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CONTENTS

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Prediction of time dependent standby failure rates for periodically tested components taking into account the operational history

The prediction of time dependent standby failure rates was studied, taking into account the operational history of a component. These studies are important for applications such as system modeling in probabilistic safety analysis to evaluate the impact of equipment aging and maintenance strategies on the risk measures (e.g. reactor core damage frequency) considered. The time dependent model for the standby failure rate is defined based on the Weibull distribution and the principles of proportional age reduction by equipment overhauls. The parameters which determine the standby failure rate are estimated, including the definition of the operational history model and likelihood function for Bayesian analysis of parameters for periodically tested standby repairable components. The operational history is provided as time axis with defined times of overhauls, surveillance tests and failures. Assessment of time dependent unavailability due to the failure of periodically tested standby components is described. As an example, the prediction of the future behavior of components for seven different operational histories is described.

Vorhersage zeitabhängiger Standby Ausfallraten für regelmäßig getestete Komponenten unter Berücksichtigung der betriebsbedingten Vorgeschichte. *Die Vorhersage zeitabhängiger Standby Ausfallraten wurden untersucht unter Berücksichtigung der betriebsbedingten Vorgeschichte einer Komponente. Diese Untersuchungen sind wichtig für Anwendungen wie z.B. die Modellierung eines Systems bei der probabilistischen Sicherheitsanalyse, um die Auswirkungen der Alterung von Komponenten und den Einfluss von Instandhaltungsstrategien auf die betrachteten Risikomaßnahmen (z.B. die Häufigkeit von Schäden am Reaktorkern) zu bewerten. Das zeitabhängige Modell für die Standby Ausfallraten wird definiert auf der Grundlage der Weibull Verteilung und den Prinzipien proportionaler Altersreduzierung durch entsprechende Revisionsarbeiten. Die Parameter, die die Standby Ausfallraten bestimmen, werden abgeschätzt unter Einbeziehung der betriebsbedingten Vorgeschichte und der Likelihood Funktion für das Bayes'sche Schätzverfahren der Parameter für regelmäßig getestete reparierbare Standby Komponenten. Die betriebsbedingte Vorgeschichte wird bestimmt als Zeitachse mit definierten Zeiten für Revisionsarbeiten, Überwachungstests und Ausfälle. Die Bestimmung der zeitabhängigen Nichtverfügbarkeit aufgrund von Ausfällen regelmäßig getestete Standby Komponenten wird beschrieben. Als Beispiel wird die Vorhersage des zukünftigen Verhaltens von Komonenten für sieben verschiedene betriebsbedingte Vorgeschichten beschrieben.*

1 Introduction

The paper studies prediction of the future behavior of a standby component based on its observed operational history, which includes the history of demands and failures, as well as overhauls. The predicted standby failure rate for future periods can be used in applications such as probabilistic safety analysis (PSA) modeling to evaluate the impact of equipment aging and maintenance strategies on the risk measures (e.g. reactor core damage frequency) considered. In this paper, a class of periodically tested standby components is considered (i.e. components which are normally in the state of readiness for operation upon request). This type of component is very often considered in PSA models of safety systems, such as Emergency Core Cooling.

First of all, the model for time dependent standby failure rates has to be defined. Then, estimation of the parameters which determine the failure rate is required. This includes definition of the operational history model and likelihood function for Bayesian analysis of parameters.

This is followed by the description of assessment of time dependent unavailability of the periodically tested standby components due to failure. Finally, an example for demonstration purposes is provided with the prediction of the future behavior for seven different operational histories.

2 Definition of time dependent standby failure rate model with inclusion of overhauls

In the work described in this paper it is assumed that failures of a standby component occur during standby periods with a (standby) certain failure rate. Failures are discovered only when a request for the component's operation is made.

For a discussion on the dependency of failure rates on equipment age, it is referred to the paper on prediction of the time-dependent (operating) failure rate for normally operating components, [1], Section 2. Some further discussion can be found, for example, in [2–5]. For a standby failure rate, the same type of time dependent effective failure rate model as in [1] is taken, based on Pulcini [7], where it is assumed that inherent time to first failure follows the Weibull distribution and principle of proportional age reduction is then employed, i.e.,

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t - \rho x}{\alpha}\right)^{\beta-1}, \quad t > x \tag{1}$$

considering that, in this case, t represents the cumulative time spent in standby. For definitions and meaning of parameters, it is referred to [1]. In the case of standby periodically tested components it is also assumed that $t = 0$ is a time point at which the component was assembled and that Eq. (1) applies only following some initial period $[0, T_1\rangle$ (this in order to allow for the possibility that the failure rate may follow another model during the initial period due to, for example, early failures).

3 Assessment of standby failure rate parameters

3.1 Bayesian analysis of parameters α, β and ρ

Assuming there is some prior knowledge of parameters a, β and ρ and that adequate operational history records for the equipment of concern exists, the parameters can be estimated by means of Bayesian analysis (e. g. [6–9]).

For estimation of the parameters a, β and ρ from Eq. (1) by means of Bayesian inference, the same analytical procedure applies as described in Section 3.1 of ref. [1].

The key of the Bayesian analysis lies in the principle of likelihood. In the following sections, first of all, a record of operating history is defined in the form needed for the Bayesian analysis of the parameters a, β and ρ and then a likelihood function is established for a periodically tested standby component.

3.2 Record of operating history

The record of operating history of a standby component with periodical tests of operability is illustrated by Fig. 1. It consists of the history of overhauls, demands for operation and failures.

The history of overhauls can be presented as:

$$T_1 = x_1 < x_2 < \ldots < x_j < x_{j+1} < \ldots < x_s \leq T_2 \tag{2}$$

The history of requests for operation, either from tests or from actual operational demands can be presented as:

$$T_1 = z_0 < z_1 < z_2 < \ldots < z_k < z_{k+1} < \ldots < z_m < T_2 \tag{3}$$

where z_k, $k = 1, \ldots, m$ denotes the time point at which k^{th} request takes place. The history of requests is accompanied by a history of failures which, although generally occurring somewhere in between the requests, take effect at the first forthcoming request. Thus, the history of failures can be presented in the form:

$$T_1 = t_1 < t_2 < \ldots < t_i < t_{i+1} < \ldots < t_n < T_2 \tag{4}$$

with the additional remark that the time point t_i, $k = 1, \ldots, m$ of each failure coincides with one of time points z_k, $k = 1, \ldots, m$ at which requests are made ($n \leq m$).

Following are assumptions used in the forthcoming analytical considerations:

1) Overhauls are instantaneous
2) Start of observation period coincides with an overhaul (i. e. $T_1 = x_1$, see Eq. (2))
3) Each failure is immediately followed by a repair, which is instantaneous and minimal (minimal repair in this context means that equipment is restored to the status it had immediately before the failure.)
4) Demonstrations of operability either from tests or from actual operational demands are instantaneous
5) Every overhaul and every repair is immediately followed by a successful test of operability (i. e. overhaul/repair is not completed until a successful test is made).

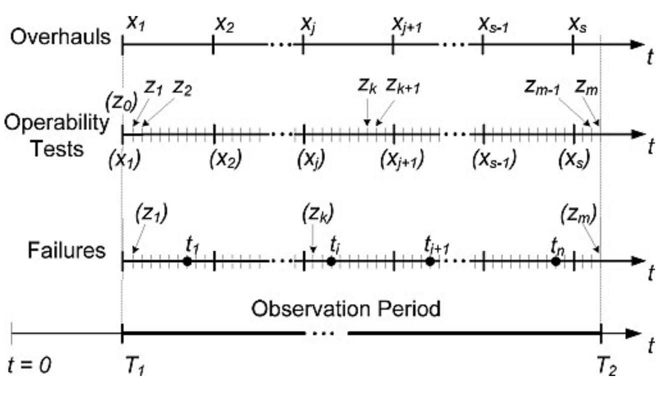

Fig. 1. Operating history record for a standby component with periodical tests of operability

3.3 Likelihood function

Regarding the time to first failure of standby equipment known to be operable at $t = t_0$, relations from Section 3.3 in [1] for the probability density function (Eq. (12) in [1]) and for the probability that the first failure occurs during the time period $\langle t_0, t]$ (Eq. (13) in [1]) apply. It should, however, be noted that t represents the time at which the failure occurs, not the time at which it is detected (by demand).

According to the considerations from the previous sections, the standby failure rate over the observation period is expressed as:

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t - \rho x_j}{\alpha}\right)^{\beta-1}, \quad x_j < t \leq x_{j+1}, \quad j = 1, \ldots, s \tag{5}$$

where $x_{s+1} = T_2$. The likelihood function presents a measure of probability that operating history (i. e. sequence of failures) shown in Fig. 1 takes place, knowing that the failure rate follows the time dependent model given by Eq. (5). In the case of a normally operating component, the exact time point at which the failure occurred is known. On the other hand, what is known in the case of a standby component is only a time interval. Due to this, a likelihood function in the case of a standby component is somewhat more complex as compared to the case of an operating component.

Let t_i be the time point at which i^{th} failure is observed (i. e. t_i is the time point of i^{th} unsuccessful request for component's operation). Let zk_i be the time point of the latest successful demonstration of operability that preceded i^{th} unsuccessful request for operation. In this case, the i^{th} failure actually occurred somewhere within the time interval $\langle zk_i, t_i]$. The illustration is provided in Fig. 2.

Having in mind that the component was operable at $t = zk_i$ and taking into account Eq. (13) in [1], a probability that a failure occurs during $\langle zk_i, t_i]$ is

$$F(t_i | zk_i) = 1 - \exp\left[-\int_{z_{k_i}}^{t_i} \lambda(\tau)\, d\tau\right] \tag{6}$$

The previous failure occurred at the time point t_{i-1}. According to the assumption postulated in Section 3.2, this failure

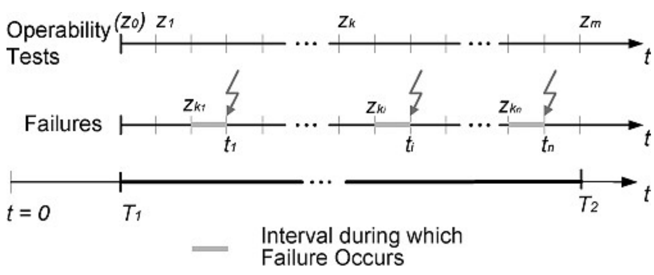

Fig. 2. Illustration of time intervals during which failures of a standby component occur

was followed by immediate and instantaneous repair, so that it is known that the component was operable at t_{i-1}^{+}. According to Eq. (13) in [1], the probability that during $\langle t_{i-1}, zk_i]$ no failure occurs is

$$R(zk_i|t_{i-1}^{+}) = 1 - F(zk_i|t_{i-1}^{+}) = \exp\left[-\int\limits_{t_{i-1}}^{z_{k_i}} \lambda(\tau)\,d\tau\right] \tag{7}$$

To be fully in accordance with the operating history shown in Fig. 1, it needs to be taken into account that the period $\langle t_n, T2]$ passed without a failure. The corresponding probability is:

$$R(T_2|t_n^{+}) = \exp\left[-\int\limits_{t_n}^{T_2} \lambda(\tau)\,d\tau\right] \tag{8}$$

The likelihood function, being the probability that n failures are observed upon requests for the component's operation in a sequence as shown in Fig. 1, can be written as:

$$l(E|\alpha,\beta,\rho) = \prod_{i=1}^{n}\left[R(z_{k_i}|t_{i-1}^{+})\,F(t_i|z_{k_i})\right]R(T_2|t_n^{+}) \tag{9}$$

where $t_0 = T_I$. By replacing R-terms and F-terms according to Eq. (6–8) and rearranging the expression, the likelihood function can be re-written in the form:

$$l(E|\alpha,\beta,\rho) = \left\{\prod_{i=1}^{n}\left[\exp\left[\int\limits_{z_{k_i}}^{t_i}\lambda(\tau)\,d\tau\right]-1\right]\right\}\exp\left[-\int\limits_{T_1}^{T_2}\lambda(\tau)\,d\tau\right] \tag{10}$$

where $\lambda(t)$ is defined by Eq. (5). It can be shown that if the time between two operability tests gets very short ($t_i - zk_i = \Delta\tau \ll$), the likelihood function for the standby component approximates the one for normally operating component. This comes as expected, since by shortening this time period the time between the onset of failure and its observation becomes shorter. In the limiting case it would become instantaneous, as with the normally operating component.

The likelihood function provided by Eq. (10) can be used for estimating parameters α, β and ρ by means of the maximum likelihood (ML) method or, if some prior knowledge exists, by Bayesian analysis. Appendix A discusses some aspects of analytical estimate of parameters α and β by means of ML in the case when overhauls impact is not included in the $\lambda(t)$ expression in an explicit manner (i.e. $\rho = 0$). With overhauls impact included, numerical methods can be applied.

If some prior knowledge of parameters α, β and ρ exists in the form of uncertainty distributions, the likelihood function (10) can be used to perform a Bayesian analysis in order to combine the prior knowledge with recorded operating experience on the principles described in Section 3.1.

4 Unavailability of standby equipment due to a failure

In PSA models, failures of the standby equipment are usually expressed through unavailability at the time of demand and failure to perform the intended mission (typically represented through a failure to operate (run) for a specified "mission time"). The relation needs to be established between the unavailability due to standby component failure and standby failure rate in order to obtain the model of dependence of unavailability on the component age.

Let t_S denote the cumulative time spent in the standby state. Having in mind Eq. (13) in [1] and Eq. (6) above, the unavailability of standby component due to failures that occur during the standby state is

$$q_S(t_S) = F(t_S|z(t_S)) = 1 - \exp\left[-\int\limits_{z(t_S)}^{t_S}\lambda_S(\tau)\,d\tau\right] \tag{11}$$

where $z(t_S)$ denotes the time point of the latest demonstration of operability before t_S. Since the standby failure rate $\lambda_S(t)$ follow, according to the assumption, the time dependent model given by Eq. (5), overhauls will influence the behavior of unavailability $q_S(t_S)$ over the time.

An additional term may be added to the right side of Eq. (11) that would represent a residual unavailability (which may account, for example, for various failures due to transitional stress and dynamic phenomena at the time of component startup). This term would be independent of the local time (i.e. the time passed since $z(t_S)$), but would generally depend on the global time or on total number of cycles (in other words, on equipment age). In the following considerations it is assumed that residual unavailability is negligible when compared to the unavailability due to failures occurring during the time spent in the standby state.

The probability of the failure to perform the intended mission, $q_R(t_R)$, if understood as a failure to run for a specified mission time period t_M, can be expressed by means of operating failure rate $\lambda_R(t_R)$, dependent on a total (i.e. cumulative) time t_R spent in operating state:

$$q_R(t_R) = F(t_R + tM|\,t_R) = 1 - \exp\left[-\int\limits_{t_R}^{t_R+t_M}\lambda_R(\tau)\,d\tau\right] \tag{12}$$

In many cases, mission time t_M is short enough so that the Eq. (12) can be approximated by:

$$q_R(t_R) \approx 1 - \exp[\lambda_R(t_R)\,t_M] \tag{13}$$

It needs to be noted that in the case of equipment that is normally in standby state, such as many safety systems in nuclear power plants, the values of the term $q_R(t_R)$ would generally be much lower (many times negligible) when compared to those of $q_S(t_S)$. The reason is that the cumulative time t_R would be very small which would result in small values of $\lambda_R(t_R)$ according to the assumed time dependent model discussed in Ref. [1].

Appendix B contains a short discussion on some aspects of estimating the unavailability $q_S(t_S)$ in the case when the overhaul impact is not included in the $\lambda(t)$ expression in an explicit manner (i.e. $\rho = 0$).

5 Example

For the purpose of demonstration, the method for predicting standby failure rates and other reliability indicators discussed above was applied to a set of examples with a periodically tested standby component. Bayesian analyses were performed based on assumed prior status of standby failure rate model parameters and various histories of overhauls and failures. Predictions were made for various reliability indicators for a future period based on prior and posterior statuses of parameters.

A standby component was considered which spent 15 years in standby condition with operability tests performed quarterly (i.e. every 3 months). Overhauls were performed on a 3-year basis. The first 3 years of operational history are not taken into account when the estimate of parameters is made. Eight different operating histories were considered in order to observe the impact of operational experience on estimates of parameters. Operating histories are defined by Table 1.

In all examples it is assumed that the improvement factor is known and that $\rho = 0.75$, which simplifies Bayesian analysis considerably. The prior status of both parameters α and β is assumed to be based on their expected values, upper and lower bounds and variances, as follows.

Table 1. Operating history for the example with a standby component

Observation Period: $\langle 3, 15]$ year; (Duration: 12 year, i.e. 4383 day) $T_1 = 1095.75$ day; $T_2 = 5478.75$ day;
Overhauls: every 3 years (1095.75 day) History of Overhauls: [1095.75 2191.5 3287.25 4383] day
Operability Tests: quarterly (91.3125 day) History of Tests: [1095.75 1187.0625 1278.375 . . . 5478.75] day
Failures: 8 different histories of failures are considered. Time point of a failure is identified through the number of the operability test at which it took effect. History 1: no failures History 2: 1 failure: 5th test History 3: 1 failure: 45th test History 4: 2 failures: 29th, 45th test History 5: 3 failures: 29th, 39th, 45th test History 6: 6 failures: 29th, 33rd, 36th, 39th, 43rd, 47th test History 7: 8 failures: 29th, 33rd, 36th, 39th, 41st, 43rd, 45th, 47th test History 8: 10 failures: 29th, 33rd, 36th, 39th, 41st, 43rd, 44th, 45th, 47th, 48th test

Let A be the random variable representing a value of considered parameter (i.e. α or β) for which it is known that it has lower bound l_A and upper bound u_A. (Probability density function for values smaller than or equal to l_A, as well as those larger than or equal to u_A is equal to zero. Due to their nature, lower bound for α is larger than zero and for β is larger than 1.) It is assumed that A is distributed over $\langle l_A, u_A \rangle$ in such a manner that it's linear transformation

$$A_0 = \frac{A - l_A}{u_A - l_A} \tag{14}$$

follows beta-distribution over $\langle 0, 1 \rangle$. In other words, A_0 has pdf of the form:

$$g_{Ao}(\alpha_0) = \begin{cases} 0, & \text{for} \quad \alpha_0 \notin \langle 0,1 \rangle \\ \dfrac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} {\alpha_0}^{a-1} (1-\alpha_0)^{b-1}, & \text{for} \quad \alpha_0 \in \langle 0,1 \rangle \end{cases} \tag{15}$$

with expectation and variance:

$$E[A_0] = \frac{a}{a+b}; \quad V[A_0] = \frac{ab}{(a+b)^2(a+b+1)} \tag{16}$$

Prior distributions for the values of parameters α (r.v. A_{prior}) and β (r.v. B_{prior}) that are used in the example set are defined in Table 2.

For each of the 8 history cases Bayesian analysis was performed in the way described in Section 3.1 with the likelihood function provided in Section 3.3 and prior distributions from Table 2. Numerical integration, as well as all other calculations, was performed by the "MATLAB" tool. Table 3 presents characteristic values of the posterior distributions obtained.

The posterior estimate of α generally decreases with increasing number of observed failures. With posterior values of β the relation is not this straightforward. The reason lies in the fact that the expected number of failures can both increase and decrease with increasing β, depending on the ratio between the characteristic life α of the component of concern and its age during the period of observation. Generally, when α is larger than T_2, the expected number of failures during the $\langle T_1, T_2]$ will decrease with an increase of β. It needs to be noted here that overhauls (through the improvement factor ρ) have the effect to increase the characteristic life of the component (by reducing its age). This is, for example, the explanation for the fact that in "History 1" (no failures) the posterior value of β is larger than its prior value. Also, by comparing the results for "History 2" and "History 3" it can be seen that not only a number of observed failures matters, but also the timing of their appearance.

Table 2. Characteristic values of prior distributions for parameters α and β

	A_{prior} (alpha)	B_{prior} (beta)
Expectation	2922 day (8 year)	3.5
Lower Bound	730.5 day (2 year)	1
Upper Bound	5113.5 day (14 year)	6
Variance	6.86E + 0.5 day^2	0.8929
a*	3	3
b*	3	3

* parameters in pdf of basic beta-distribution

Table 3. Characteristic values of posterior distributions for parameters α and β

	Posterior Status for α		Posterior Status for β	
	Expectation (day)	Variance (day^2)	Expectation	Variance
History 1	3426.4	3.96E + 05	3.689	0.839
History 2	3071.6	4.40E + 05	2.685	0.637
History 3	2984.3	3.95E + 05	3.522	0.834
History 4	2624.1	3.46E + 05	3.181	0.793
History 5	2315.7	2.69E + 05	3.034	0.736
History 6	1714.5	6.28E + 04	3.505	0.669
History 7	1541.5	3.40E + 04	3.694	0.587
History 8	1444.4	2.06E + 04	4.074	0.507

Various indications of component reliability were calculated for the "future period" in order to predict the component's behavior. The future period for which predictions were made is defined by Table 4. The "future", per assumption, starts immediately after the "history" ends. The same periodicity of overhauls is assumed.

Table 4. Definition of the future period for which reliability is to be predicted

Observation Period: $\langle 15, 30]$ year; (Duration: 15 year, i.e. 5478.75 day) $T_1 = 5478.75$ day; $T_2 = 10957.5$ day;
Operability Tests: quarterly (91.3125 day) Tests: [5478.75 5570.0625 5661.375 . . . 10957.5] day
Overhauls: every 3 years (1095.75 day) Overhauls: [5478.75 6574.5 7670.25 8766 9861.75] day

Fig. 3 through 7 comparatively present predictions of various indications based on the prior and posterior parameter values for the case of "History 4". As can be seen, based on the observed history, distribution of time to first failure shifts toward lower values. Unavailability due to failure (i. e. the probability that a component is in a failed state at the time of requested operation) is higher than based on prior parameter values and, consequently, larger number of failures is expected.

6 Summary and concluding remarks

Standby failure rate and unavailability predictions for future periods can be used in PSA models for evaluating impacts of equipment aging and maintenance strategies (e. g. frequency and scope of overhauls) on the risk measures considered. Failure of a standby component, such as a motor-driven pump, to perform intended function upon request typically represents a failure of one train of safety system in response to an initiating event.

In most of the cases, the quantification of risk by PSA model assumes that failure rates and other reliability parameters are time invariant, i. e. PSA models are used to produce long-term averaged results. Time dependent standby failure

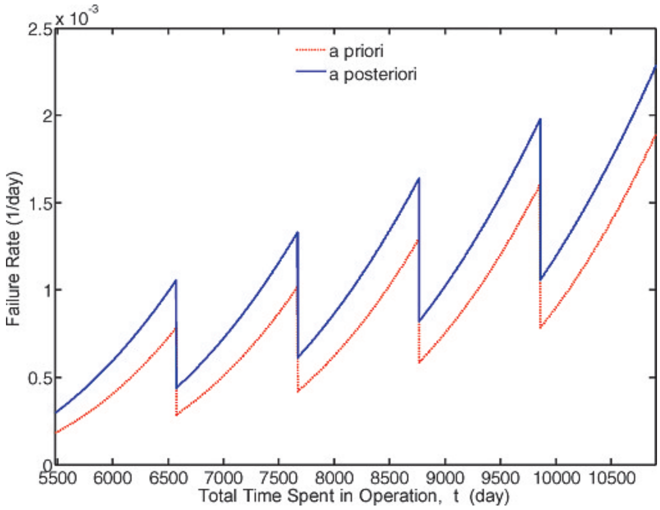

Fig. 3. Predicted standby failure rate for "History 4"

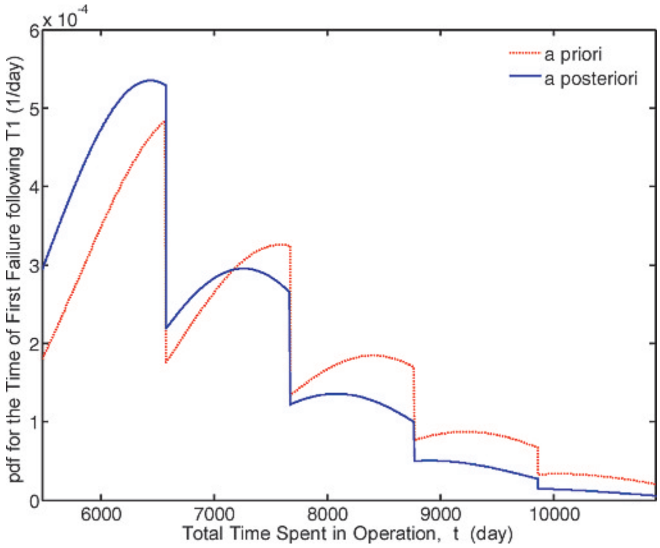

Fig. 4. Predicted pdf for the time of first failure following T_1 for "History 4"

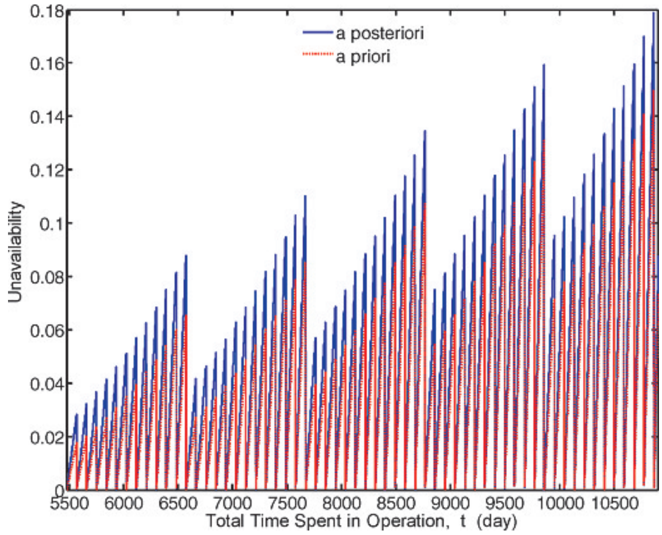

Fig. 5. Predicted instantaneous unavailability for "History 4"

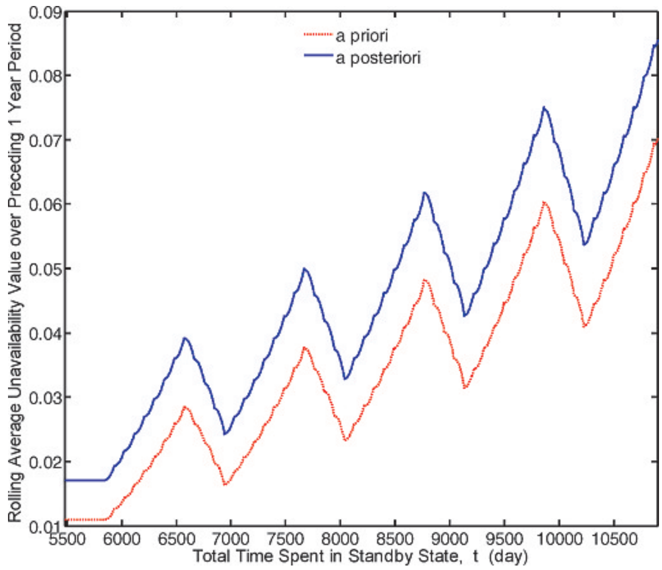

Fig. 6. Rolling average value of predicted unavailability over the preceding one-year period for "History 4"

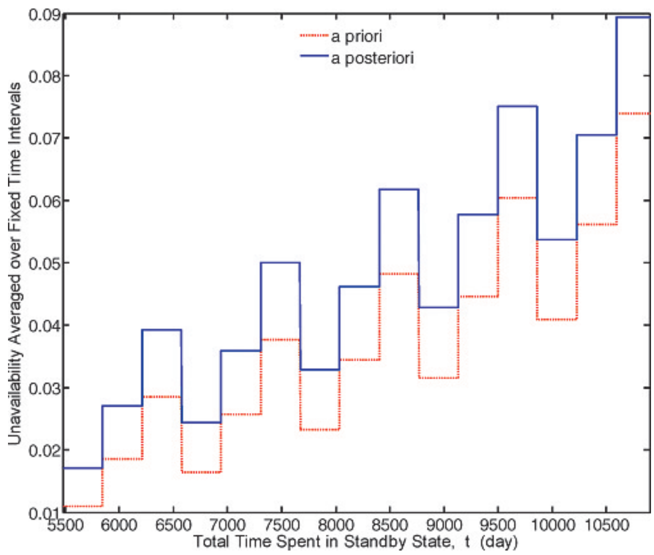

Fig. 7. Predicted unavailability averaged over the fixed time intervals for "History 4"

rate predictions obtained by methods discussed in the paper can be used in a way that the failure rate is averaged over appropriately selected future periods of time (e.g. several years). These “constant” standby failure rates (over specified time period) can then be applied in a PSA model and the risk measures (e.g. reactor core damage frequency) considered can be computed for the time period of concern.

Corresponding methods were also established for normally operating components in order to enable prediction of unconditional failure intensities or frequencies of failures, taking into account the history of failures and overhauls. The work is under progress by which predicted failure rates and probabilities for future time periods are applied to a full-scope PSA model of a nuclear power plant in order to observe an increase in the estimated reactor core damage frequency toward the end of plant life, as compared to the time invariant estimate.

One issue still open is the availability of prior distributions for time dependent standby failure rate model parameters, since most of data bases used by PSA models assume constant failure rates. This issue requires additional research work.

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Appendix A

On the maximum likelihood estimate of parameters α and β in the case of a standby component without explicit inclusion of overhaul impact ($\rho = 0$)

In cases where overhauls are not explicitly included in the model (i.e. $\rho = 0$), expression (5) for the failure rate collapses to:

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}, \quad T_1 < t \leq T_2 \tag{A.1}$$

By using, for simplicity, the following notation:

$$\vartheta_i = \int_{z_{k_i}}^{t_i} \lambda(\tau)\, d\tau = \int_{z_{k_i}}^{t_i} \frac{\beta}{\alpha}\left(\frac{\tau}{\alpha}\right)^{\beta-1} d\tau = \left(\frac{t_i}{\alpha}\right)^{\beta} - \left(\frac{z_{k_i}}{\alpha}\right)^{\beta}, \quad i = 1, \ldots, n$$

$$I_\lambda = \int_{T_1}^{T_2} \lambda(\tau)\, d\tau = \left(\frac{T_2}{\alpha}\right)^{\beta} - \left(\frac{T_1}{\alpha}\right)^{\beta} \tag{A.2}$$

the likelihood function given by Eq. (10) takes the form:

$$l(E|\alpha,\beta) = \exp(-I_\lambda)\prod_{i=1}^{n}[\exp(\vartheta_i) - 1] \tag{A.3}$$

The consideration can be simplified by assuming that for each i the following applies:

$$\vartheta_i = \int_{z_{k_i}}^{t_i} \lambda(\tau)\, d\tau \ll 1, \quad i = 1, \ldots, n \tag{A.4}$$

In this case the likelihood function is approximated by:

$$l(E|\alpha,\beta) \approx \exp(-I_\lambda)\prod_{i=1}^{n}\vartheta_i \tag{A.5}$$

By imposing the requirement $\frac{\partial l(E|\alpha,\beta)}{\partial \alpha} = 0$ onto Eq. (A.5), the following is obtained:

$$I_\lambda = n \tag{A.6}$$

Taking into account the second equation in (A.2), this comes to:

$$\left(\frac{T_2}{\alpha}\right)^{\beta} - \left(\frac{T_1}{\alpha}\right)^{\beta} = n \tag{A.7}$$

It can be shown that this expression corresponds to the one for normally operating component. It needs to have it in mind that Eq. (A.7) applies only when (A.4) is valid, which ensures that observation of failure will follow quickly after its occurrence. In this manner the conditions are similar to those that apply in the case of the normally operating component, which explains the fact that the two expressions are identical.

The expression (A.7) enables the determination of the parameter α when β is known. When neither parameter is known, additional requirement $\frac{\partial l(E|\alpha,\beta)}{\partial\alpha}=0$ can be imposed on (A.5). In this manner a system of equations is obtained, which can be solved numerically.

A.1 Time invariant failure rate

In the special case when failure rate is assumed constant over time, i. e. *β=1*, expression (5) collapses to:

$$\lambda(t)=\frac{1}{\alpha},\quad T_1<t\leq T_2 \tag{A.8}$$

The assumption (A.4) now it corresponds to:

$$\frac{t_i-z_{k_i}}{\alpha}\ll 1,\quad i=1,\ldots,n \tag{A.9}$$

The corresponding likelihood function is then approximated by:

$$l(E|\alpha)\approx\exp\left(-\frac{T_2-T_1}{\alpha}\right)\frac{1}{\alpha^n}\prod_{i=1}^{n}(t_i-z_{k_i}) \tag{A.10}$$

and ML-estimator for α is obtained from $\frac{\partial l(E|\alpha,\beta)}{\partial\alpha}=0$ as:

$$\frac{1}{\hat{\alpha}}=\frac{n}{T_2-T_1} \tag{A.11}$$

for which it can be shown that it is identical to the corresponding expression for normally operating component. Thus, as long as the time between the onset of failure and its observation is much shorter than the characteristic life of the equipment of concern, ML-estimate for the constant standby failure rate can be obtained as a ratio between the number of observed failures and the duration of the period of observation. This kind of estimate can be found, for example, in Ref. [10].

Appendix B

On estimating unavailability due to a failure in standby state in the case when overhaul impact is not explicitly included (*ρ* = 0)

Assuming *ρ = 0*, expression (11) for unavailability comes to:

$$q_S(t_S)=1-\exp\left\{-\left[\left(\frac{t_S}{\alpha}\right)^{\beta}-\left(\frac{z(t_S)}{\alpha}\right)^{\beta}\right]\right\} \tag{B.1}$$

If (A.4) is the case, this further collapses to:

$$q_S(t_S)\approx\left(\frac{t_S}{\alpha}\right)^{\beta}-\left(\frac{z(t_S)}{\alpha}\right)^{\beta} \tag{B.2}$$

Assuming that parameters α and β are known, expressions (B.1) and (B.2) can be used to determine instantaneous values of unavailability due to a failure in the standby state over time.

In the case of a time invariant failure rate, i. e. $\beta = 1$, the expression (B.1) further reduces to:

$$q_S(t_S)=1-\exp\left[-\frac{t_S-z(t_S)}{\alpha}\right] \tag{B.3}$$

while, assuming that $t_s-z(t_s)\ll\alpha$ (which corresponds to (A.4)), Eq. (B.3) becomes:

$$q_S(t_S)\approx\frac{t_S-z(t_S)}{\alpha} \tag{B.4}$$

If it can further be assumed that the time interval between two consecutive operability tests is constant which, accounting also for the assumption on immediate repair, comes to

$$t_s-z(t_s)=\Delta z \tag{B.5}$$

then the unavailability $q_S(t_S)$ would come to be the same at each particular request for operation:

$$q_s=\frac{\Delta z}{\alpha} \tag{B.6}$$

In such circumstances it is possible to estimate q_S from the number of requests for operation and the number of observed failures during the observation period $\langle T_1,T_2]$, namely:

$$T_2-T_1=m\,\Delta z \tag{B.7}$$

where m is the total number of requests for operation during the observation period $\langle T_1,T_2]$. In this case the likelihood function for observed history of considered component provided by (A.5) can be rewritten as:

$$l(E|\alpha)\approx\exp\left(-\frac{m\,\Delta z}{\alpha}\right)\prod_{i=1}^{n}\frac{\Delta z}{\alpha}=\exp(-q_s)]^m q_s^n \tag{B.8}$$

The ML-estimator for the unavailability q_S is obtained from $\frac{\partial l(E|q_s)}{\partial q_s}=0$ as:

$$\hat{q}_s=\frac{n}{m} \tag{B.9}$$

Estimator of this type for the unavailability due to a failure is very often used in PSA and reliability analyses, e. g. [11].