

Novel Low-Sensitivity, Third-Order LP Active Leap-Frog Filter

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Abstract – This paper presents the realization of third-order low-pass (LP) active-*RC* filters using new simplified Leap-Frog (LF) topology. It has the reduced number of components, reduced complexity and straightforward design procedure. It is well known that the passive ladder-RLC filters, terminated with equal resistors at both ends have very low sensitivity to component tolerances in the pass-band. If active-*RC* filter are built using structures which simulate ladder-RLC using signal-flow-graphs (SFG), their sensitivities are significantly decreased. The leap frog structure follows readily from this simulation and has minimum sensitivity. The new structure presented here is a simplified LF structure with the elements calculated directly from the transfer function coefficients. As an illustration of the efficiency of the proposed modified LF filter, the sensitivity analysis with Monte Carlo runs using PSpice is performed for a third-order Butterworth and Chebyshev LP filters with various pass-band ripples.

Low-Sensitivity, Active-*RC*, Leap-Frog Filter

I. INTRODUCTION

In spite of popular digitization of large number of electronic systems there is an unavoidable part of every mixed analog and digital signal processing device, so called 'analog front end'. In the analog front end selective filters are usually most frequently met. Continuous-time circuits have advantages over discrete-time circuits in that they do not produce sampling noise, because they do not possess any A/D and D/A converters, require no anti-aliasing filters, require less power, possess a wider bandwidth, and finally are much simpler to realize. The filters that are intended to be realized on chips must not have inductors, that is, they are realized as active-*RC* filters. On the other hand analog filters have many drawbacks in IC design such as power consumption, a need for large chip area, etc. The most important disadvantage of active-*RC* filters is *high sensitivity* to component tolerances of the circuit which emphasize the accuracy problem. Because of this problem every new proposal of low-sensitivity circuit is very welcome.

In this paper we present a new topology of third-order allpole active-*RC* circuit, which has low sensitivity to component variations and low complexity because of minimum number of components. The new filter has the 'leap-frog' structure and will be compared to others, broadly used filters that realize the same, third-order, low-pass allpole transfer function. We compare performance of four low-pass active-*RC* filters: (i) New leap-frog filter (ii) Standard leap-frog filter (signal-flow graph simulation of *LC* ladder filters), (iii) Sallen and Key biquad (single-amplifier biquad), and (iv) Tow-Thomas biquad (multi-amplifier biquad).

Using the examples of Butterworth, and Chebyshev (with lower and higher pass-band ripple) approximations, we illustrate the sensitivity of four filters using Monte Carlo runs with PSpice. It turns out that the new leap-frog filter has better performance when compared to other three low-pass filter topologies regarding sensitivity and design complexity.

II. LOW-SENSITIVITY CIRCUITS

A. New leap-frog filter

Consider the third-order voltage transfer function of an allpole low-pass filter given in terms of the polynomial coefficients a_i ($i=0, 1, 2$)

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{k \cdot a_0}{s^3 + a_2 s^2 + a_1 s + a_0}, \quad (1)$$

and in factored form consider the negative-real pole γ and the complex conjugate pole pair given in terms of the pole frequency ω_p and the pole Q, q_p ,

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{k \cdot \gamma \omega_p^2}{(s + \gamma) \left(s^2 + \frac{\omega_p}{q_p} s + \omega_p^2 \right)}. \quad (2)$$

Note that k represents the dc-gain. Third-order transfer functions in (1) can be realized by the new leap-frog third-order filter section shown in Fig. 1.

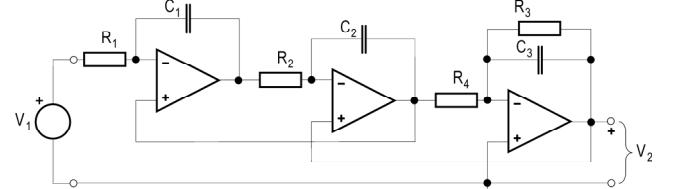


Fig. 1. New leap-frog third-order filter section.
 Transfer function coefficients for the section in Fig. 1 are given by

$$\begin{aligned} k &= \frac{R_3}{R_4}; \quad a_0 = \frac{1}{R_1 C_1 R_2 C_2 R_3 C_3}; \\ a_1 &= \frac{1 + R_3 / R_4}{R_2 C_2 R_3 C_3} + \frac{1}{R_1 C_1 R_2 C_2}; \quad a_2 = \frac{1 + R_3 / R_4}{R_3 C_3} + \frac{1}{R_2 C_2}. \end{aligned} \quad (3)$$

Using abbreviations

$$\alpha_3 = \frac{R_3}{R_4}; \quad \omega_1 = \frac{1}{R_1 C_1}; \quad \omega_2 = \frac{1}{R_2 C_2}; \quad \omega_3 = \frac{1}{R_3 C_3}, \quad (4)$$

we can rewrite (3) in the form

$$\begin{aligned} k &= \alpha_3; \quad a_0 = \omega_1 \omega_2 \omega_3; \\ a_1 &= (1 + \alpha_3) \omega_2 \omega_3 + \omega_1 \omega_2; \\ a_2 &= (1 + \alpha_3) \omega_3 + \omega_2. \end{aligned} \quad (5)$$

From (5) the solutions for ω_1 , ω_2 and ω_3 readily follow

$$\begin{aligned} \omega_3^3 - 2a_2\omega_2^2 + (a_1 + a_2^2)\omega_2 - a_1 a_2 + a_0(1 + \alpha_3) &= 0 \Rightarrow \omega_2 \\ \omega_3 &= (a_2 - \omega_2)/(1 + \alpha_3); \\ \omega_1 &= (\omega_2^2 - a_2\omega_2 + a_1)/\omega_2. \end{aligned} \quad (6)$$

Normalized element values of new leap-frog filter in Fig. 1 are calculated using the following design steps:

- i) Choose or determine $\alpha_3=k$;
- ii) Choose $R_1=R_2=R_3=1$ and calculate $R_4=R_3/\alpha_3$;
- iii) Calculate ω_2 , ω_3 and ω_1 using (6);
- iv) Calculate $C_1=1/\omega_1$; $C_2=1/\omega_2$; $C_3=1/\omega_3$.

The procedure for the design is very simple. The most complex task is finding ω_2 by solving the third-order polynomial in (6).

As an illustration of the step-by-step design procedure above consider three examples of third-order LP filter transfer functions: (i) Butterworth, (ii) Chebyshev with the pass-band ripple $A_{\max}=0.5$ dB and (iii) Chebyshev with $A_{\max}=1.0$ dB. A Butterworth filter has 'maximally flat' amplitude response and corresponds to the limit case of no ripple in the filter pass-band. Compared to a Chebyshev filter of equal order, it has lower pole Qs. It is well known that the higher the ripple the higher are the pole Qs, and consequently the sensitivities, as well. Since we investigate sensitivities to component tolerances, those three approximations examples are suitable because they provide three different pole Q-factors: i.e. $q_p=1$ (low-Q example for Butterworth filter), $q_p=1.2$ and 2.017 (medium- and high-Q examples for Chebyshev filter examples). Coefficients a_i ($i=0, 1, 2$) and the corresponding dc-gain and pole parameters, for these examples are given in Table I. All filters are realized by the circuit shown in Fig. 1 and their transfer function magnitudes $\alpha(\omega)=20\log|H(j\omega)|$ [dB] are shown in Fig. 7 in Section 3.

Table I Pole parameters of filter examples.

Type	k	γ	q_p	ω_p	a_2	a_1	a_0
Butter	1	1	1	1	2	2	1
Cheby 0.5	1	0.626456	1.706189	1.068853	1.25291	1.5349	0.71569
Cheby 1.0	1	0.494171	2.017720	0.997098	0.98834	1.23841	0.49131

Normalized element values of new leap-frog filters are given in Table II.

Table II Element values of novel leap-frog filters.

Type	R_1	R_2	R_3	R_4	C_1	C_2	C_3
Butterworth	1	1	1	1	0.3522	2.19154	1.29567
Chebyshev 0.5	1	1	1	1	0.13715	5.44699	1.87034
Chebyshev 1.0	1	1	1	1	0.10701	8.24621	2.30661

As can be seen, we require *three* opamps to realize the new leap-frog third-order filter in Fig. 1.

B. Signal-flow graph simulation of LC ladder filters

Third-order transfer function in (1) can be realized by the passive-RLC ladder filter shown in Fig. 2. To be realized using modern IC design, the original passive-RLC ladder filter is converted into an inductorless ladder circuit consisting of only adders, subtractors, and integrators with the help of signal-flow graphs (SFG) [1][2].

This well-known method is best illustrated by the following example. The corresponding SFG of the doubly terminated third-order low-pass allpole filter in Fig. 2 is shown in Fig. 3(a).

The SFG in Fig. 3(a) represents a set of linear algebraic equations that define the Kirchhoff's laws and voltage-to-current relations of all branches in the passive-RLC network.

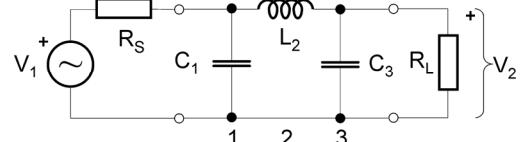


Fig. 2. LP passive-RLC low-pass filter prototype.

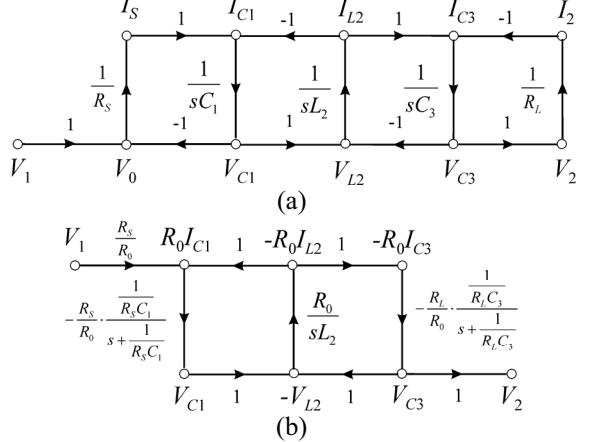


Fig. 3. Signal-flow-graph of the passive-RLC simulation.

Circuit realization using SFG, can be simplified if we use integrators for the summing purpose, as well. Therefore, after some manipulation, the final SFG is shown in Fig. 3(b). The corresponding active-RC circuit is shown in Fig. 4, and it is known as the (standard) *leap-frog* circuit. We require *four* opamps to realize the third-order leap-frog filter in Fig. 4.

Passive ladder-LC filters, terminated with equal resistors at both ends have very low sensitivity to component tolerances in the pass-band [3]. This property is retained in the leap-frog topology and because of this performance we propose a new, simpler and also very low-sensitive leap-frog filter.

Normalized element values of ladder-RLC filters following from filter tables [4][5] or filter programs are given in Table III. They are directly used in active-RC circuit in Fig. 4.

Table III Element values of ladder-RLC filters.

Type	R_1	C_1	L_2	C_3	R_2
Butterworth	1	1	2	1	1
Chebyshev 0.5	1	1.5963	1.0967	1.5963	1
Chebyshev 1.0	1	2.0236	0.9941	2.0236	1

C. Single-amplifier biquad (SAK)

Third-order transfer function in (1) can also be realized by the filter section shown in Fig. 5 [6].

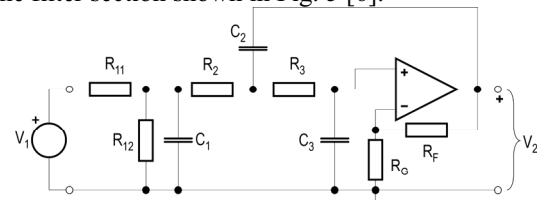


Fig. 5. Low-pass single-amplifier active-RC filter.

Transfer function coefficients for the section in Fig. 5 are

$$\begin{aligned} k &= \alpha\beta, & a_0 &= \frac{1}{R_1 R_2 R_3 C_1 C_2 C_3}; \\ a_1 &= \frac{R_1 C_1 + (R_1 + R_2 + R_3)C_3 + (1 - \beta)C_2(R_1 + R_2)}{R_1 R_2 R_3 C_1 C_2 C_3}, & (7) \\ a_2 &= \frac{R_2 C_3 (R_1 C_1 + R_3 C_2) + R_1 R_3 C_3 (C_1 + C_2) + (1 - \beta)R_1 R_2 C_1 C_2}{R_1 R_2 R_3 C_1 C_2 C_3}, \end{aligned}$$

with

$$\alpha = \frac{R_{12}}{R_{11} + R_{12}}; \quad R_1 = \frac{R_{11} R_{12}}{R_{11} + R_{12}}; \quad \beta = 1 + \frac{R_F}{R_G}. \quad (8)$$

The circuit in Fig. 5 belongs to the class of active- RC filters with single amplifier and positive feedback (Sallen and Key type). Normalized element values of SAK filters readily follow using well-known design procedure in [6] and are given in Table IV. The design procedure presented in [6] is optimal because it provides low-sensitivity SAK filters. To minimize the sensitivity of the transfer function magnitude to passive components, the 'capacitive tapering' (by factor three, i.e. $C_2 = C_1/3$; $C_3 = C_1/9$) with values $R_2 \approx R_3$ is used.

Table IV Element values of single-amplifier filters (SAK).

Type	R_1	R_2	R_3	C_1	C_2	C_3	α	β
Butterworth	1.13636	4.82684	4.92247	1	0.33	0.11	0.85178	1.17401
Chebyshev 0.5	1.8315	4.4987	4.5787	1	0.33	0.11	0.70124	1.42604
Chebyshev 1.0	2.3202	4.8855	4.8482	1	0.33	0.11	0.67836	1.4742

D. Tow-Thomas biquad

Third-order low-pass allpole transfer functions in (1) can also be realized using a biquad circuit and an RC section for realization of a real pole shown in Fig. 6.

The circuit shown in Fig. 6 has proved to be advantageous for various reasons including its good dynamic-range properties, and its excellent tuning properties. It uses the Tow-Thomas biquad or the multi-amplifier biquad [7]. To realize the third-order allpole transfer function the circuit needs four opamps.

Transfer function parameters for the filter in Fig. 6 are

$$k = \frac{R_3}{R_4}; \quad \omega_p = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}}; \quad q_p = R_1 \sqrt{\frac{C_1}{R_2 R_3 C_2}}; \quad \gamma = \frac{1}{R_7 C_3}. \quad (9)$$

Step-by-step design process can be performed as follows:

- i) Choose $R_2 = 1$, $C_1 = C_2 = C_3 = 1$;
- ii) Calculate $R_3 = 1/\omega_p^2$ (to realize and tune ω_p);
- iii) Calculate $R_1 = q_p/\omega_p$ (to realize and tune q_p);
- iv) Calculate $R_4 = 1/(\omega_p^2 k)$ (to realize and tune k);
- v) Calculate $R_7 = 1/\gamma$ (to realize γ).

From (9) it is possible to formulate the non-iterative ('orthogonal') tuning procedure which is also notified in design steps i) to v) above.

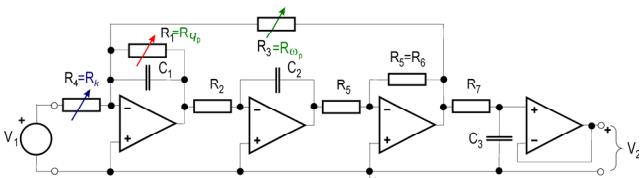


Fig. 6. Third-order Tow-Thomas biquad.

For the three examples defined by parameters in Table I normalized element values of Tow-Thomas biquad, obtained using the above design steps are given in Table V.

Table V Element values of Tow-Thomas Biquad.

Type	R_1	$R_{2,5-6}$	R_3	R_4	R_7	C_{1-3}
Butterworth	1	1	1	1	1	1
Chebyshev 0.5	1.5963	1	0.8753	0.8753	1.5963	1
Chebyshev 1.0	2.0236	1	1.0058	1.0058	2.0236	1

III. SENSITIVITY ANALYSIS

In order to perform the sensitivity analysis we compared the four circuits in the Section 2 that realize the transfer functions defined by the parameters in Table I. We use PSpice with Monte Carlo runs (having zero-mean Gaussian distribution and 1% standard deviation of all passive components) [8]. The corresponding magnitudes and MC runs are shown in Fig. 7.

Sensitivity of the new leap-frog filter (NLF) is shown in the row no. 1). For Butterworth approximation it has a slightly better sensitivity than the standard Leap-frog (LF), but worse than SAK and TT biquad. However the Chebyshev approximation examples give different sensitivity figures. The SAK circuit has the worst sensitivities and TT circuit the best. NLF circuit is slightly better than LF, and worse than TT. One can conclude that for the approximation with lower pole Q-factors such as Butterworth, SAK circuit is more suitable from the sensitivity point. However for higher pole Q approximations, such as Chebyshev, it is less suitable, and TT circuit gives better sensitivity results.

One more aspect should be pointed out when the LF and NLF sensitivities are compared to other circuits. It can be seen that in LF and NLF circuits the dispersions of the frequency responses are equally distributed along the passband, meaning that overall filter gain of both circuits is more sensitive than in other circuits, but the shape of the frequency response in the passband remains rather unchanged. In other two circuits the sensitivities are emphasized at the bandedges, meaning that the shape of the frequency response characteristic is deformed more than in LF and NLF. In many applications where this shape is more important than the overall filter gain, LF and NLF circuit should have an advantage.

The circuits can also be compared from the signal dynamics point. It can be easily concluded that in this view, SAK circuit has the best performance, since it has only one amplifier and the overall filter gain can be easily established. LF and TT circuits have more amplifiers, but they can be easily optimized for the best dynamic performance. NLF circuit, in the presented configuration, is not suitable for optimization, and that is the main drawback of this circuit.

Finally, the advantage of the new leap-frog circuit is, beside low sensitivity, the minimal number of passive components compared to the rest of the circuits, and less opamps than in LF and TT circuits.

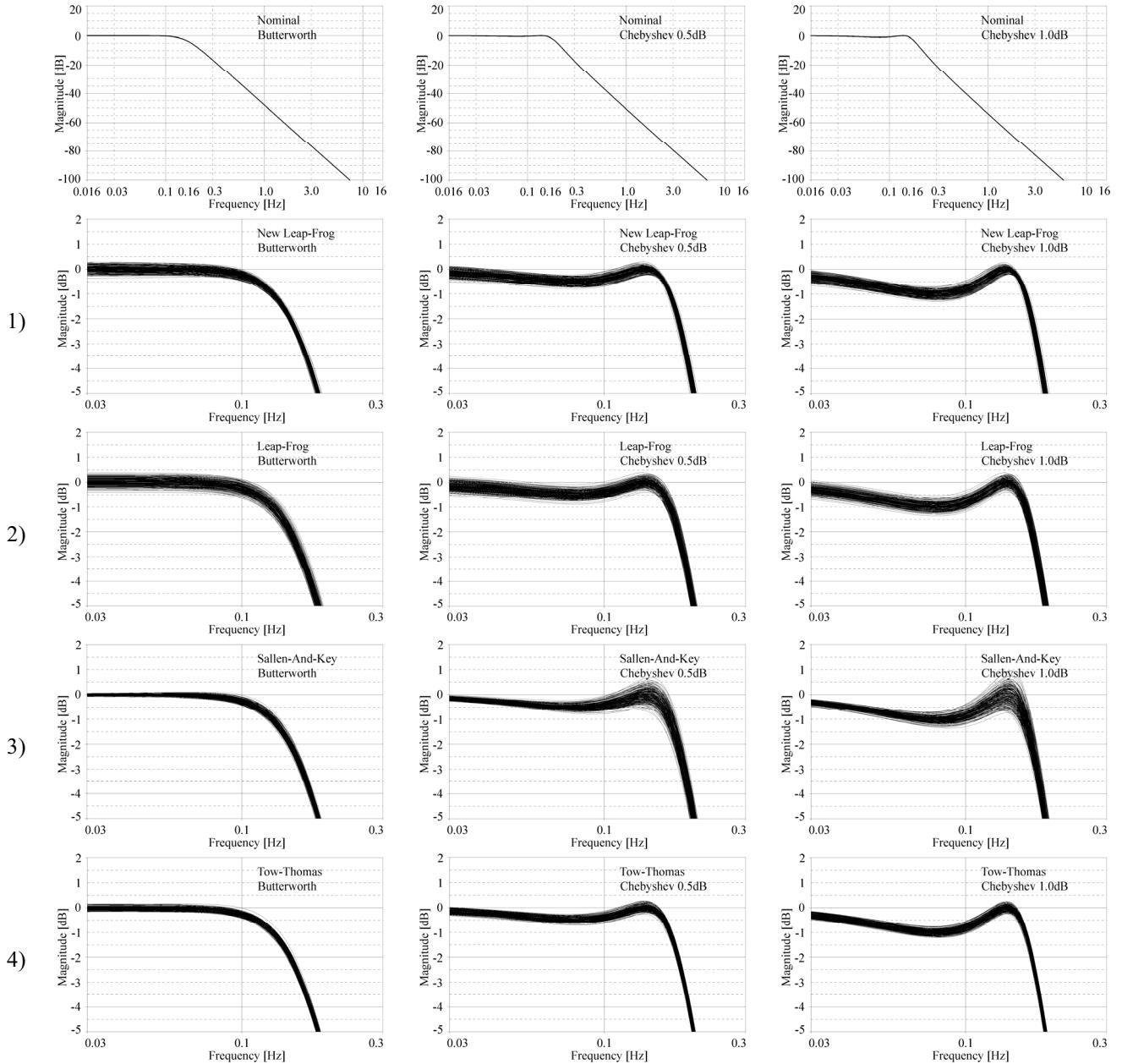


Fig. 7. Magnitudes and Monte Carlo runs of examples.

IV. CONCLUSIONS

This paper presents the new leap-frog topology for the design of allpole third-order LP filter. The presented topology has very low sensitivities to component tolerances, even lower than the standard leap-frog circuit. Besides, it has canonical number of components and straightforward design procedure. Furthermore, the new leap-frog topology can be generalized to realize n^{th} -order filter (the same is true for the standard leap-frog).

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