Fuzzy logic and its application in technical systems

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Introduction

- Humans have a remarkable capability to reason and make decisions in an environment of uncertainty, imprecision, incompleteness of information, and partiality of knowledge, truth and class membership.
- The principal objective of **fuzzy logic** is formalization of this capability.
- Examples
  - The weather is cloudy and mostly cloudy
  - The water is warm
  - This girl is very tall
  - The concert starts around 10pm
- The fuzzy set theory and possibility theory represent very convenient framework for dealing with knowledge described in this way [Zadeh]

Fuzzy logic

- The term *fuzzy logic* emerged in the development of the theory of fuzzy sets by Lotfi Zadeh (1965)
- A type of logic that recognizes more than simple true and false values
  - With fuzzy logic, propositions can be represented with degrees of truthfulness
  - For example, the statement, *today is sunny,* might be
    - 100% true if there are no clouds,
    - 80% true if there are a few clouds,
    - 50% true if it's hazy and
    - 0% true if it rains all day.

Fuzzy logic

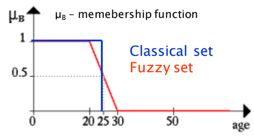
- Two main directions in fuzzy logic have to be distinguished (Zadeh 1994).
  - *Fuzzy logic in the broad sense* (older, better known, heavily applied but not asking deep logical questions) serves mainly as apparatus for fuzzy control, analysis of vagueness in natural language and several other application domains.
  - It is one of the techniques of *soft-computing*, i.e. computational methods tolerant to suboptimality and impreciseness (vagueness) and giving quick, simple and *sufficiently good* solutions.

Fuzzy logic

- *Fuzzy logic in the narrow sense* is symbolic logic with a comparative notion of truth developed fully in the spirit of classical logic (syntax, semantics, axiomatization, truth-preserving deduction, completeness, etc.; both propositional and predicate logic).
  - It is a branch of *many-valued logic* based on the paradigm of *inference under vagueness.*
  - This fuzzy logic is a relatively young discipline, both serving as a foundation for the fuzzy logic in a broad sense and of independent logical interest, since it turns out that strictly logical investigation of this kind of logical calculi can go rather far.

Fuzzy sets

- **Fuzzy sets** are sets whose elements have degrees of membership
- Fuzzy set theory permits the gradual assessment of the membership of elements in a set
  - this is described with the aid of a membership function valued in the real unit interval [0, 1], instead of set {0, 1} in classical (crisp) sets
- Example: **B** fuzzy set of young people



Fuzzy sets

- A *degree of membership* of element x in fuzzy set **A** is expressed by membership function
- Let X be the universal set (universe of discourse), and its elements are denoted by x
- Classical set A might be expressed as

$$A = ((x, \mu_A(x)) \mid x \in X, \mu_A(x) : X \to \{0,1\})$$

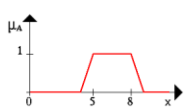
- Fuzzy set **A** is expressed as

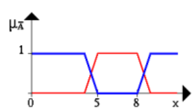
$$\boldsymbol{A} = ((x, \mu_A(x)) \mid x \in X, \mu_A(x) : X \to [0,1])$$

Fuzzy sets

- Basic operations on fuzzy sets: complement, intersection and union
  - A complement of a fuzzy set **A**⊆X, **A**^{c} ⊆X is

$$\mu_{A^c}(x) = 1 - \mu_A(x), \forall x \in X$$

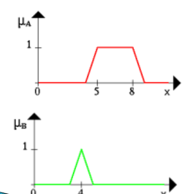


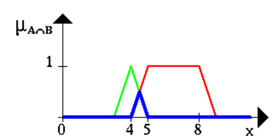


Fuzzy sets

- An intersection of two fuzzy sets **A** i **B**, **A,B** ⊆ X, **A**∩**B** ⊆ X is

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)), \forall x \in X$$

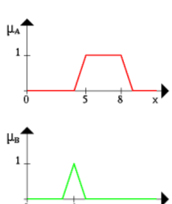


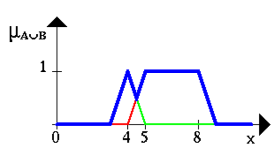


Fuzzy sets

- A union of two fuzzy sets **A** i **B**, **A,B** ⊆ X, **A**∪**B** ⊆ X is

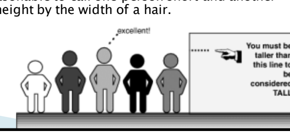
$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)), \forall x \in X$$

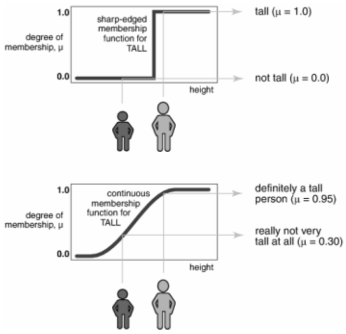




Example

- One of the most commonly used examples of a fuzzy set is the set of tall people.
- In this case, the universe of discourse is all potential heights, say from 120 cm to 240 cm, and the word tall would correspond to a curve that defines the degree to which any person is tall.
- If the set of tall people is given the well-defined (crisp) boundary of a classical set, you might say all people taller than 185 cm are officially considered tall.
  - However, such a distinction is clearly absurd.
  - It may make sense to consider the set of all real numbers greater than 185 because numbers belong on an abstract plane, but when we want to talk about real people, it is unreasonable to call one person short and another one tall when they differ in height by the width of a hair.





Subjective interpretations and appropriate units are built right into fuzzy sets.
If you say "She's tall," the membership function tall should already take into account whether you are referring to a six-year-old or a grown woman.

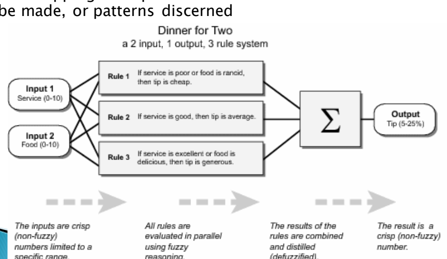
http://www.mathworks.com/access/helpdesk/help/toolbox/fuzzy/bp78l6_-1.html

Fuzzy sets

- If -then rules
  - Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic
  - If-then rule statements are used to formulate the conditional statements that comprise fuzzy logic
- A single fuzzy if-then rule assumes the form
- if *x* is *A* then *y* is *B*
  - where *A* and *B* are linguistic values defined by fuzzy sets on the ranges (universes of discourse) X and Y, respectively
  - the if-part of the rule "*x* is *A*" is called the *antecedent* or premise, while the then-part of the rule "*y* is *B*" is called the *consequent* or conclusion.
  - An example of such a rule might be:
  - *If service is good then tip is average*
  - The consequent of a rule can also have multiple parts
  - e.g. *If temperature is cold then hot water valve is open and cold water valve is shut*

Fuzzy Inference System

- Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic
- The mapping then provides a basis from which decisions can be made, or patterns discerned



Fuzzy inference system

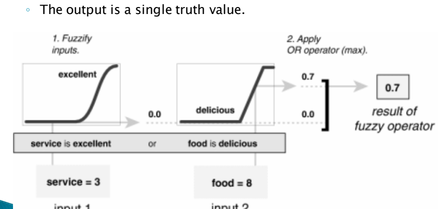
- Fuzzy inference process comprises of five parts:
  1. fuzzification of the input variables
  2. application of the fuzzy operator (AND or OR) in the antecedent
  3. implication from the antecedent to the consequent
  4. aggregation of the consequents across the rules
  5. defuzzification

- **EXAMPLE**
- **Step 1: Fuzzify Inputs**
  - The first step is to take the inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions
  - This example is built on three rules, and each of the rules depends on resolving the inputs into a number of different fuzzy linguistic sets: service is poor, service is good, food is rancid, food is delicious, and so on.
  - Before the rules can be evaluated, the inputs must be fuzzified according to each of these linguistic sets. For example, to what extent is the food really delicious?

The following figure shows how well the food at the hypothetical restaurant (rated on a scale of 0 to 10) qualifies, (via its membership function), as the linguistic variable delicious.
In this case, we rated the food as an 8, which, given your graphical definition of delicious, corresponds to μ = 0.7 for the delicious membership function.

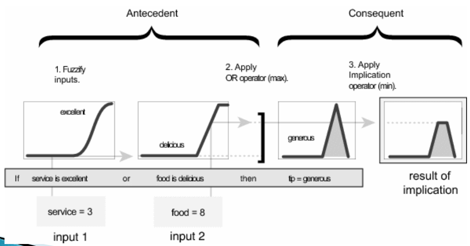
In this manner, each input is fuzzified over all the qualifying membership functions required by the rules.

- **Step 2: Apply Fuzzy Operator**
  - After the inputs are fuzzified, you know the degree to which each part of the antecedent is satisfied for each rule
  - If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule
  - This number is then applied to the output function
  - The input to the fuzzy operator is two or more membership values from fuzzified input variables
  - The output is a single truth value.

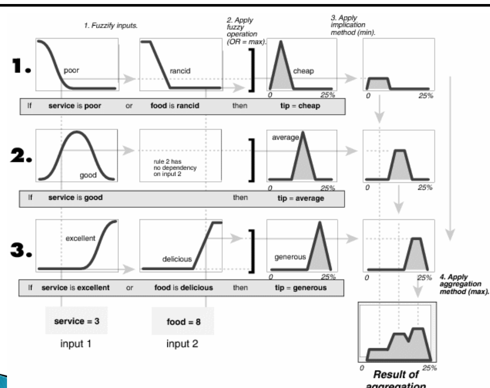


The fuzzy OR operator from rule 3. simply selects the maximum of the two values, 0.7, and the fuzzy operation for rule 3 is complete.

- **Step 3. Apply Implication Method**
  - The consequent is reshaped using a function associated with the antecedent (a single number)
  - The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set
  - Implication is implemented for each rule.

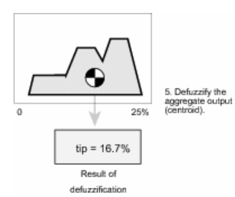


- **Step 4. Aggregate All Outputs**
  - Decisions are based on the testing of all of the rules in a FIS
  - The rules must be combined in some manner in order to make a decision
  - An aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set
  - An ggregation only occurs once for each output variable, just prior to the fifth and final step, defuzzification.
    - The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule.
    - The output of the aggregation process is one fuzzy set for each output variable.



- **Step 5. Defuzzify**
  - The input for the defuzzification process is a fuzzy set (the aggregate output fuzzy set) and the output is a single number.
  - As much as fuzziness helps the rule evaluation during the intermediate steps, the encompasses a range of output values, and so must be defuzzified in order to resolve a single output value from the set.
  - Perhaps the most popular defuzzification method is the centroid calculation, which returns the center of area under the curve

This example is solved with Mamdani method. Mamdani's fuzzy inference method is the most commonly seen fuzzy methodology. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed in 1975 by Ebrahim Mamdani as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators.



History

- In 1965, Lotfi A. Zadeh of the University of California at Berkeley published "Fuzzy Sets," which laid out the mathematics of fuzzy set theory and, by extension, fuzzy logic.
  - Zadeh had observed that conventional computer logic couldn't manipulate data that represented subjective or vague ideas, so he created fuzzy logic to allow computers to determine the distinctions among data with shades of gray, similar to the process of human reasoning.
- The first industrial application,
  - A cement kiln built in Denmark, coming on line in 1975
  - Sendai railway (the first subway system was built which worked with a fuzzy logic)- fuzzy systems were used to control accelerating, braking, and stopping when the line opened in 1987.
  - It was a big success and resulted in a fuzzy boom.

History

- Fuzzy boom
  - Universities as well as industries got interested in developing the new ideas
  - First, this was mainly the case in Japan.
    - Since the religion in Japan accept that things can contain parts of their opposites, it wasn't such a frightening idea as in most other parts of the world.
  - And fuzzy logic promised lots of money to the industries, which was of course welcome too
  - Today, almost every intelligent machine has fuzzy logic technology inside it.

History

- One of the most important applications of Fuzzy Logic was Yamaichi Fuzzy Fund.
- This is the premier financial application for trading systems.
  - It handles 65 industries and a majority of the stocks listed on Nikkei Dow and consists of approximately 800 fuzzy rules.
  - Rules are determined monthly by a group of experts and modified by senior business analysts as necessary.
  - The system was tested for two years, and its performance in terms of the return and growth exceeds the Nikkei Average by over 20%
  - While in testing, the system recommended "sell" 18 days before the black Monday in 1987 (October19) , when Dow Jones declined 23%
- The system went to commercial operations in 1988.

History

- The reason of this decision is, that forecasts on stock prices are complex "real world" problems, hardly manageable with mathematical models: fluctuations are influenced by, besides quantitative data, also by lots of "semantic" factors, or anyway factors that only modern adaptive "soft computing" approaches can cope with (relevant politics, law, psychological facts etc)
- A typical rule that could be inserted in a fuzzy logic system is:

"If the FED keeps high rates, and the official discount rate is low, than the short term rates in USA will rise a lot".

- The number of these rules is very high (thousands), and needs really powerful computers to evaluate the respective degree of truth, day after day, to give an advice reasonably fast.

History

- Following such demonstrations, the Japanese became infatuated with fuzzy systems, developing them for both industrial and consumer applications.
- In 1988 they established the Laboratory for International Fuzzy Engineering (LIFE), a cooperative arrangement between 48 companies to pursue fuzzy research.
- Japanese companies developed a wide range of products using fuzzy logic, ranging from washing machines and vacuum cleaner to autofocus cameras and industrial air conditioners.
- Some work was also performed on fuzzy logic systems in the US and Europe,
- However, little has been said about the technology in recent years, which implies that it has either become such an ordinary tool that it is no longer worth much comment, or it turned out to be an industrial fad that has now generally died out.

Implementations of fuzzy logic in technical systems

- An successful attempt to incorporate human thinking in technical systems
- In technical science
  - Western world also accepts fuzzy logic
  - Important journals and conferences:
    - 1978 – Fuzzy Sets and Systems (An International Journal in Information Science and Engineering)
      - Official Publication of the International Fuzzy Systems Association (IFSA
    - 1992 – IEEE (Institut for Electrical and Electronics Engineers) organized first conference "Fuzzy systems", and in 1993 Journal IEEE Transactions on Fuzzy Systems
- Areas where fuzzy sets are often used are: medicine, economics, psychology
- Fuzzy sets have been applied more in engineering than in science
- In natural science (physics, chemistry, biology, ecology) applications of fuzzy set theory are particularly rare
- A few areas in which fuzzy sets have already been applied to varios degrees are: reliability analysis of large-scale software systems, image and speech understanding, transportation, earthquake studies and robotics

Example – fuzzy washing machine

- The greatest sucess of fuzzy set theory has been in the engineering area of control systems
- One area in which fuzzy controllers quickly established an excellent reputation in the early 1990s is the broad area of consumer products.
- Among the earliest consumer products equipped with fuzzy controllers were *washing machines.*
- Next example describes the basic idea of the use of fuzzy controllers in washing machines.
  - In a conventional washing machine, the time of each run is set by the user.
  - If insufficient time is set for a given load of clothes, they are not properly washed
  - If the washing time is overly long, the time and energy are wasted and the machine as well as the clothes are unnecessarily worn out
- Different types of washing machines are commercially available and their control capabilities vary quite substantially.
- In next example is described the operation of a very simple fuzzy controller, whose purpose is to determine the proper operating time of the washing machine for each load of clothes.

Example – fuzzy washing machine

- Once the washing machine is loaded with dirty laundry, it begins to calculate how dirty the laundry is and how long it would take to wash it. If a machine takes ten minutes to wash clothes, it calculates how dirty the laundry is. If the load is 100% dirty, then it adds two minutes per piece of dirty laundry to the wash cycle that would have taken ten minutes originally. If the laundry is 50% dirty, then it would add 50% of two minutes. This means a minute to the ten-minute wash cycle. If the laundry is greasy then an additional two minutes are added to the cycle. If the laundry is dirty and greasy, then the machine factors in 4 additional minutes to the entire load.
- A fuzzy machine would also have to take into account the amount of soap it would require for dirty clothes, greasy clothes and dirty and greasy clothes. This requires complex calculations which the machine is programmed to do. The machine would also require drawing sufficient water to accommodate all the calculations into a wash cycle.
- The fuzzy washing machines have now begun to approximate subtle human functions.
- Fuzzy logic has made functions of washing machines more economical, energy-saving and less time-consuming.

Example – fuzzy washing machine

- The operating time of a washing machine depends on two properties of each given load of clothes
  - It depends of how dirty the clothes are
  - It depends on the type of soil
- The degree of dirtiness is measured by a special sensor via the degree of water transparency
  - The less transparent the water, the dirtier the clothes
- The type of soil is determined by measuring the time needed, after the machine has started, to reach a state in which the water transparency remains constant
  - This time is called saturation time and is different for different types of soil

Fuzzy Set Theory (Foundations and Applications) – G.J.Klir, U.H.St. Clair, B. Yuan, Prentice Hall 1997

Example – fuzzy washing machine

- Assume that
  - the *degree of dirtiness d* is expressed by number in interval [0, d_{max}]
  - And that we deal only with three level of dirtiness (expressed in natural language): *high, medium and low*

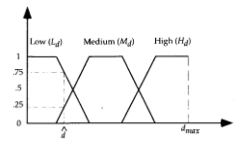


Figure 10.1 Fuzzy numbers representing three levels of dirtiness: *low* (L_d), *medium* (M_d), *high* (H_d). These are states of linguistic variable $\mathcal{D}$.

- Assume that
  - the *saturation time s* is expressed by number in interval [0, s_{max}]
  - And that we have only three saturation time (expressed in natural language): *short, medium and long*

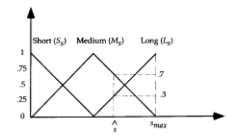


Figure 10.2 Fuzzy numbers representing *short* (S_d), *medium* (M_s), *long* (L_d) saturation time. These are states of linguistic variable $\mathcal{S}$.

The required washing time should be some mathematical function of a degree of diritiness and a saturation time.
It is impossible to determine this function exactly.

Example – fuzzy washing machine

- By using a fuzzy controller we can approximate this function with relative ease on the basis of human intuition and experience.
- We need to determine another linguistic variable representing the reguired washing time t∈[0, t_{max}]

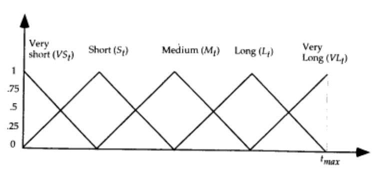


Figure 10.3 Fuzzy numbers characterizing the required washing time. These are states of linguistic variable $\mathcal{T}$.

Example – fuzzy washing machine

- Now, we need to express knowledge of experienced users by conditional fuzzy propositions of the form
- If *D*=____ and *S*=___ then *T*=___
- Where appropriate states of the three linguistic variables are placed into the lines
- The result is nine distinct conditional fuzzy propositions e.g.
  - If $D=L_d$ and $S=S_s$ then $T=VS_t$
  - If $D=M_d$ and $S=M_s$ then $T=M_t$
  - If $D=H_d$ and $S=L_s$ then $T=VL_t$

	t	S_s	M_s	L_s
$\mathcal{D}$	L_d	VS_t	S_t	M_t
	M_d	S_t	M_t	L_t
	H_d	M_t	L_t	VL_t

Figure 10.4 Inference rules for fuzzy washing machine.

A fuzzy controler is connected to a washing machine

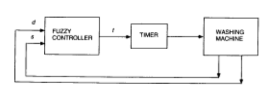


Figure 10.5 Fuzzy control of washing time.

- $\hat{d}$ and $\hat{s}$ are measured values e.g.
  - 0.75, 0.25 and 0 for $\hat{d}$
  - 0, 0.7 and 0.3 for $\hat{s}$
  - Only rules for which the compatibilities of the measured values with both antecedents are positive, take place in determining the value of the controlled variable (rules that fire)

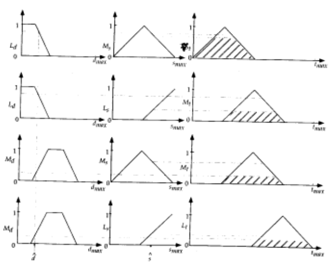


Figure 10.6 Inference rules that fire for $d=\hat{d}$ and $s=\hat{s}$.

- We obtain overall conclusion by taking the union of all the individual conclusions (the fuzzy set $C_{\hat{d},\hat{s}}(t)$), whose membership function is defined for each x∈[0, t_{max}]

$$C_{\hat{d},\hat{s}}(t) = \max\{\min[L_d(\hat{d}), M_s(\hat{s}), VS_t(t)], \min[L_d(\hat{d}), L_s(\hat{s}), S_t(t)], \min[M_d(\hat{d}), M_s(\hat{s}), M_t(t)], \min[M_d(\hat{d}), L_s(\hat{s}), L_t(t)]\}$$

The graph of this function is shown in Fig. 10.7.

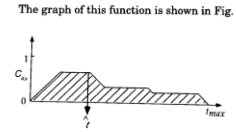


Figure 10.7 The fuzzy set which represents the overall conclusion for the measured values $\hat{d}$ and $\hat{s}$ and its defuzzified value $\hat{t}$.

Center of gravity deffuzification method

$$a = \frac{\int_{a_1}^{a_2} xA(x)dx}{\int_{a_1}^{a_2} A(x)dx} := \frac{\int_0^{t_{max}} xC_{\hat{d},\hat{s}}(t)dt}{\int_0^{t_{max}} C_{\hat{d},\hat{s}}(t)dt}$$

This value is the desirable operating time of the washing machine as determined by the fuzzy controller for conditions d^ and s^
The washing timer is set to this value