\right)\right)}\right) = \quad (14)$$

$$= \frac{1}{\mathrm{j} \cdot \pi \cdot k} \cdot e^{-\mathrm{j} \cdot k \cdot \Theta_\mathrm{s}} \cdot \sum_{n=-\infty}^{\infty} J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \left(e^{\mathrm{j} \cdot \left(n-k\right) \cdot \frac{\pi}{2}} \cdot e^{\mathrm{j} \cdot \left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)} - e^{-\mathrm{j} \cdot \left(n-k\right) \cdot \frac{\pi}{2}} \cdot e^{-\mathrm{j} \cdot \left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)}\right)$$

Where Θ_r, a product of t_r and ω_r, is the phase shift of the reference signal, and Θ_S, a product of t_S and ω_S, is the phase shift of the carrier signal. J_n are Bessel functions of the first kind of the $n^{\text{-th}}$ order, and f_r is the frequency of the reference signal.

c_0 is calculated by applying the L'Hospital rule, due to zeros in the numerator and denominator of the fraction of c_k after insertion of $k = 0$ and integer n in (14). Instead of calculating c_k for all cases of the integer n and $k = 0$, a_0 can be calculated according to (3), after representing the Fourier series in trigonometric form.

By inserting (14) into (2) the modulated signal can be represented as the Fourier series in complex form:

$$f_\mathrm{rm}(t) = c_0 + \frac{1}{\mathrm{j} \cdot \pi} \cdot e^{-\mathrm{j} \cdot k \cdot \Theta_\mathrm{s}} \cdot \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=-\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k} \cdot J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \left(e^{\mathrm{j} \cdot (n-k) \cdot \frac{\pi}{2}} \cdot e^{\mathrm{j} \cdot \left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)} - e^{\mathrm{j} \cdot (k-n) \cdot \frac{\pi}{2}} \cdot e^{-\mathrm{j} \cdot \left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)}\right) e^{\mathrm{j} \cdot k \cdot \omega_\mathrm{s} \cdot t} =$$

$$= c_0 + \frac{1}{\mathrm{j} \cdot \pi} \cdot e^{\mathrm{j} \cdot k \cdot \Theta_\mathrm{s}} \cdot \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=-\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k} \cdot J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \left\{\left[\cos\left(\left(n-k\right) \cdot \frac{\pi}{2}\right) + \mathrm{j} \cdot \sin\left(\left(n-k\right) \cdot \frac{\pi}{2}\right)\right] \cdot e^{\mathrm{j} \cdot \left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)} - \right. \quad (15)$$

$$\left. - \left[\cos\left(\left(n-k\right) \cdot \frac{\pi}{2}\right) - \mathrm{j} \cdot \sin\left(\left(n-k\right) \cdot \frac{\pi}{2}\right)\right] \cdot e^{-\mathrm{j} \cdot \left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)}\right\} \cdot e^{\mathrm{j} \cdot \left(k \cdot \omega_\mathrm{s} \cdot t - k \cdot \Theta_\mathrm{S}\right)}$$

(15) can be written as:

$$f_\mathrm{rm}(t) = c_0 + \frac{1}{\mathrm{j} \cdot \pi} \cdot \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=-\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k} \cdot J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \left\{\cos\left(\left(n-k\right) \cdot \frac{\pi}{2}\right) \cdot \left[\cos\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) + \mathrm{j} \cdot \sin\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) - \right.\right.$$

$$- \cos\left(-n \cdot \omega_\mathrm{r} \cdot t + n \cdot \Theta_\mathrm{r}\right) + \mathrm{j} \cdot \sin\left(-n \cdot \omega_\mathrm{r} \cdot t + n \cdot \Theta_\mathrm{r}\right)\big] + \mathrm{j} \cdot \sin\left(\left(n-k\right) \cdot \frac{\pi}{2}\right) \cdot \big[\cos\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) + \quad (16)$$

$$\left.\left. + \mathrm{j} \cdot \sin\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) + \cos\left(-n \cdot \omega_\mathrm{r} \cdot t + n \cdot \Theta_\mathrm{r}\right) - \mathrm{j} \cdot \sin\left(-n \cdot \omega_\mathrm{r} \cdot t + n \cdot \Theta_\mathrm{r}\right)\right]\right\} \cdot e^{\mathrm{j} \cdot \left(k \cdot \omega_\mathrm{s} \cdot t - k \cdot \Theta_\mathrm{S}\right)}$$

The sine is an odd and the cosine is an even function. The following applies:

$$\sum_{n=-\infty}^{\infty} \cos\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) - \cos\left(-n \cdot \omega_\mathrm{r} \cdot t + n \cdot \Theta_\mathrm{r}\right) = \sum_{n=-\infty}^{\infty} \cos\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) - \cos\left[-\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right)\right] = \quad (17)$$

$$= \sum_{n=-\infty}^{\infty} \cos\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) - \cos\left(n \cdot \omega_\mathrm{r} \cdot t - n \cdot \Theta_\mathrm{r}\right) = 0$$

(18)

$$\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)+\sin(-n\cdot\omega_r\cdot t+n\cdot\Theta_r)=\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)+\sin[-(n\cdot\omega_r\cdot t-n\cdot\Theta_r)]=$$
$$=\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)-\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)=0$$

$$\sum_{n=-\infty}^{\infty}\cos(n\cdot\omega_r\cdot t-n\cdot\Theta_r)+\cos(-n\cdot\omega_r\cdot t+n\cdot\Theta_r)=\sum_{n=-\infty}^{\infty}\cos(n\cdot\omega_r\cdot t-n\cdot\Theta_r)+\cos[-(n\cdot\omega_r\cdot t-n\cdot\Theta_r)]=$$
$$=\sum_{n=-\infty}^{\infty}\cos(n\cdot\omega_r\cdot t-n\cdot\Theta_r)+\cos(n\cdot\omega_r\cdot t-n\cdot\Theta_r)=2\cdot\sum_{n=-\infty}^{\infty}\cos(n\cdot\omega_r\cdot t-n\cdot\Theta_r) \tag{19}$$

$$\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)-\sin(-n\cdot\omega_r\cdot t+n\cdot\Theta_r)=\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)-\sin[-(n\cdot\omega_r\cdot t-n\cdot\Theta_r)]=$$
$$=\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)+\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r)=2\cdot\sum_{n=-\infty}^{\infty}\sin(n\cdot\omega_r\cdot t-n\cdot\Theta_r) \tag{20}$$

It can be written:

$$f_{\mathrm{rm}}(t)=c_0+\frac{1}{\mathrm{j}\cdot\pi}\cdot\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{n=-\infty}^{\infty}\frac{2}{k}\cdot J_n\left(\frac{\pi}{2}\cdot k\right)\cdot\mathrm{j}\cdot\sin\left((n-k)\cdot\frac{\pi}{2}\right)\cdot e^{\mathrm{j}\cdot(n\cdot\omega_r\cdot t-n\cdot\Theta_r)}\cdot e^{\mathrm{j}\cdot(k\cdot\omega_s\cdot t-k\cdot\Theta_S)}=$$
$$=c_0+\sum_{\substack{k=-\infty\\k\neq0}}^{\infty}\sum_{n=-\infty}^{\infty}\frac{2}{k\cdot\pi}\cdot J_n\left(\frac{\pi}{2}\cdot k\right)\cdot\sin\left((n-k)\cdot\frac{\pi}{2}\right)\cdot e^{\mathrm{j}\cdot[n\cdot(\omega_r\cdot t-\Theta_r)+k\cdot(\omega_s\cdot t-\Theta_S)]} \tag{21}$$

Resolution of real and imaginary parts of the coefficients in (21) due to the expression (21) in the trigonometric form is carried out analogously to those in the section „Introduction“. The expression (21) in the trigonometric form is:

$$f_{\mathrm{rm}}(t)=a_0+\sum_{k=1}^{\infty}\sum_{n=-\infty}^{\infty}\frac{4}{k\cdot\pi}\cdot J_n\left(\frac{\pi}{2}\cdot k\right)\cdot\sin\left((n-k)\cdot\frac{\pi}{2}\right)\cdot\cos[n\cdot(\omega_r\cdot t-\Theta_r)+k\cdot(\omega_s\cdot t-\Theta_S)] \tag{22}$$

The coefficient a_0 is calculated according to (3), analogously to calculation of the coefficients c_k:

$$a_0(i)=\frac{1}{T_r}\cdot\left(\int_{-\frac{T_s}{2}+t_s+(i-1)\cdot T_s}^{\frac{T_s}{4}\cdot(f_r(t-t_r)-1)+t_s+(i-1)\cdot T_s}1\cdot\mathrm{d}t+\int_{-\frac{T_s}{4}\cdot(f_r(t-t_r)-1)+t_s+(i-1)\cdot T_s}^{\frac{T_s}{2}+t_s+(i-1)\cdot T_s}1\cdot\mathrm{d}t-\int_{\frac{T_s}{4}\cdot(f_r(t-t_r)-1)+t_s+(i-1)\cdot T_s}^{-\frac{T_s}{4}\cdot(f_r(t-t_r)-1)+t_s+(i-1)\cdot T_s}1\cdot\mathrm{d}t\right)=$$
$$=\frac{T_S}{T_r}\cdot\cos(\omega_r\cdot t-\Theta_r) \tag{23}$$

$$a_0=\sum_{i=1}^{l}a_0(i)=\cos(\omega_r\cdot t-\Theta_r) \tag{24}$$

(25)

By insertion of (24) into (22) the modulated signal can be represented as:

$$f_{\mathrm{rm}}(t) = \cos(\omega_{\mathrm{r}} \cdot t - \Theta_{\mathrm{r}}) +$$
$$+ \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{4}{k \cdot \pi} \cdot J_n\left(\frac{\pi}{2} \cdot k\right) \cdot \sin\left((n-k) \cdot \frac{\pi}{2}\right) \cdot \cos\left[n \cdot (\omega_{\mathrm{r}} \cdot t - \Theta_{\mathrm{r}}) + k \cdot (\omega_{\mathrm{S}} \cdot t - \Theta_{\mathrm{S}})\right]$$

The Bessel function of the first order for the argument $\frac{\pi}{2}$ is the form:

$$J_n\left(\frac{\pi}{2} \cdot k\right) = \sum_{l=0}^{\infty} (-1)^l \cdot \frac{\left(\frac{\pi}{2}\right)^{n+2\cdot l}}{l! \cdot 2^{n+2\cdot l} \cdot \Gamma(n+l+1)} \tag{26}$$

By growth of order of the Bessel function n, its value decreases exponentially. The Bessel function of the ninth order is by five orders of magnitude smaller in comparison to the Bessel function of the first and the second order, for the same argument. In [1] the amounts for the Bessel function for $|n| > 9$ are neglected.

The harmonics of the modulated signal are shown in the Fig. 4., for the frequency of the carrier signal f_{S} = 2000 Hz and the frequency of the reference signal of f_{r} = 50 Hz. The frequency of the carrier signal is dominant in comparison to the frequency of the reference signal and the harmonics $k \neq 0$ and $n \neq 0$ from (13) are repeated every $k \cdot f_{\mathrm{S}}$, as in the Fig. 4.

Harmonics for $k \neq 0$ and $n \neq 0$ are called carrier and sideband harmonics in [1]. Carrier and sideband harmonics are a consequence of two frequencies contained in the waveform of the voltage of the PWM-converter frequency of the modulating signal and carrier signal.

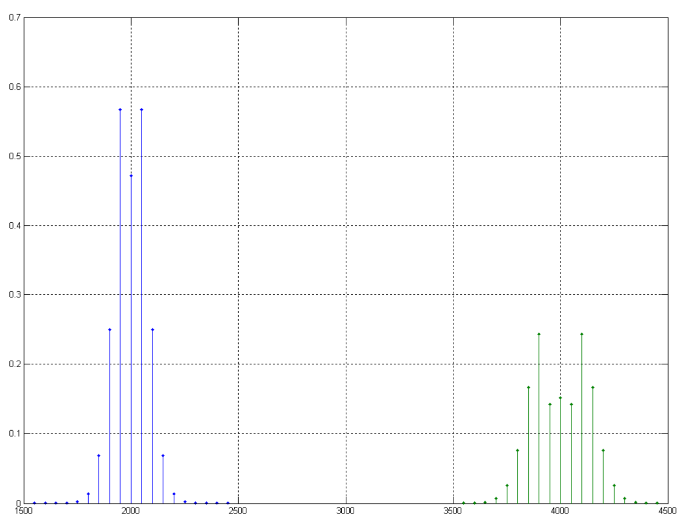

Fig. 4. Amplitude - frequency properties of the PWM-converter output voltage in the case of the triangular carrier signal

2. BIPOLAR PULSE WIDTH MODULATION IN THE CASE OF SAWTOOTH CARRIER SIGNAL

The calculation of the Fourier series for modulated signal by bipolar pulse width modulation in the case of a sawtooth carrier signal waveform was carried out in the same manner as in the section for the bipolar modulation of the pulse width in the case of triangular carrier signal waveform.

Coefficients c_k(i) for the modulated signal Fourier series $f_{\mathrm{rm}}(t)$ are:

$$\begin{aligned} c_k(i) &= \frac{1}{T_\mathrm{r}} \cdot \left(\int\limits_{-\frac{T_\mathrm{S}}{2}+t_\mathrm{S}+(i-1)\cdot T_\mathrm{S}}^{\frac{T_\mathrm{S}}{2}\cdot f_\mathrm{r}(t-t_\mathrm{r})+(i-1)\cdot T_\mathrm{S}+t_\mathrm{S}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_\mathrm{S}\cdot t}\cdot \mathrm{d}t - \int\limits_{\frac{T_\mathrm{S}}{2}\cdot f_\mathrm{r}(t-t_\mathrm{r})+(i-1)\cdot T_\mathrm{S}+t_\mathrm{S}}^{\frac{T_\mathrm{S}}{2}+t_\mathrm{S}+(i-1)\cdot T_\mathrm{S}} e^{-\mathrm{j}\cdot 2\pi\cdot k\cdot f_\mathrm{S}\cdot t}\cdot \mathrm{d}t \right) = \frac{\mathrm{j}\cdot T_\mathrm{S}}{\pi\cdot k\cdot T_\mathrm{r}} \cdot e^{-\mathrm{j}\cdot k\cdot \Theta_\mathrm{S}} \cdot \left(e^{-\mathrm{j}\cdot\pi\cdot k\cdot (f_\mathrm{r}(t-t_\mathrm{r}))} - e^{\mathrm{j}\cdot k\cdot\pi} \right) = \\ &= \frac{\mathrm{j}\cdot T_\mathrm{S}}{\pi\cdot k\cdot T_\mathrm{r}} \cdot e^{-\mathrm{j}\cdot k\cdot\Theta_\mathrm{S}} \cdot \left(\sum_{n=-\infty}^{\infty} J_\mathrm{n}(\pi\cdot k)\cdot e^{-\mathrm{j}\cdot\left(n\cdot(\omega_\mathrm{r}\cdot t-\Theta_\mathrm{r})+n\cdot\frac{\pi}{2}\right)} - e^{\mathrm{j}\cdot k\cdot\pi} \right) \end{aligned} \tag{27}$$

Coefficients c_k are:

$$\begin{aligned} c_k &= \sum_{i=1}^{l} c_k(i) = l\cdot c_k(i) = l\cdot \frac{\mathrm{j}\cdot T_\mathrm{S}}{\pi\cdot k\cdot T_\mathrm{r}} \cdot e^{-\mathrm{j}\cdot k\cdot\Theta_\mathrm{S}} \cdot \left(\sum_{n=-\infty}^{\infty} J_\mathrm{n}(\pi\cdot k)\cdot e^{-\mathrm{j}\cdot\left(n\cdot(\omega_\mathrm{r}\cdot t-\Theta_\mathrm{r})+n\cdot\frac{\pi}{2}\right)} - e^{\mathrm{j}\cdot k\cdot\pi} \right) = \\ &= \frac{\mathrm{j}}{\pi\cdot k} \cdot e^{-\mathrm{j}\cdot k\cdot\Theta_\mathrm{S}} \cdot \left(\sum_{n=-\infty}^{\infty} J_\mathrm{n}(\pi\cdot k)\cdot e^{-\mathrm{j}\cdot\left(n\cdot(\omega_\mathrm{r}\cdot t-\Theta_\mathrm{r})+n\cdot\frac{\pi}{2}\right)} - e^{\mathrm{j}\cdot k\cdot\pi} \right) \end{aligned} \tag{28}$$

The coefficient c_0 has a zero in the numerator and denominator and is calculated by applying the L'Hospital rule. Instead of calculating the coefficient c_0, the coefficient a_0 can be calculated according to (3), after the Fourier series is represented in trigonometric form.By inserting (28) into (2) the modulated signal it can be represented as a Fourier series in complex form:

$$\begin{aligned} f_\mathrm{rm}(t) &= c_0 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \left[\sum_{n=-\infty}^{\infty} \frac{\mathrm{j}}{\pi\cdot k}\cdot J_\mathrm{n}(\pi\cdot k)\cdot e^{-\mathrm{j}\cdot\left(n\cdot(\omega_\mathrm{r}\cdot t-\Theta_\mathrm{r})+n\cdot\frac{\pi}{2}\right)} - e^{\mathrm{j}\cdot k\cdot\pi} \right] \cdot e^{\mathrm{j}\cdot k\cdot(\omega_\mathrm{s}\cdot t-\Theta_\mathrm{S})} = \\ &= c_0 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{n=-\infty}^{\infty} \frac{\mathrm{j}}{\pi\cdot k}\cdot J_\mathrm{n}(\pi\cdot k)\cdot e^{-\mathrm{j}\cdot n\cdot\frac{\pi}{2}} \cdot e^{\mathrm{j}\cdot[k\cdot(\omega_\mathrm{s}\cdot t-\Theta_\mathrm{S})-n\cdot(\omega_\mathrm{r}\cdot t-\Theta_\mathrm{r})]} - \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{\mathrm{j}}{\pi\cdot k}\cdot e^{\mathrm{j}\cdot k\cdot\pi}\cdot e^{\mathrm{j}\cdot k\cdot(\omega_\mathrm{s}\cdot t-\Theta_\mathrm{S})} = \\ &= c_0 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\mathrm{j}}{\pi\cdot k}\cdot J_\mathrm{n}(\pi\cdot k)\cdot e^{\mathrm{j}\cdot n\cdot\frac{\pi}{2}} \cdot e^{\mathrm{j}\cdot[k\cdot(\omega_\mathrm{s}\cdot t-\Theta_\mathrm{S})-n\cdot(\omega_\mathrm{r}\cdot t-\Theta_\mathrm{r})]} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{\mathrm{j}}{\pi\cdot k}\cdot e^{\mathrm{j}\cdot k\cdot(\omega_\mathrm{s}\cdot t-\Theta_\mathrm{S})}\cdot\left(J_0(\pi\cdot k) - e^{\mathrm{j}\cdot k\cdot\pi}\right) \end{aligned} \tag{29}$$

Representation of the Fourier series in trigonometric form is done analogously to the case of the triangular carrier signal. Coefficient a_0(i) is:

$$a_0(i) = \frac{1}{T_\mathrm{r}} \cdot \left(\int\limits_{-\frac{T_\mathrm{S}}{2}+t_\mathrm{S}+(i-1)\cdot T_\mathrm{S}}^{\frac{T_\mathrm{S}}{2}\cdot f_\mathrm{r}(t-t_\mathrm{r})+(i-1)\cdot T_\mathrm{S}+t_\mathrm{S}} 1 \cdot \mathrm{d}t - \int\limits_{\frac{T_\mathrm{S}}{2}\cdot f_\mathrm{r}(t-t_\mathrm{r})+(i-1)\cdot T_\mathrm{S}+t_\mathrm{S}}^{\frac{T_\mathrm{S}}{2}+t_\mathrm{S}+(i-1)\cdot T_\mathrm{S}} 1 \cdot \mathrm{d}t \right) = \frac{T_\mathrm{S}}{T_\mathrm{r}}\cdot f_\mathrm{r}(t-t_\mathrm{r}) \tag{30}$$

Coefficient a_0 is:

$$a_0 = \sum_{i=1}^{l} a_0(i) = f_\mathrm{r}(t - t_\mathrm{r}) = \cos\left(\omega_\mathrm{r} \cdot t - \Theta_\mathrm{r}\right) \tag{31}$$

The modulated signal represented as a Fourier series in trigonometric form is:

$$\begin{aligned} f_\mathrm{rm}(t) &= a_0 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2}{\pi \cdot k} \cdot J_\mathrm{n}(\pi \cdot k) \cdot \left\{ \sin\left(n \cdot \frac{\pi}{2}\right) \cdot \cos\left[k \cdot (\omega_\mathrm{s} \cdot t - \Theta_\mathrm{S}) - n \cdot (\omega_\mathrm{r} \cdot t - \Theta_\mathrm{r})\right] - \right. \\ &\left. - \cos\left(n \cdot \frac{\pi}{2}\right) \cdot \sin\left[k \cdot (\omega_\mathrm{s} \cdot t - \Theta_\mathrm{S}) - n \cdot (\omega_\mathrm{r} \cdot t - \Theta_\mathrm{r})\right] \right\} + \sum_{k=1}^{\infty} \frac{2}{\pi \cdot k} \cdot \left(\cos(m \cdot \pi) - J_0(\pi \cdot k)\right) \cdot \sin\left(\omega_\mathrm{s} \cdot t - \Theta_\mathrm{S}\right) \\ &= \cos(\omega_\mathrm{r} \cdot t - \Theta_\mathrm{r}) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2}{\pi \cdot k} \cdot J_\mathrm{n}(\pi \cdot k) \cdot \left\{ \sin\left(n \cdot \frac{\pi}{2}\right) \cdot \cos\left[k \cdot (\omega_\mathrm{s} \cdot t - \Theta_\mathrm{s}) - n \cdot (\omega_\mathrm{r} \cdot t - \Theta_\mathrm{r})\right] - \right. \\ &\left. - \cos\left(n \cdot \frac{\pi}{2}\right) \cdot \sin\left[k \cdot (\omega_\mathrm{s} \cdot t - \Theta_\mathrm{s}) - n \cdot (\omega_\mathrm{r} \cdot t - \Theta_\mathrm{r})\right] \right\} + \sum_{k=1}^{\infty} \frac{2}{\pi \cdot k} \cdot \left[\cos(m \cdot \pi) - J_0(\pi \cdot k)\right] \cdot \sin\left(\omega_\mathrm{s} \cdot t - \Theta_\mathrm{s}\right) \end{aligned} \tag{32}$$

Spectrum of the modulated signal sampled by pulse width modulation in case of a sawtooth carrier signal differs from the spectrum of the modulated signal sampled by a triangular carrier signal in the contents of the accompanying harmonics. This is a consequence of changed integration boundaries while calculating Fourier series' coefficients c_k.

As well as in the case of pulse width modulated with a triangular carrier signal, harmonics of the modulated signal have a frequency equal to the sum of the multiples of the frequency of the carrier signal and the multiples of the reference signal.

CONCLUSION

The analysis of the frequency harmonics of the converter output voltage with the bipolar pulse width modulation was carried out in time domain. The applied calculation method for the spectrum is performed without a need to know the double variable controlled waveform Fourier series. Changing the boundaries of integration while calculating the members of the Fourier series applies to the procedure for calculation of the frequency spectrum for any form of the carrier signal and the reference signal. It was demonstrated that the signal spectrum sampled by various forms of the carrier signals differ in the contents of the carrier and sideband harmonics. Appearance of the carrier and sideband harmonics in the output voltage spectrum of the converter is the result of two frequencies contained in the waveform of the converter voltage.

LITERATURE

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