Mobile robot pose tracking by correlation of laser range finder scans in Hough domain

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***Abstract* — This paper proposes a correlation scan matching method based on matching consecutive readings of laser range finder. The method uses the Hough transform and the Hough spectrum for determining the relative rotation. By calculating also translation in the Hough domain, the method can also be successfully used in non-perpendicular environments. This paper also looks at validity of utilizing the probabilistic Hough transform with regard to the complexity and the accuracy of the algorithm. Proposed scan matching methods are used for mobile robot pose tracking. Developed algorithms are validated by simulation and experimental tests.**

I. Introduction

A mobile robot, exploring a new unknown space, uses its sensors to collect information about the environment in order to solve the pose tracking problem. Usually, encoder sensor measurements are used to estimate mobile robot's pose relative to its start pose. However, odometry based pose tracking can be used only for short distances due to the estimation error which grows with the travelled distance. In order to improve the results exteroceptive sensors are used (laser range finder, cameras, sonars, …) for periodical corrections of odometry data or for solving pose tracking problem independently.

In the later case, when only laser range finder is used, the pose tracking problem can be solved by scan matching methods. By matching two successive scans, e.g. the current scan against the referent scan, the relative difference (relative translation and rotation) of current pose with respect to referent pose can be determined. Scan matching transforms the pose tracking problem into a problem of alignment of two sets of data which represent the environment around the mobile robot visible at two successive poses.

The method presented in this paper is an extension and adaptation of the correlation based approach to scan matching [1, 2, 3, 4]. In correlation methods, features of environment are represented by histograms, which contain information about the relative pose change. The relative translation and rotation are determined by calculating the cross-correlation functions of corresponding histograms and finding their maximum value.

In this paper we introduced two improvements over standard correlation method. The first improvement is with respect to sensor noise. Namely, the most delicate step in the correlation method is determining the maximum value in the angle-histogram (main axis direction). Sensor noise can distort the angle-histogram which is then reflected in inaccurate information obtained by the maximum value. In order to alleviate this problem we introduced the Hough transform (HT) and the Hough spectrum (HS) concept for determining the main axis direction and relative rotation. The second improvement is with regard to the requirements of a correlation method to the environment shape. The correlation method works only in the perpendicular environment, but by determining the relative translation in Hough domain we achieved independence from perpendicularity. The calculation of HT significantly affects the complexity of our method and therefore we investigated also the validity of utilizing the probabilistic Hough transform (PHT).

The rest of the paper is organized as follows. In section II the correlation method is described. HT and its properties are briefly described in section III. In sections IV and V we present improvements and extensions by exploiting HT. Simulation and experimental results are given in section VI. Finally, conclusions are drawn in section VII.

II. Correlation Based Approach to Scan Matching

A. Problem definition

Let S_{ref} denote the referent scan taken at pose P_{ref}, and S_{curr} denote the current scan taken at pose P_{curr} (see Fig. 1.a), the assignment is to determine the relative difference between pose P_{curr} and pose P_{ref}. It is necessary to determine the transformation TM, which consists of rotation ω and translation T, to overlap common parts of current and referent scans (see Fig. 1.b).

Let P_1 denote a point on S_{ref} and P_2 denote a corresponding point on S_{curr} then the transformation TM is described by

$$P_1 = R_\omega P_2 + T \tag{1}$$

where R_ω is the rotation matrix

$$R_\omega = \begin{bmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{bmatrix} \tag{2}$$

and T is translation

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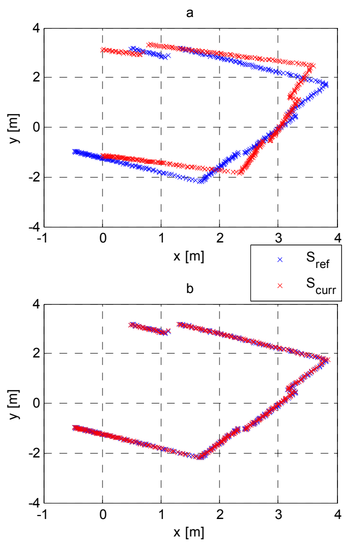

Figure 1. Matching two laser scans.

$$T = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \quad (3)$$

where T_x and T_y are translation in x and y axes respectively. If (1) is applied to every common point in scans, after applying the transformation TM, S_{curr} overlaps with S_{ref} as shown in Fig. 1:

$$S_{ref} = TM(\omega, T_x, T_y)\left[S_{curr}\right] \quad (4)$$

B. *Correlation based scan matching*

The correlation method consists of two main steps: the histograms creation and correlation. Since the mobile robot's pose change consists of rotation – a change in orientation, and translation – a change in x and y position, it is necessary to create three histograms for both the reference and the current laser scans. Because the features of the environment are represented by histograms, it is assumed that the environment contains enough line segments.

To obtain independency for determining the translation, an angle-histogram is created first. If we connect the adjacent points of the scan, the obtained lines approximate the surface of structures, while the relative angle between the lines and the x-axis approximates the orientation of the surface (Fig. 2). The relative angle is calculated as follows

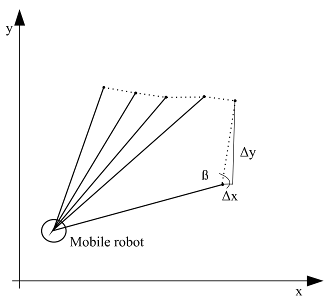

Figure 1. Calculating angles for angle-histogram.

$$\beta = \arctan\left(\frac{\Delta y}{\Delta x}\right) \quad (5)$$

The angle-histogram of the current scan can be seen as a shifted angle-histogram of the reference scan for some value of an angle. This shift is independent of translation and it is equal to the relative rotation ω.

Scans are now rotated for its main axis direction (angle with the highest value in the angle-histogram) so as to achieve the alignment of scans with the x-axis. Only if the environment is perpendicular the independence will be obtained for the position coordinates. Now, similar shifts can be viewed in x- and y-histograms, and the relative translation T can be determined.

When the appropriate histograms of referent and current scans are created it is necessary to find the shift between them. Because of the sensing noise, calculating the shift by comparing the peaks in histograms is not a good solution. Since the histograms present discrete functions, a discrete cross-correlation function can be calculated to determine the shift

$$C(j) = \sum_{i=1}^{n} h_{curr}(i) h_{ref}((i+j)\,\%\,n) \quad (6)$$

where C presents the cross-correlation function for a shift amount of j, h_{ref} and h_{curr} present the two histograms that are compared, and n is the number of histogram classes. Index j, which denotes the maximal value of the cross-correlation function, is reciprocal to the shift of the histograms.

III. Hough transform of the scans

A. *Discrete Hough Transform of the scans*

Hough Transform is defined as a transformation from the input space to the parameter space. In our case the input space is a finite number of points $S = \{u_i\}$ on the scan. The parameter space was chosen to represent the lines in $\Re^2$, using the polar representation of lines

$$\rho = x\cos\theta + y\sin\theta \quad (7)$$

where ρ is the distance of the normal from the line to the origin, and θ is the angle between x-axis and the normal. The polar representation of lines is used for computational reasons and to avoid adverse situations when the line is parallel to the x-axis.

Now we can observe how the relative rotation and translation between the referent and the current scans are transformed into the Hough domain. S_{curr} and S_{ref} are interconnected by transformation $TM(\omega,T)$. If $HT_{ref}(\rho,\theta)$ denotes the Hough transform of S_{ref} and $HT_{curr}(\rho,\theta)$ the Hough transforms of S_{curr}, we have

$$HT_{ref}(\theta,\rho) = HT_{curr}(\theta+\omega, \rho + (\cos\theta \quad \sin\theta)T) \quad (8)$$

In the implementation of the algorithm, we use the discrete Hough transform (DHT), which is easy to calculate. The loss of information which occurs due to the use of DHT is small, but depends on the sampling $\Delta\theta$ in interval $\left[0, 2\pi\right\rangle$ and $\Delta\rho$ in interval $\left[0, \rho_{\max}\right]$, where ρ_{max} is the maximal value in the laser sensor readings. Thus, the parameter space (ρ, θ) is discretized with a finite number of rows n_ρ and columns n_θ. The DHT procedure begins with the declaration of accumulator matrix H and initializing its values to zeros. For each point on scans the corresponding accumulators are incremented; respectively each point "votes" for the parameter pair (ρ_j, θ_i) of every line which passes through it

$$p_{ik} = x_k \cos\theta_i + y_k \sin\theta_i \tag{9}$$

where j is determined with $\rho_j - \frac{\Delta\rho}{2} \le \rho_{ik} < \rho_j + \frac{\Delta\rho}{2}$. The maximum in the accumulator array is presented with a certain pair (ρ_i, θ_i). This pair represents the strongest straight line in scan while local maxima represent lines that are less expressed.

Once DHT has been computed, the most prominent lines in laser scan can be used to determine orientation of the surfaces in the mobile robot environment.

B. *The probabilistic discrete Hough transform*

In [6] the authors have researched how the reduction of points, which are used as input for the HT, affects the probability of detecting straight lines in digital images. In this paper we will observe how the reduction of points, which are used as input, affects the complexity and accuracy of determining relative rotation ω.

The complexity of calculating DHT of scan with M points is $O(M \cdot n_\theta)$, and the complexity of computation HS is $O(n_\theta \cdot n_\rho)$. By reducing the dimensions n_ρ and n_θ of the accumulator array we directly impact the loss of information and sensitivity of the method. Thus, only the M remains as the dominant variable.

Starting from the assumption that the majority of lines can be detected, even if all points of the scan are not used, we will use a random subset of points m ($m < M$) as input for DHT and thus reduce the complexity of the algorithm and execution time. Now, the complexity of computation DHT is $O(m \cdot n_\theta)$. Significant savings in the execution time will be made when m is much smaller than M. The resulting probabilistic DHT (PDHT) is slightly impaired. There is, naturally, tradeoff between the size of the subset of points and the accuracy of the algorithm, and the best ratio depends on the specific application.

C. *The Hough spectrum*

In [5] the concept of Hough spectrum (HS) is introduced. HS is defined as a vector of elements, obtained by summing the squared values in columns of accumulator array H. More precisely, if H denotes accumulator array with n_ρ rows and n_θ columns, then HS is computed as follows

$$HS(k) = \sum_{i=1}^{n_\rho} H(i,k)^2, \quad 1 \le k \le n_\theta \tag{10}$$

HS is invariant to translation and circularly shifted on rotation.

IV. CORRELATION OF HOUGH SPECTRUM

In the general correlation method [1] the relative angle β is computed directly from the data. The resulting angle-histogram is not reliable when a significant sensing noise is present. As a result, after rotating scan for the main axis directions, the points are not aligned with the x-axis and the desired independences for the creation of x- and y-histograms are not obtained.

Fig. 3 shows an example of scan with significant sensing noise and the corresponding angle-histogram. As we can see, the sensing noise can distort the angle-histogram which is than reflected in inaccurate information obtained by the maximum value. The consequence is that wrong value of relative rotation is determined.

Since HS contains information on the orientation of the surfaces in the mobile robot environment, it can be used as "new" angle-histogram. The maximum value of HS also provides information on orientation of the majority of surface in the environment. The shift between HS of the referent and the current scans is equal to the relative rotation ω.

Fig. 4 shows HS of a scan with significant sensing noise from Fig. 3 (HS has been reduced in $\left[0, \pi\right\rangle$ range). When comparing the predetermined angle-histogram with HS, a difference is immediately noticed. Despite the present sensing noise, HS still contains clear information about orientation of the surfaces in the environment.

Therefore, by calculating HS we gain a method for determining the relative rotation that is robust to sensing noise. For future reference we name this method as the Hough Spectrum Correlation (HSC) method or the Probabilistic Hough Spectrum Correlation (PHSC) method depending on used HT (DHT or PDHT).

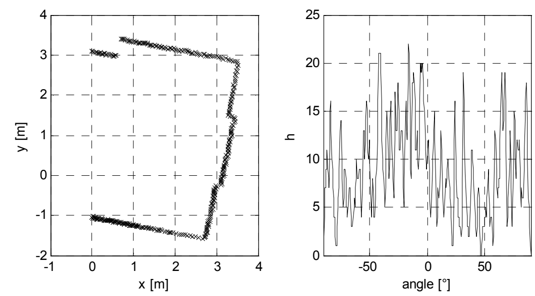

Figure 3. A scan with significant sensing noise (left) and the corresponding angle-histogram (right).

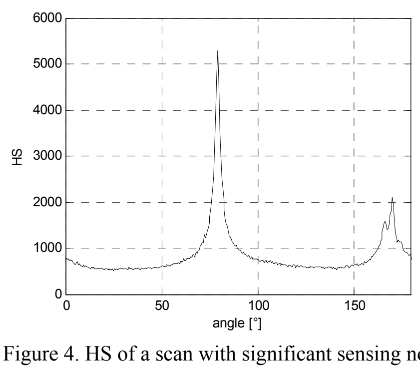

Figure 4. HS of a scan with significant sensing noise.

V. SCAN MATCHING IN NON-PERPENDICULAR ENVIRONMENTS

Another significant disadvantage of the correlation method is the assumption that the environment is perpendicular (as it was in Figs. 1 and 3). In order to apply the method efficiently in environments which do not contain perpendicular surfaces (but still contain line features), we must change the procedure for calculating the relative translation T. In this paper we propose calculation of the relative translation T in the Hough domain.

HS of scan, taken in an environment which does not contain perpendicular surfaces, has several local maxima which are not 90 degrees apart from each other. After calculating HS and determining the relative rotation ω, we now have the value for which we need to circularly translate the columns of $\mathrm{HT}_{\mathrm{curr}}$ to overlap with the corresponding columns of $\mathrm{HT}_{\mathrm{ref}}$. After applying translation we have

$$\mathrm{HT}_{\mathrm{ref}}(\hat{\theta}, \rho) = \mathrm{HT}_{\mathrm{curr}}(\hat{\theta}, \rho + (\cos\hat{\theta} \quad \sin\hat{\theta})T) \qquad (11)$$

where $\hat{\theta}$ is an arbitrary value.

By applying the cross-correlation function on corresponding columns of $\mathrm{HT}_{\mathrm{curr}}$ and $\mathrm{HT}_{\mathrm{ref}}$, we can determine the shift $d(\hat{\theta})$, which is actually the projection of T in $\hat{\theta}$ direction.

$$(\cos\hat{\theta} \quad \sin\hat{\theta})T = d(\hat{\theta}) \qquad (12)$$

By correlation of numerous columns, the projection of translation T in different directions will be determined. In order to ascertain the value of the resulting translation T (in the direction of the x-axis and y-axis), the following linear system is built (see Fig. 5):

$$\begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ \cos\theta_2 & \sin\theta_2 \\ \vdots & \vdots \\ \cos\theta_n & \sin\theta_n \end{bmatrix} T = \begin{bmatrix} d(\theta_1) \\ d(\theta_2) \\ \vdots \\ d(\theta_n) \end{bmatrix} \qquad (13)$$

wherefrom T is obtained by using the least-square method.

The selection of directions $\hat{\theta}$ is arbitrary. All columns of DHT can be used, but this would significantly increase the complexity of the algorithm and execution time. Such an approach is unsuitable for efficient implementation. We propose selecting only those columns in which the local maxima of HS are found. Due to the above mentioned selection, additional savings in execution time are achieved and the cross-correlation function will have a clearly defined peaks. There are situations, however, in which the previously defined linear system is ill-posed, e.g. when a mobile robot explores a long corridor and only one direction is selected. We will assume, however, that the environment has more than one linear feature, which is in consent with the fact that the mobile robot is exploring an indoor space.

For a future reference we name this method as the Hough Domain Correlation (HDC) method. We do not look at validity of utilizing PHT for this method, because of a significant error that occurs when the translation is determined in such a way.

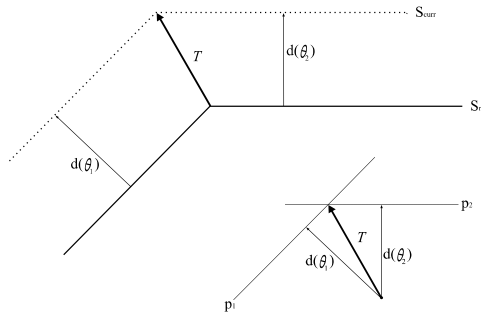

Figure 5. Graphic representation of the linear system (13) for n=2.

VI. RESULTS

The simulation and experimental verification of the developed algorithms were performed using the Player/Stage project. Simulation experiments were conducted on a computer equipped with 1.67 GHz processor and 512 MB of RAM. The experimental results were obtained with a Pioneer 3-DX mobile platform equipped with the LMS200 SICK laser range finder.

Simulation experiments are also performed to test the basic functionality of methods in situations when sensing noise is present. The methods were tested in a perpendicular and non-perpendicular environment. We also observed the effect of using PHT with regard to the execution time and the accuracy of the algorithm. At the end, we present the experimental results of mobile robots pose tracking based on scan matching.

A. *Scan matching – simulated data*

For each case we have run a total of 100 experiments. The shifted laser scans were obtained by applying randomly generated value for rotation and translation, where ω has been uniformly distributed in [-15, 15] degrees and T has been uniformly distributed in [-30, 30] cm. The noise is modeled as uniform noise with maximum values of ±2.5 cm or ±2.5 cm. The statistical results are listed in appropriate Tables I and II.

In cases when the sensing noise is not present both HSC and HDC method give good results. In cases when there is sensing noise in the laser scans, the HSC method proves to be more robust as it searches only the maximum value of HS, while HDC method searches all local maxima, which makes it hard to determine the right value of the threshold.

TABLE I.
STATISTICAL ANALYSIS, PERPENDICULAR ENVIRONMENT, SENSING NOISE ±2.5 CM.

	HSC			HDC		
	$\Delta\omega$ [°]	ΔT_x [m]	ΔT_y [m]	$\Delta\omega$ [°]	ΔT_x [m]	ΔT_y [m]
RMS	0.1677	0.0036	0.0053	0.1825	0.0039	0.0057
Mean	-0.0016	$6.5\cdot10^{-5}$	$1.0\cdot10^{-4}$	-0.01825	$1.6\cdot10^{-4}$	$-1.9\cdot10^{-4}$
Std.	0.1677	0.0036	0.0053	0.1816	0.0039	0.0057
Min	-0.3410	-0.0098	-0.0142	-0.3920	-0.0083	-0.0137
Max	0.3740	0.0089	0.0125	0.4110	0.0104	0.0133

TABLE II.
STATISTICAL ANALYSIS, PERPENDICULAR ENVIRONMENT, SENSING NOISE ± 5 CM.

	HSC			HDC		
	$\Delta\omega$ [°]	ΔT_x [m]	ΔT_y [m]	$\Delta\omega$ [°]	ΔT_x [m]	ΔT_y [m]
RMS	0.2197	0.0060	0.0073	0.2344	0.0239	0.0280
Mean	-0.0266	$-6.2\cdot10^{-4}$	$9.8\cdot10^{-4}$	-0.0205	-0.0020	$8.2\cdot10^{-4}$
Std.	0.2181	0.0059	0.0072	0.2335	0.0238	0.0280
Min	-0.4790	-0.0152	-0.0202	-0.6230	-0.0636	-0.0532
Max	0.5730	0.0156	0.0155	0.4600	0.1123	0.2071

Despite the fact that the HSC method assumes the perpendicular environment, the obtained results in the non-perpendicular environment shown in Fig. 6 are of the same quality as with the HDC method as can be seen in Table III. By assuming that the scans were taken at close poses, we can define bounds in which the relative shifts are searched, which then lead to additional robustness. However, the HSC method cannot be successfully applied in all non-perpendicular environments.

The experiments in perpendicular environment were repeated, but now results obtained using the HT and the PHT were compared. As shown in Figs. 7 and 8, the execution time and the standard deviation of rotation error were calculated for different values of random subset m. We can conclude from these results that in cases when sensing noise is present, error significantly increases when the percentage of points decreases. But, in cases when sensing noise is not present, the quality of results slightly decreases while the execution time declines from 150 ms to 60 ms.

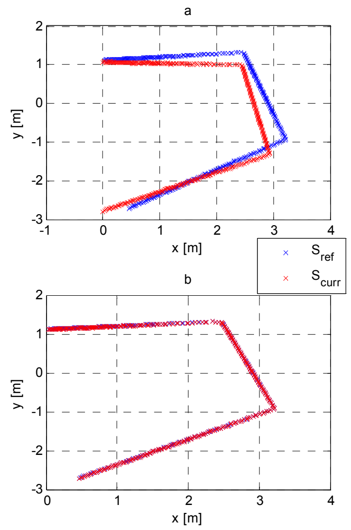

Figure 6. Matching two laser scans in non-perpendicular environment.

TABLE III.
STATISTICAL ANALYSIS, NON-PERPENDICULAR ENVIRONMENT, NO SENSING NOISE.

	HSC			HDC		
	$\Delta\omega$ [°]	ΔT_x [m]	ΔT_y [m]	$\Delta\omega$ [°]	ΔT_x [m]	ΔT_y [m]
RMS	0.1481	0.0031	0.0036	0.1371	0.0026	0.0029
Mean	-0.0404	$4.5\cdot10^{-6}$	$-2.5\cdot10^{-4}$	-0.0089	$-4.0\cdot10^{-4}$	$-5.5\cdot10^{-4}$
Std.	0.1425	0.0030	0.0036	0.1368	0.0026	0.0029
Min	-0.3230	-0.0072	-0.0080	-0.3110	-0.0064	-0.0049
Max	0.2590	0.0073	0.0100	0.2340	0.0060	0.0093

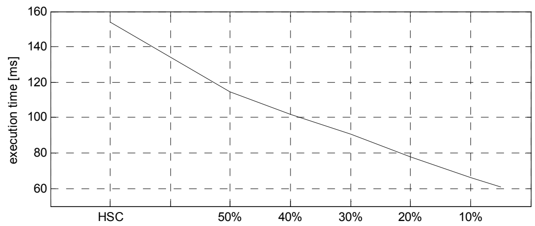

Figure 7. Execution time of HSC and PHSC methods.

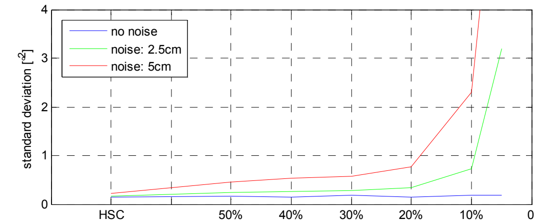

Figure 8. Rotation errors with HSC and PHSC methods.

B. *Pose tracking – simulated data*

In the following experiments, we tested pose tracking approaches based on the HSC (PHSC) and HDC methods. When the scan matching was computed for longer time period for keeping track on robots pose, accumulated error became significant with the increase of the traveled distance. Thus, we propose the execution of scan matching algorithm only when the mobile robot has moved enough from previous position. The reduction filter was used to reduce the negative effects to the "voting" process in the HT of increased density of laser scan points when mobile robot was close to a wall.

Fig. 9 and Table IV shows that in perpendicular environment PHSC method gives comparable results to the HSC method even when small percentage of points is used. Fig. 10 and Table V shows that the HDC method yields better results than HSC method in non-perpendicular environments.

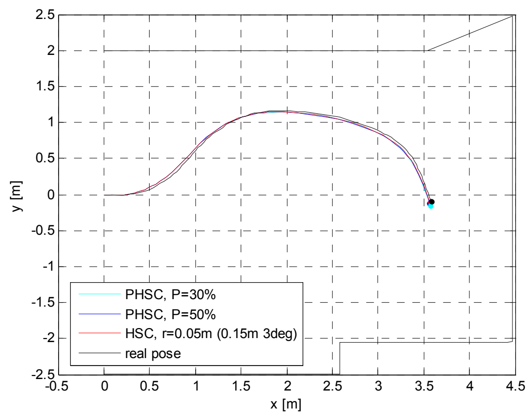

Figure 9. Pose tracking in perpendicular environment.

TABLE IV.
POSE TRACKING IN PERPENDICULAR ENVIRONMENT.

	HSC	PHSC, 30%	PHSC, 50%	Real position
x [m]	3.5690	3.578	3.5650	3.581
y [m]	-0.1130	-0.1600	-0.1300	-0.090
Θ [rad]	-1.2650	-1.300	-1.274	-1.288

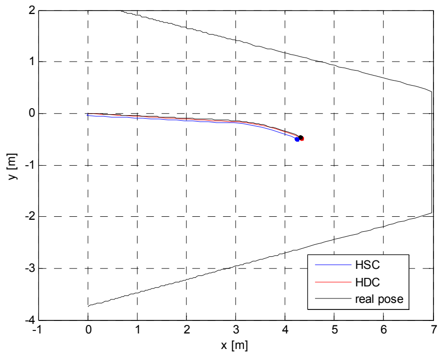

Figure 10. Pose tracking in nonperpendicular environment.

TABLE V.
POSE TRACKING IN NON-PERPENDICULAR ENVIRONMENT.

	HSC	HDC	Real position
x [m]	4.2390	4.328	4.321
y [m]	-0.5010	-0.485	-0.4630
Θ [rad]	-0.4800	-0.4800	-0.487

C. *Pose tracking – experimental results*

Figs. 11 and 12 and Table VI and VII shows the experimental results obtained with the Pioneer 3-DX mobile platform equipped with the LMS200 SICK laser range finder in perpendicular and non-perpendicular environments, respectively. The HSC and HDC method presented in this paper were compared with the MbICP method presented in [7]. It can be seen that both proposed methods outperform MbICP method in perpendicular and in non-perpendicular environments.

VII. CONCLUSION

In this paper the correlation scan matching methods based on matching consecutive readings of laser range finder were presented. In order to improve the general scan matching method, we use the Hough Transform and Hough Spectrum so as to determine the relative rotation and translation. The developed scan matching methods can be successfully applied also in non-perpendicular environments and with the present sensing noise. Also, significant savings in execution time can be made by using the Probabilistic Hough Transform. Developed algorithms were tested by simulations and experiments.

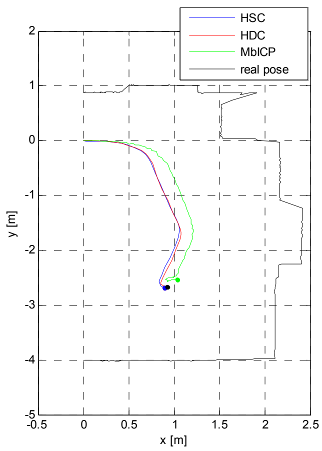

Figure 11. Pose tracking in perpendicular environment (experiment).

TABLE VI.
POSE TRACKING IN PERPENDICULAR ENVIRONMENT (EXPERIMENT).

	HSC	HDC	MbICP	Real position
x [m]	0.891	0.894	1.040	0.9197
y [m]	-2.678	-2.686	-2.536	-2.668
Θ [rad]	-1.667	-1.676	-1.674	-1.7209

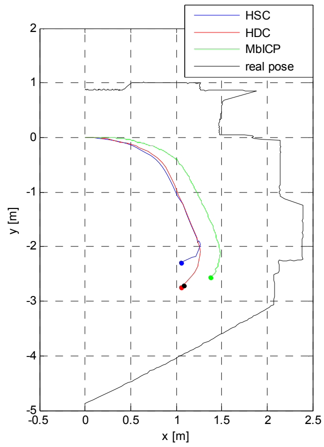

Figure 12. Pose tracking in non-perpendicular environment (experiment).

TABLE VII.
POSE TRACKING IN NON-PERPENDICULAR ENVIRONMENT (EXPERIMENT).

	HSC	HDC	MbICP	Real position
x [m]	1.051	1.059	1.375	1.090
y [m]	-2.301	-2.754	-2.571	-2.710
Θ [rad]	-2.094	-2.112	-2.039	-2.076

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