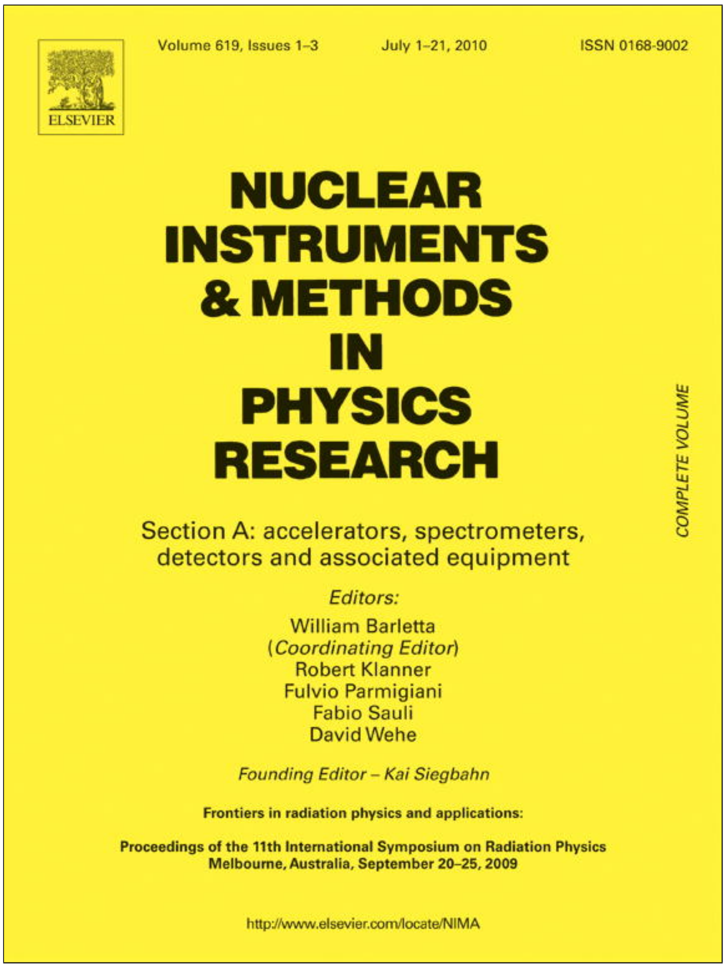

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Polarization effects in the nuclear excitation by positron–electron annihilation

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ABSTRACT

In this paper polarization effects in the nuclear excitation by positron–electron annihilation (NEPEA) process are considered. We calculated cross-sections for the NEPEA with wave functions of bound electron and incoming positron in point nuclear Coulomb field. The results are compared to predictions which are derived using Born approximation. Results for the annihilation on the K-electron are presented. We found that cross-section for magnetic transition is much more pronounced for orientation of electron spin in positron direction. For electric transition cross-sections with parallel spin orientation are larger than for antiparallel, at least for small positron energy.

1. Introduction

There have been several theoretical calculations of cross-section for the nuclear excitation by positron–electron annihilation (NEPEA) process. The first theoretical prediction [1] showed that cross-section of the process can be calculated by a two step model. Despite different methods of calculations, a large discrepancy between theoretical predictions and experimental results still persists. In order to explain the observed discrepancies several authors have offered different theoretical models. Pisk et al. [2] made calculation in the one step model based on Born approximation, while Kaliman et al. [3] improved the analysis by using wave functions for the bound electron and positron in nuclear Coulomb field [4], as well as screened Coulomb field [5]. Ljubičić et al. [6] try to explain discrepancies by applying a model of indistinguishable quantum oscillators.

In the present work we examine the effect of spin orientation of positron and electron in the NEPEA process. We present results for annihilation of K-electron in Born approximation and in point nuclear Coulomb field.

2. Cross-section calculation

The cross-section for the NEPEA process via 2^L-pole transition can be written in a form

$$\sigma^{(i)} = \frac{g\Gamma_0}{\Gamma}\frac{E}{p}\frac{4\pi^2\alpha}{\omega}\frac{1}{2L+1}\sum_{M=-L}^{L}|B_{LM}^{(i)}|^2 \tag{1}$$

where index i stands for m-magnetic or e-electric type of transition, E and p are positron energy and momentum, ω is nuclear transition energy, α is the fine structure constant. In Eq. (1) g is the statistical weight, and Γ_0 is the ground state transition width of the excited level which has the total width Γ. Here $B_{LM}^{(i)}$ is

$$B_{LM}^{(i)} = \sqrt{32\pi}\mathrm{i}\omega\frac{1}{\sqrt{L(L+1)}}\sum_{jl}A_{jl}^{(i)}R_{jl}^{(i)} \tag{2}$$

where $R_{jl}^{(i)}$ is a radial part, $A_{jl}^{(i)}$ is an angular part of the matrix element, and j, l denote positron total and orbital angular momentum. Radial parts are

$$R_{jl}^{(m)} = [l',\lambda']R_5 - [l,\lambda]R_6$$

$$R_{jl}^{(e)} = L\left[R_1 + R_2 + \left(1-\frac{1}{L}[l,\lambda']\right)R_3 - \left(1-\frac{1}{L}[l',\lambda]\right)R_4\right] \tag{3}$$

where R_i are radial integrals defined in [3] Eqs. (11) and (13). We defined [,] with

$$[l,\lambda] = j(j+1) - l(l+1) - J(J+1) + \lambda(\lambda+1) \tag{4}$$

where J, λ are bound electron total and orbital quantum numbers. Also, $l' = 2j - l$, $\lambda' = 2J - \lambda$.

We chose the direction of the positron as z-axis, and after some angular momentum algebra, we got for the angular term

$$A_{jl}^{(i)} = (-1)^{\mu-1/2}\frac{2j+1}{4\pi}\frac{1}{2}[1-(-1)^{l+\lambda+L+1}] \sqrt{\left(J+\frac{1}{2}\right)(2L+1)}\begin{pmatrix} j & L & J \\ 1/2 & 0 & -1/2 \end{pmatrix}\begin{pmatrix} j & L & J \\ \nu & -M & \mu \end{pmatrix} \tag{5}$$

where ν is z-projection of positron spin, and μ projection of electron total angular momentum. We have constraints on

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Table 1
Amplitudes for different orientation of spins in positron annihilation on *s* electron.

μ	ν	$\sum_{jl} A_{jl} R_{jl}$
1/2	1/2	$k\sqrt{L(L+1)}(R_+ - R_-)$
$-1/2$	$-1/2$	$k\sqrt{L(L+1)}(R_+ + R_-)$
$-1/2$	1/2	$-k((L+1)R_+ + LR_-)$
1/2	$-1/2$	$k(-(L+1)R_+ + LR_-)$

Constant k is given by $k = 1/(4\pi\sqrt{2L+1})$, and $R_\pm$ are given by Eq. (6).

positron angular momentum from $3-j$ coefficients. For the s electron ($\lambda = 0, J = \frac{1}{2}, \lambda' = 1$) we have constraints $j = L \pm \frac{1}{2}$, $l = L$ for magnetic transition or $l' = L$ for electric transition, and A_{jl}, $R_\pm$ are given by

$$A_{jl} = (-1)^{L+\mu+1/2}\frac{2j+1}{4\pi\sqrt{2}}\begin{pmatrix} j & L & 1/2 \\ \nu & -M & \mu \end{pmatrix}$$

$$R_+^{(m)} = -L(R_5 + R_6)$$

$$R_-^{(m)} = (L+1)(R_5 + R_6)$$

$$R_+^{(e)} = R_1 + R_2 + 2R_3$$

$$R_-^{(e)} = \sqrt{\frac{L}{L+1}}\left(R_1 + R_2 - \frac{1}{L}R_3 - \frac{2L+1}{L}R_4\right). \tag{6}$$

Radial integrals in $R_\pm$ have to be calculated with $j = L \pm \frac{1}{2}$. Amplitudes for different orientation of electron and positron spins are shown in Table 1.

For the annihilation of K-electron, it is possible to get analytical (compact) expression for radial integrals in Born approximation (valid for $\alpha Z \ll 1$). In that case $R_2 = R_3 = R_5 = 0$, and

$$R_1 = -2i^L\sqrt{\frac{E-m}{2E}}(m\alpha Z)^{3/2}R_L \tag{7a}$$

$$R_4 = -2i^L\sqrt{\frac{E+m}{2E}}(m\alpha Z)^{3/2}R_{L-1} \tag{7b}$$

$$R_6 = -2i^{L+1}\sqrt{\frac{E+m}{2E}}(m\alpha Z)^{3/2}R_L \tag{7c}$$

$$R_L = \frac{p^L}{\omega^{L+1}}\mathcal{F}_1, \quad |\mathcal{F}_1|^2 = \frac{1}{(\omega^2-p^2)+(2\alpha Z\omega)^2}. \tag{7d}$$

In Eqs. (7) m is the mass of the electron (positron) and Z is the atomic number.

3. Results and discussion

Cross-sections for the annihilation of K-electron in Born approximation and for small positron energy are shown in Table 2. Cross-sections for the magnetic transition for the annihilation of electron with spin pointing opposite of the direction of positron momentum are strongly suppressed, see Figs. 1 and 3.

For the electric transition, all cross-sections are of the same order (because $R_4 \gg R_1 \to R_- \gg R_+$ for most energies). In this approximation, only relative spin projection of electron and positron is important and cross-sections for parallel orientation are larger than for antiparallel by the factor $(L+1)/L$, i.e.

$$\sigma_{++} = \sigma_{--} = \frac{L+1}{L}\sigma_{+-} = \frac{L+1}{L}\sigma_{-+} \tag{8}$$

where $\pm$ means spin up(down) and the first is for electron, the second for positron. These conclusions are confirmed for small positron energy and $E1$ transition, as shown in Fig. 2. Approximative formula in Table 2 and conclusions in Eq. (8) cannot be used for higher positron energies because R_1 is not small in comparison to R_4. In case of higher L and larger energies, it can happen that $\sigma_{-+} > \sigma_{++}$ (Fig. 3 lower panel).

Table 2
Cross-sections for the positron annihilation on K-electron in Born approximation for different spins.

μ	ν	$\sigma^{(m)}/K$	$\sigma^{(e)}/K$
1/2	1/2	$\lvert R_L\rvert^2$	$\frac{L}{L+1}\lvert R_{L-1}\rvert^2$
$-1/2$	$-1/2$	$\frac{\lvert R_L\rvert^2}{(2L+1)^2}$	$\frac{L}{L+1}\lvert R_{L-1}\rvert^2$
$-1/2$	1/2	0	$\left(\frac{L}{L+1}\right)^2\lvert R_{L-1}\rvert^2$
1/2	$-1/2$	$\frac{4L(L+1)}{(2L+1)^2}\lvert R_L\rvert^2$	$\left(\frac{L}{L+1}\right)^2\lvert R_{L-1}\rvert^2$

Constant is given by $K = (g\Gamma_0/\Gamma)((E+m)/p)16\pi\alpha\omega(m\alpha Z)^3$, and R_L by Eq. (7d). In an electric transition it is supposed that positron kinetic energy is small, that implies $R_1 \ll R_4$.

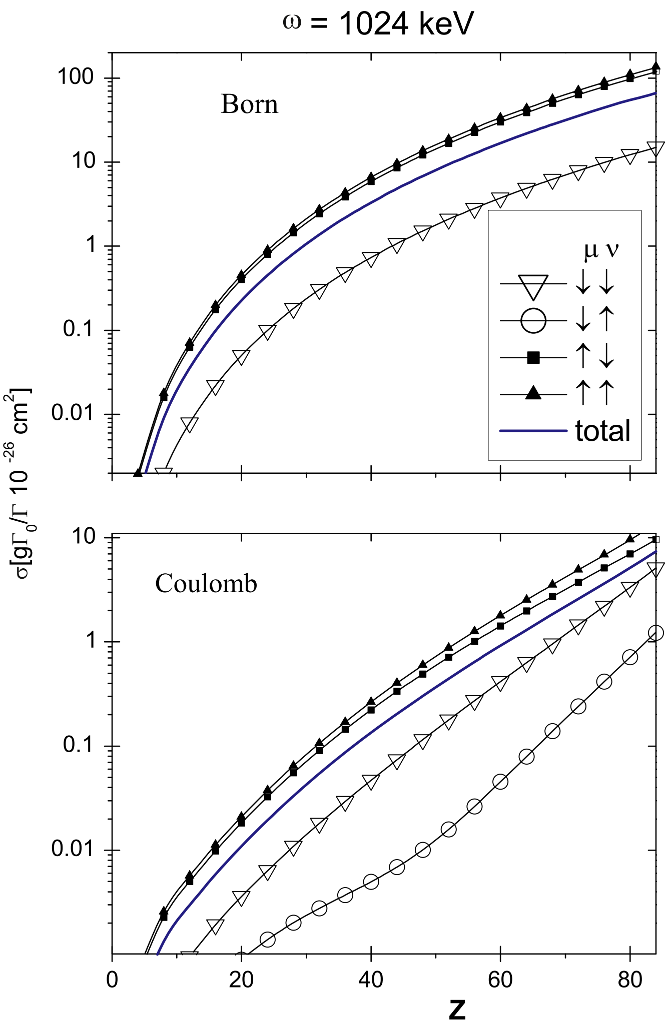

Fig. 1. The Z dependence of the cross-sections for NEPEA of K-electron for different spin projections, in Born approximation (upper panel), and with Coulomb wave functions (lower panel) for M1 transition. The nuclear energy level $\omega = 1024$ keV is assumed.

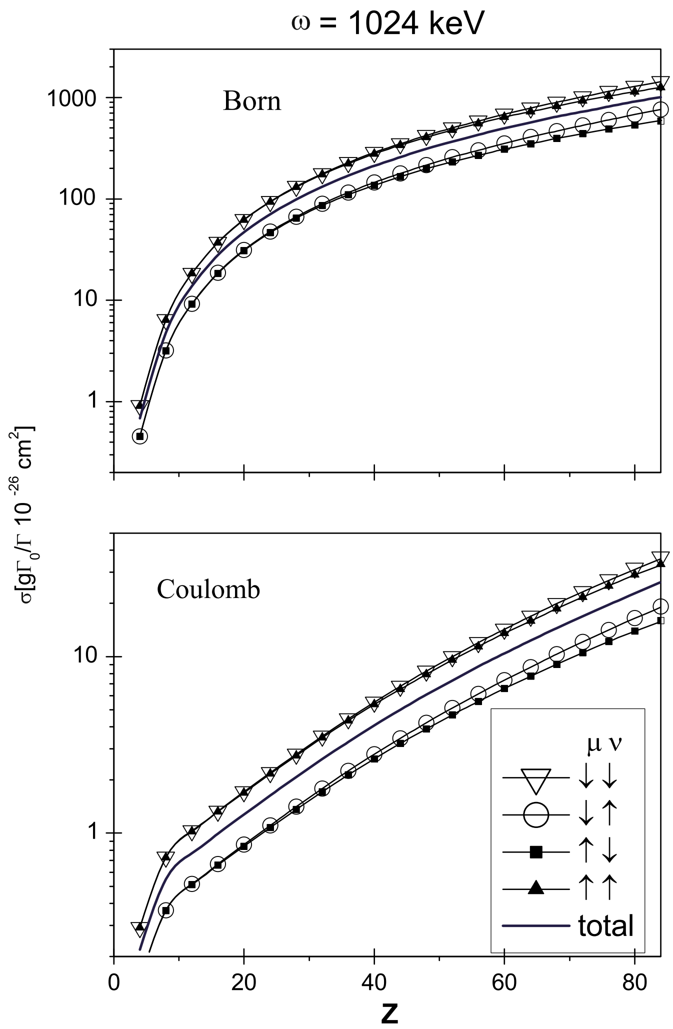

Fig. 2. Same as Fig. 1 for electric (E1) transition.

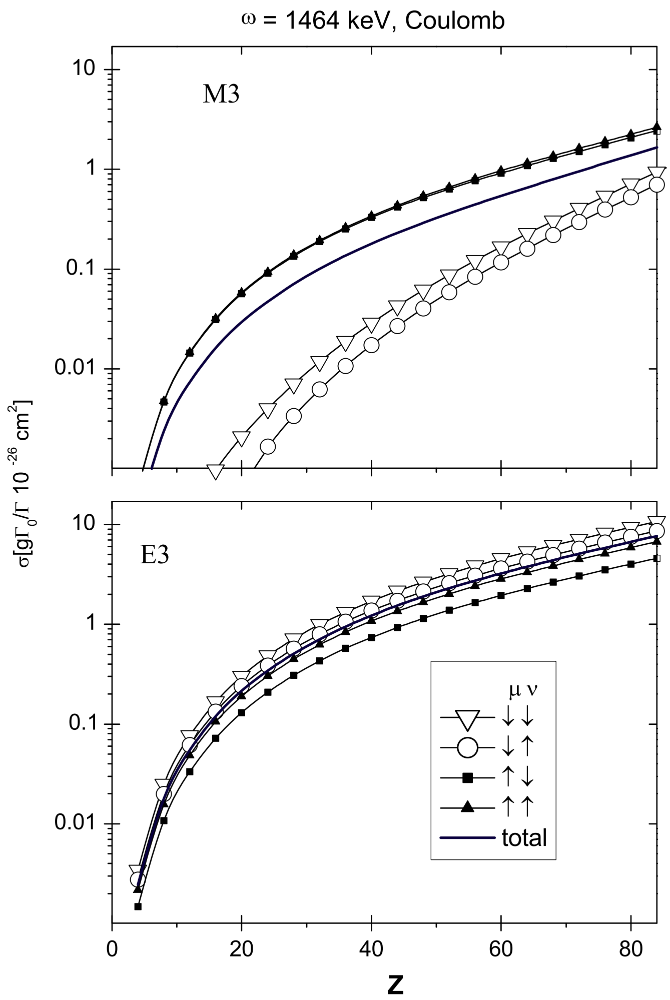

Fig. 3. The Z dependence of the cross-sections for NEPEA of K-electron for different spin projections with Coulomb wave functions for M3 and E3 transition. The nuclear energy level $\omega = 1464\,\text{keV}$ is assumed.

Effects of the spin orientation are well described in Born approximation. There are similar discrepancies between the calculation in Born approximation and with wave functions in point nuclear Coulomb field for the total cross-sections and for different spin orientations of bound electron and incoming positron.

4. Conclusion

We investigated effects of the spin orientation in the NEPEA process. Magnetic transition cross-section for the annihilation of electron with the spin in positron direction is much more pronounced than for the opposite direction. Electric transition cross-section for parallel spin orientation is larger than for the antiparallel, at least for small positron energy and small L.

In Born approximation polarization effects are faithfully described and discrepancies are similar as in total cross-section. Spin dependencies are qualitatively similar to that obtained in point nuclear Coulomb field.

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