Poles of the Zagreb analysis partial-wave T matrices

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The Zagreb analysis partial-wave T matrices included in the *Review of Particle Physics* [by the Particle Data Group (PDG)] contain Breit-Wigner parameters only. As the advantages of pole over Breit-Wigner parameters in quantifying scattering matrix resonant states are becoming indisputable, we supplement the original solution with the pole parameters. Because of an already reported numeric error in the S_{11} analytic continuation [Batinić *et al.*, Phys. Rev. C **57**, 1004(E) (1997); arXiv:nucl-th/9703023], we declare the old BATINIC 95 solution, presently included by the PDG, invalid. Instead, we offer two new solutions: (A) corrected BATINIC 95 and (B) a new solution with an improved S_{11} πN elastic input. We endorse solution (B).

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On the basis of the Carnegie-Mellon-Berkeley (CMB) coupled-channel model of Ref. [1], the Zagreb group in 1995 produced a set of isospin-1/2 partial-wave amplitudes S_{11}–G_{17} [2]. We used a three-channel model with πN, ηN, and a third, effective, $\pi^2 N$ channel, and fitted the model parameters to (i) the Karslruhe-Helsinki partial waves for πN elastic scattering [3] (824 data points), and to (ii) the complete available experimental database for the $\pi N \to \eta N$ process (839 data points). Since 1996, this analysis has been included in the *Review of Particle Physics*, a world compilation of nucleon resonances by the Particle Data Group (PDG) as the BATINIC 95 solution.

It is important to mention that at the time when these results were published, only Breit-Wigner parameters were accepted as a reliable quantification of resonance properties, and pole parameters served only as auxiliary quantities. In accordance with the general trend, and in spite of the fact that pole parameters have also been collected by the PDG, in [2] we unfortunately chose *not* to publish both sets.

However, the important fact, already formulated by Höhler in 2000 [4], that Breit-Wigner parameters are inherently model dependent and that the quantification of resonance properties should be made at the level of T-matrix pole parameters, has only recently been fully recognized [5]. It became imperative for partial-wave analyses to provide pole positions. We now realize that our decision not to publish the available pole parameters for the BATINIC 95 solution was a serious omission; the purpose of this Brief Report is to correct this.

There are several reasons why our 14-year-old results are of current interest. The two most prominent are as follows:

(i) No really new experimental data for meson production have appeared since 1995 (there are some recent measurements of the $\pi N \to \eta N$ process at low energies [6], but the results overlap with old data), so repeating the analysis is unnecessary;

(ii) the BATINIC 95 partial waves are based on an active set of codes which are currently used for an entirely different purpose (to analyze the analytic structure of the world collection of partial-wave amplitudes), so in case new data appear (possibly from MIPP at Fermilab [7] or J-PARC in Tokyo [8]) one may use the formalism to update the solution.

The main intent of this Brief Report is to present the pole parameters for our solutions, and so enable the physics community to compare our results with the poles of the world collection of other active partial-wave analyses (VPI/GWU [9], EBAC [10], Jülich [11], Bonn [12], Mainz [13], and Giessen [14]).

We would also like to use this opportunity to update the BATINIC 95 solution. Soon after publishing the original paper, we discovered a minor numerical error in the S_{11} analytic continuation formalism and reported it in an Erratum [15]. However, as we were allowed to publish only graphs of the corrected partial-wave amplitudes without the resonance parameters, the erroneous values of the Breit-Wigner parameters collected by the PDG could not be replaced, because the PDG publishing policy requires that all included results must be published in a reviewed journal, and our corrected numerical values have been given only in the arXiv preprint [16]. Publication of corrected Breit-Wigner parameter values in this Brief Report is the chance to remedy this situation, and to declare the BATINIC 95 solution, presently included in PDG [17], invalid.

In its place, we offer two new solutions: (A) a corrected BATINIC 95; and (B) a new solution with an improved normalization of the low-energy S_{11} πN elastic input. In spite of finding them statistically almost equivalent, we endorse solution (B) because of its better embedding into low-energy πN physics.

For the convenience of the reader, we repeat the essence of our model.

The Zagreb partial-wave analysis [18] is based on the CMB coupled-channel approach, which is fully analytic, and manifestly unitary [1]. In the CMB analysis, a central role is played by the unitary, normalized, partial-wave T matrix $\mathbf{T}(s)$, a matrix in the channel indices:

$$\mathbf{T}(s) = \sqrt{\mathrm{Im}\,\mathbf{\Phi}(s)} \cdot \boldsymbol{\gamma}^{\mathrm{T}} \cdot \mathbf{G}(s) \cdot \boldsymbol{\gamma} \cdot \sqrt{\mathrm{Im}\,\mathbf{\Phi}(s)}. \qquad (1)$$

Two main ingredients of the model are the channel propagator $\mathbf{\Phi}(s)$, the diagonal matrix in channel indices that takes care of

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channel-related singularities, and the bare resonant propagator $\mathbf{G}_0(s)$, the diagonal matrix in resonant indices incorporating real first-order poles related to resonances (and background). Resonant contributions are generated by "dressing" bare propagator poles from the N^* energy domain with self-energy terms. The nonresonant background contribution is quite reliably [1] simulated by (usually two) subthreshold poles, but another background pole may also be placed above the energy region of interest. The dressed resonance propagator $\mathbf{G}(s)$ is given by solving the (Schwinger-Dyson) equation $\boldsymbol{G}^{-1}(s) = \boldsymbol{G}_0^{-1}(s) - \boldsymbol{\Sigma}(s)$, where the self-energy term $\boldsymbol{\Sigma}(s)$ is built from the channel propagator as $\boldsymbol{\gamma} \cdot \boldsymbol{\Phi}(s) \cdot \boldsymbol{\gamma}^{\mathrm{T}}$. The parameter matrix $\boldsymbol{\gamma}$ is, generally, a nonsquare matrix in resonance $\otimes$ channel indices and is obtained from a least-squares fit to experimental data or generated partial waves. In addition to $\boldsymbol{\gamma}$ parameter matrices, the values of the bare propagator real poles are simultaneously acquired from the same fit.

The channel propagator matrix $\boldsymbol{\Phi}(s)$ is assembled from channel propagator functions $\phi(s)$. The dominant singularity in the resonant region, apart from those of the resonances themselves, is the physical (channel opening) branching point s_0. In the CMB approach, contributions from other singularities (such as the left-hand cut, the nucleon pole, etc.) are given partly by the design of the imaginary part of the channel propagator, while the rest are taken care of by the background.

The imaginary part of the channel propagator is given by

$$\mathrm{Im}\,\phi(x) = \frac{[q(x)]^{2L+1}}{\sqrt{x}\left\{Q_1 + \sqrt{Q_2^2 + [q(x)]^2}\right\}^{2L}}, \tag{2}$$

where $q(x)$ is the standard two-body center-of-mass momentum for a particular meson-baryon channel, and Q_1 and Q_2 are the CMB model parameters with values equal to the π meson or, in our case [18], the channel meson mass. Here L is the orbital angular momentum number of the given partial wave.

Analyticity of the channel propagator function $\phi(s)$ is ensured by the once-subtracted dispersion relation

$$\mathrm{Re}\,\phi(s) = \frac{s - s_0}{\pi}\ \mathrm{P}\int_{s_0}^{\infty} \frac{\mathrm{Im}\,\phi(x')\,dx'}{(x'-s)(x'-s_0)}, \tag{3}$$

where P stands for the Cauchy principal value. The physical (unitarity) branch cut is, thus, chosen to go from the branching point s_0 to positive infinity. The variable x' is used in the integral rather than s' to indicate that the integration path is on the real axis.

The model, in principle, contains N_C channels, N_P bare propagator poles s_i (N_R resonant and two background poles), and all channel-to-resonance mixing matrix real parameters γ. Once the number of channels, N_C, and the number of bare propagator poles, N_P are chosen, the model contains a total of $N_P + (N_P \times N_C)$ free parameters per partial wave. All parameters of the model, the nonsquare parameter matrix γ, and the values of the real bare propagator poles s_i, are concurrently obtained from a least-squares fit of the unitary, normalized $T(s)$ to the input.

In 1995, the model was restricted to three channels, and the optimal number of resonances per partial wave. The input, consisting of πN elastic and $\pi N \to \eta N$ processes, was separated into two structurally different parts: (a) instead of taking the huge πN elastic database, we have fitted the KH80 partial waves (824 points, i.e., 103 points per partial wave) [3]; and (b) all existing $\pi N \to \eta N$ differential and total cross section data (839 points). The number of πN elastic partial-wave energy values was balanced to be as close as possible to the number of available $\pi N \to \eta N$ data points in order to ensure equal weighting of the fit to both channels.

The fitting was done simultaneously for all partial waves S_{11}–G_{17}, and contains 132 free parameters. We started with a minimal set of poles: $N_R = 1$ resonant poles, and two for the background. We then increased the number of resonant poles until a satisfactory fit was achieved, that is, until the quality of the fit, as measured by the reduced χ^2 value, could not be improved. In addition, a visual resemblance of the fitted curve to the data set as a whole was used as a rule of thumb: we rejected all solutions that had a tendency to accommodate for rapidly varying data points, regardless of the resulting χ^2 value.

As stated at the beginning of the paper, because of a numerical error in the S_{11} analytic continuation in Refs. [15,16], we would like to declare the old BATINIC 95 solution, presently recognized by the PDG, invalid. Instead, we offer two new solutions: (A) a corrected BATINIC 95, and (B) a new solution with an improved S_{11} πN elastic input.

Both solutions are obtained from the same $\pi N \to \eta N$ database, but with different πN elastic input. The corrected solution uses the standard KH80 πN elastic set; the resonance parameters obtained are presented in Table I. Table II shows resonance parameters obtained from the new solution that uses the KH80 set with a modified low-energy part of the S_{11} partial wave. As some normalization problems have been established and reported for the S_{11} low-energy points (up to 1500 MeV c.m. energy) in the KH80 solution [19,20], this part of the KH80 partial wave has been replaced with the more reliable VPI/GWU solution SM95 [9]. This procedure was of course approved and encouraged by Höhler [21]. For a detailed description of the new solution, see Ref. [18].

The quality of the fit, as measured by the χ^2_{red}, is compatible and satisfactory for both solutions; see Table III.

Here, χ^2 is defined in the standard way:

$$\chi^2_{\mathrm{red}} = \chi^2/(824 + 839 - 132 - 1).$$

The statistical difference between the two solutions is insignificant.

(A) The corrected solution with standard KH80. As the difference between the erroneous and corrected Breit-Wigner parameters is small (within one standard deviation), the corrected solution retains all the features of the original one. Hereafter we summarize the main features of all Zagreb solutions. At a first glance it might seem natural to compare our results only with the KH80 resonance and pole parameters. However, in spite of the fact that we qualitatively reproduce *all* KH80 resonances, we believe that we should go beyond this.

Even if we could ideally reproduce the KH80 πN elastic partial waves (and indeed we are very close to doing so), our pole parameters could, even substantially, differ from those of KH80.

TABLE I. Breit-Wigner and pole parameters of the corrected BATINIC 95 solution [15]. The Breit-Wigner parameters are given in columns 3–7, and the pole parameters of this solution are given in columns 8–10.

State	L_{2I2J}	Mass and width					Pole parameters		
		Mass (MeV)	Width (MeV)	x_π (%)	x_η (%)	x_{π^2} (%)	Real (MeV)	$-2 \times$ Imaginary (MeV)	πN elastic residue $(\|r\|, \theta)$
$N(1535)$	S_{11}	1550(9)	204(39)	39(8)	57(7)	4(3)	1502(21)	216(47)	(84, 4)
$N(1650)$	S_{11}	1659(11)	213(20)	77(9)	13(7)	10(4)	1652(11)	216(20)	(110, −62)
$N(2090)$	S_{11}	1792(23)	360(49)	35(7)	19(10)	46(10)	1778(24)	371(54)	(68, −175)
$N(1440)$	P_{11}	1442(17)	438(125)	62(4)	0(0)	38(4)	1366(11)	155(12)	(46, −86)
$N(1710)$	P_{11}	1718(16)	195(18)	28(20)	5(7)	67(20)	1697(14)	187(16)	(31, 10)
$N(????)$	P_{11}	1737(11)	159(26)	33(29)	12(9)	55(29)	1724(11)	155(23)	(29, −149)
$N(2100)$	P_{11}	2136(41)	340(86)	16(5)	83(6)	1(1)	2100(45)	330(78)	(32, −71)
$N(1720)$	P_{13}	1722(19)	247(29)	18(3)	2(2)	80(4)	1692(24)	234(24)	(20, −108)
$N(1520)$	D_{13}	1523(8)	133(12)	55(5)	0.1(0.1)	45(5)	1507(9)	123(9)	(36, −6)
$N(1700)$	D_{13}	1821(23)	141(37)	9(6)	20(5)	71(9)	1808(25)	135(33)	(7, −31)
$N(2080)$	D_{13}	2047(65)	507(122)	17(6)	8(3)	75(6)	1963(48)	554(103)	(49, −63)
$N(1675$	D_{15}	1679(9)	152(8)	35(4)	0.1(0.2)	65(4)	1658(9)	136(7)	(25, −16)
$N(2100)$	D_{15}	2216(27)	480(16)	13(4)	0.1(0.3)	87(4)	2143(31)	436(13)	(26, −71)
$N(1680)$	F_{15}	1680(7)	142(7)	67(3)	0.2(0.2)	33(3)	1666(7)	135(6)	(44, −19)
$N(1990)$	F_{17}	2256(455)	1926(7444)	3(6)	2(4)	95(9)	2014(41)	318(144)	(8, −24)
$N(2190)$	G_{17}	2167(89)	505(274)	14(12)	0.2(1)	86(12)	2072(35)	401(135)	(34, −17)

First, due to an extension of the number of channels, they can differ quantitatively. For the same reason, our solutions can also contain new resonances. As we have already shown in [22], fitting only the elastic P_{11} GWU/VPI single-energy partial wave [9] was insufficient to fully constrain all possible resonances. The P_{11} (1710) resonance was left undetected until the ηN channel was included. The new resonance strongly determines the behavior of the ηN channel, but only marginally influences the πN elastic channel, so analysis of only the elastic channel leaves it undetected. This finding is not a flaw in our model, it is an example of the general rule that fitting one particular channel *in principle cannot* reveal precise information about resonances that *dominantly* couple to *other* channels. To "see" these resonances precisely, other channels should be added and fitted. The resulting shift of the poles describing the inelastic channels to new, better-constrained positions also substantially moves the remaining poles. In Hoehler's partial-wave analysis (PWA) the inelasticity constraint is imposed by introducing t-channel dispersion relations, but it does not include the ηN channel. Therefore, information about the

TABLE II. Breit-Wigner and pole parameters of the *new* BATINIC 98 [18] solution. The Breit-Wigner parameters are given in columns 3–7, and the pole parameters of this solution are given in columns 8–10.

State	L_{2I2J}	Mass and width					Pole parameters		
		Mass (MeV)	Width (MeV)	x_π (%)	x_η (%)	x_{π^2} (%)	Real (MeV)	$-2 \times$ Imaginary (MeV)	πN elastic residue $(\|r\|, \theta)$
$N(1535)$	S_{11}	1553(8)	182(25)	46(7)	50(7)	4(2)	1521(14)	190(28)	(68, 12)
$N(1650)$	S_{11}	1652(9)	202(16)	79(6)	13(5)	8(3)	1646(8)	204(17)	(100, −65)
$N(2090)$	S_{11}	1812(25)	405(40)	32(6)	22(10)	46(9)	1797(26)	420(45)	(60, −164)
$N(1440)$	P_{11}	1439(19)	437(141)	62(4)	0(0)	38(4)	1363(11)	151(13)	(44, −88)
$N(1710)$	P_{11}	1729(16)	180(17)	22(24)	6(8)	72(23)	1711(15)	174(16)	(24, 20)
$N(????)$	P_{11}	1740(11)	140(25)	28(34)	12(9)	60(35)	1729(11)	137(23)	(22, −146)
$N(2100)$	P_{11}	2157(42)	355(88)	16(5)	83(5)	1(1)	2120(47)	346(80)	(33, −59)
$N(1720)$	P_{13}	1720(18)	244(28)	18(3)	0.4(1)	82(4)	1691(23)	233(23)	(20, −109)
$N(1520)$	D_{13}	1522(8)	132(11)	55(5)	0.1(0.1)	45(5)	1506(9)	122(9)	(35, −7)
$N(1700)$	D_{13}	1817(22)	134(37)	9(6)	14(5)	77(9)	1806(23)	129(33)	(7, −34)
$N(2080)$	D_{13}	2048(65)	529(128)	17(7)	8(3)	75(7)	1957(49)	467(106)	(53, −65)
$N(1675)$	D_{15}	1679(9)	152(8)	35(4)	0.1(0.1)	65(4)	1658(9)	137(7)	(25, −16)
$N(2100)$	D_{15}	2217(27)	481(17)	13(4)	0.2(1)	87(4)	2144(31)	438(13)	(26, −71)
$N(1680)$	F_{15}	1680(7)	142(7)	67(3)	0.4(0.2)	33(3)	1666(8)	135(6)	(44, −19)
$N(1990)$	F_{17}	2262(470)	2036(8235)	3(6)	2(4)	95(8)	2015(39)	320(143)	(8, −24)
$N(2190)$	G_{17}	2125(61)	381(160)	18(12)	0.1(0.3)	82(12)	2063(32)	330(101)	(34, −19)

TABLE III. Quality of the fits measured by the χ^2 values.

Solution	χ^2	$\chi^2_{\pi N,\pi N}$	$\chi^2_{\frac{d\sigma}{d\Omega}(\pi N\to\eta N)}$	$\chi^2_{\sigma_{\rm tot}(\pi N\to\eta N)}$	$\chi^2_{\rm red}$
(A)	1490	989	408	93	0.97
(B)	1500	1007	396	97	0.98

ηN channel is not included in the KH80 pole positions. In our case, as we fitted the $\pi N \to \eta N$ data in addition to the KH80 elastic channel partial waves, we expect that our pole positions are affected by the ηN channel, and we must obtain a solution that corresponds to a compromise between the elastic and ηN channel data. As a result, our poles and the KH80 poles are in principle different. Therefore, we prefer to make our comparison with PDG averages, as they effectively include *all* channels.

Our findings are as follows:

(i) All 4* resonances are found by the fit, and almost all parameter values (Breit-Wigner parameters, pole positions, and residuals) lie well within PDG limits. The exceptions are $N(1535)$ S_{11} and $N(1520)$ D_{15}, whose parameters are slightly outside the PDG limits.

(ii) All 3* resonances are found and only $N(1700)$ D_{13} has parameters slightly outside the PDG limits.

(iii) Some of the 2* resonances are found, and their parameters are notably different from those of the PDG.

(iv) Some 1* resonances are also found, and their parameters are also notably different from those of the PDG.

The advantage of our approach is is that all parameters, those that agree with those of the PDG and those that do not, are obtained in a simultaneous, multichannel, multiresonance fit. Therefore, we expect our resonance parameters to be significantly more reliable than those of the PDG where there is disagreement.

(B) The new solution with modified KH80 input in the S_{11} partial wave. The Breit-Wigner and pole parameters of this solution are very similar to those of solution (A), so all features of the first solution are preserved.

In conclusion, in addition to Breit-Wigner parameters we present for the first time pole positions for two Zagreb PWA solutions: model (A) from Refs. [15,16], and model (B) from Ref. [18].

In spite of a strong statistical similarity of the two solutions, *we endorse solution (B)*. The reason is simple. Even when the input set for solution (B) is obtained in a somewhat artificial way (simply replacing the debatable low-energy part of the KH80 S_{11} solution [19,20] with that of the VPI/GWU SM95 solution), the fit is still based on an S wave with fewer normalization problems in the lower-energy region; for an expanded discussion see [18]. So we believe this solution takes into account much more low-energy pion-nucleon physics, such as the ηN S-wave scattering length and σ term physics, which depend on a precise low-energy normalization.

The need for the results presented here was clearly established during discussions at the recent ECT* workshop “The Fifth International Pion-Nucleon PWA Workshop and Interpretation of Baryon Resonances,” Trento, 2009, and was confirmed in Seattle at “The Jefferson Laboratory Upgrade to 12 GeV Workshop (INT 09-3),” 2009. We also acknowledge the support of the DAAD-MZOŠ Croatian-German bilateral agreement contract between Rudjer Bošković Institute and Forschungszentrum Jülich, titled “Excitation and Decay of Baryon Resonances in a Meson-Theoretical Approach.”

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