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Paper 133

Some Discrete Properties of the Granular Contents of Silos

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Abstract

In this paper, the discrete element method (DEM) will be used to model the discharge stage in silo, and the emphasis will be made on phenomena that occur during the discharge stage rather than on algorithms used to model the problem. These discrete numerical approaches comprise three main parts: (I) interaction model, (II) determination of the interacting bodies, and (III) numerical integration of the governing equations. Some discrete properties and the diagram of horizontal pressure for a system of balls in the model of a typical silo will be investigated.

Keywords: discrete element method, silo, discharge stage, interaction model, spatial sorting and searching, horizontal pressures, contact forces.

1 Introduction

Storage and flow of granular materials in hoppers and silos is important in many process engineering applications. Silos are industrial structures which experience a significant percentage of damage and collapse in comparison with other engineering structures: over 1000 silos, bins and hoppers fail in North America each year [1]. More or less validated differential equations (versions of Jansen-Koenen equation) and their exact or approximate solutions exist for filling stages and their content at rest [2] when principles of continuum mechanics excluding inertial forces are acceptable. In the course of the discharge stages the usual state of the content is that of a non-uniform, relatively slow flow of material, characterized by arching and a large number of collisions between particles and therefore, by high dissipation of energy which leads to potential instabilities in solving equations derived from thermo-dynamical or hydro-dynamical analogies [3]. In this paper, the discrete element method (DEM) will be used to model the discharge stage in silo, and the emphasis will be made on phenomena that occur during the discharge stage rather than on algorithms used to model the problem.

DEM is today widely used in modelling of multi-body granular systems [4]. There are variations on the technique but essentially the principle is as follows: it tracks, in a time stepping simulation, the trajectory and rotation of each element in a system to evaluate its position and orientation, and then to calculate the interactions between the elements themselves and also between the elements and their environment. The interactions will then subsequently affect the element positions. In general, elements are individual particles, but they could represent clusters as well [5]. These discrete numerical approaches comprise three main parts:

(I) interaction model,

(II) determination of the interacting bodies,

(III) numerical integration of the governing equations.

2 Interaction model

The simplest shape of the particles, the ball, was opted among the various geometric solids [6]. However, to avoid crystallization, balls were given varying radii, as shown in equation (1):

$$r_i = r_{\min} + (r_{\max} - r_{\min})\, rnd() \tag{1}$$

where r_{max} and r_{min} are predefined extreme radii and *rnd()* is the random number generator with a uniform distribution on the unit segment. More complex shapes can be realized by connecting two or more balls with some overlap.
If friction is omitted, rotational degrees of freedom do not need to be taken into account. Equations of motion of the centroid of the *i-th* particle are then (2):

$$\ddot{\mathbf{u}}_i(t) = \mathbf{M}_i^{-1}\mathbf{f}_i(t) + \mathbf{g}\,, \tag{2}$$

where $\ddot{\mathbf{u}}_i(t)$ is the acceleration of the centroid, $\mathbf{M}_i$ the diagonal mass matrix and $\mathbf{g}$ is the acceleration vector due to the gravity. The total applied force vector $\mathbf{f}_i(t)$ on the centroid of the particle *i*, interacting with the $k_i(t)$ particles, is given by equation (3):

$$\mathbf{f}_i(t) = \sum_{\substack{j=1 \\ j\neq i}}^{k_i(t)} f_{i,j}(t)\mathbf{n}_{i,j}(t) \tag{3}$$

The short range interaction force $\mathbf{f}_{i,j}(t)$ between the particles *i* and *j* is modelled by the linear spring and the viscous damper in parallel (viscoelastic Kelvin or Voigt body) if the balls overlap. The linear spring (Hookean body) was used to model the balls if they are within the reach of cohesion and move apart (Figure 1).

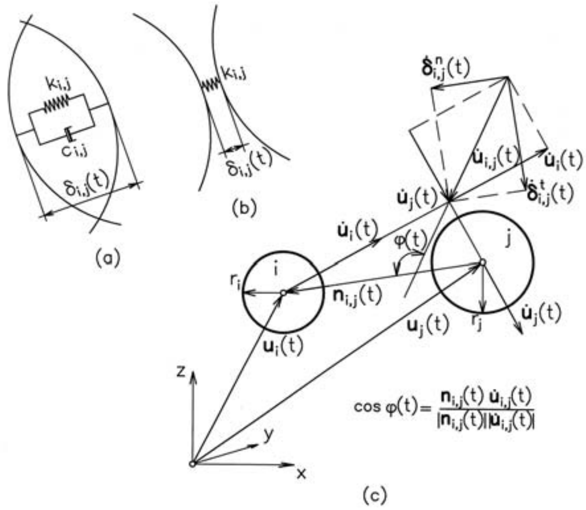


Figure 1: Rheological model of interaction forces between balls: (a) during overlap, (b) during cohesion, (c) definition of vectors

Maximum overlap or minimum distance between two balls is given by equation (4):

$$\delta_{i,j}(t) = r_i + r_j - \left|\mathbf{u}_i(t) - \mathbf{u}_j(t)\right| \tag{4}$$

Where $\mathbf{u}_i(t)$, $\mathbf{u}_j(t)$ are position vectors of the ball centres. Clearly, the overlap $\delta_{i,j}(t)>0$ is the numerical/geometrical counterpart of the squeezing of the balls during contact. The unit vector $\mathbf{n}_{i,j}$ (t) on the line joining centres of the balls is defined by equation (5):

$$\mathbf{n}_{i,j}(t) = \frac{\mathbf{u}_{i,j}(t)}{\left|\mathbf{u}_{i,j}(t)\right|} = \frac{\mathbf{u}_i(t) - \mathbf{u}_j(t)}{\left|\mathbf{u}_i(t) - \mathbf{u}_j(t)\right|} \tag{5}$$

System of equations (2) for i=1,…,n(t) is an approximate description of the large displacements and strains problem. Although material linearity is assumed, the geometric nonlinearity still remains. Because of the frequent collisions, the paths, velocities and accelerations are not smooth functions. Not only the magnitudes, but also the types of the interaction forces between particles depend on the particles positions and velocities and therefore change intensively in time. Described nonlinear problem has no analytical solution and some step by step technique should be used to numerically integrate equations of motion, the approach used was the predictor-corrector method [7].

What is more, neighbours of the *i*-th particle, need to perform the summation in (3), are not known in advance, but as the particle system is in permanent motion, must be determinate in each time step.

3 On spatial sorting and searching

The neighbour id defined here as a particle which is close enough to the observed particle so that any of the before mentioned short range interactions can be "activated".

Contact detection is defined as determination of contact or overlap among members of a set of *n* geometric entities in an *m*-dimensional space. The straightforward algorithm to find interacting particle pairs is to simply test each particle against every other in nested loops, but this is a very time consuming process for systems with many discrete elements (say 10 000 or more). Here, the so called spatial search is based on the *fixed cubes scheme*.

These more advanced algorithms usually consist of two (possibly overlapping) phases called spatial or neighbour searching and contact or geometric resolution.

Spatial searching is the identification of the potential neighbours, while *contact resolution* determines whether candidate pairs actually interact. As the number of potential neighbours is small, the computational cost of contact resolution depends almost only on the complexity of the geometric representation of particles.

In this paper, a variation of the grid based spatial sorting and searching algorithm was developed.

The main idea of the fixed cube scheme is to cover the search space with cubes and sort balls in them (Figure 2a). Then, during the calculation of forces, contact resolution is made only through the contents of the cubes which intersect the observed ball, and not through the whole region of the silo model (Figure 2b).

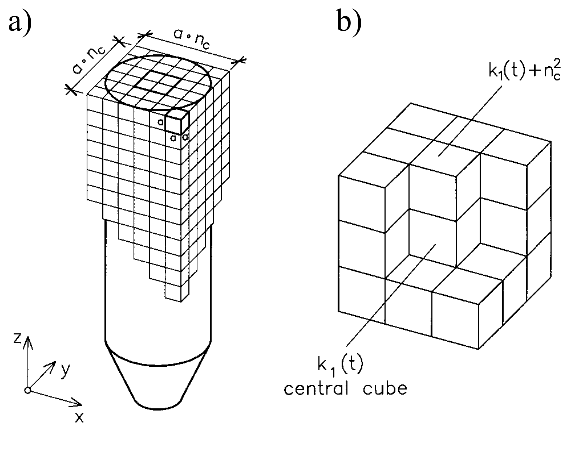


Figure 2: Fixed cubes scheme: (a) covering the region of calculation, (b) central cube and its 26 neighbors (6 cubes are omitted for clearness)

More detailed description of fixed cube scheme, special configurations and ambiguous cases are described in [8].

After spatial sorting is completed, it is quite simple to solve the geometry needed for determining interaction forces between balls *i* and *j*. Two examinations, defined by expressions (6) and (7), are needed to find if balls overlap or possibly stick to each other:

$$\delta_{i,j}(t) > 0 \tag{6}$$

$$\frac{-N^{c}_{max;i,j}}{k_{i,j}} \leq \delta_{i,j}(t) \leq 0 \tag{7}$$

where distance $\delta_{i,j}(t)$ is given by (4), $N^{c}_{max;I,j}$ is the maximum cohesion force and $k_{i,j}$ is the stiffness of the collision model.

4 Boundary conditions

The model of the silo wall is made of fixed overlapping balls with randomized radii, to prevent crystallization of balls representing silage material.
This model also imitates friction due to roughness and geometric imperfections on the surface of the wall. Thus, in the absence of an expensive model of friction between balls (characterized by a friction coefficient), the aim was to simulate at least the geometric part of this phenomena [8].

5 Discussion of some results

Based on the preceding discussions, a computer program was written in Absoft F77 and various regimes during silo exploitation were simulated. The problem data of the presented example are given in Table 1.

r_{min}	0,215 m
r_{max}	0,225 m
n^{cont}_{max}	22741
n_{wall}	3821
γ	1250 kg/m^3
k_c	10^7 N/m
k_{wall}	10^9 N/m
c_c	10^8 Ns/m
c_{wall}	10^7 Ns/m
$\Delta t_{filling}$	10^{-4} s
$\Delta t_{discharge}$	10^{-5} s
$\Delta t_{"hammer"}$	10^{-5} s

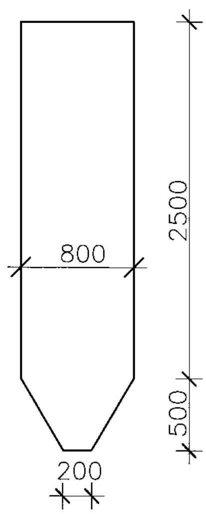


Table 1: Main data of the model

In the following sections only some of the observed phenomena, which are believed to be interesting, will be discussed.

5.1 Material behaviour

It is known that granular material is highly compressive and subjected to consolidation over time (even if a single grain is considered rigid), because of the rearrangement and better packing of grains in the heap.
It was observed that averaged heap density is exceeded by up to 14% during filling and 17% during discharge or material "hammer" (right in Figure 3).
The greatest values are found little above connection between the cylinder and hopper. This is a highly compressed zone because of the intensive arching activities.
The heap surface is of concave shape during discharge (Figure 3) because of the non-uniform horizontal velocity profiles. Parts of the contents close to wall are slower due to the model of geometric friction and arching influence (funnel flow).

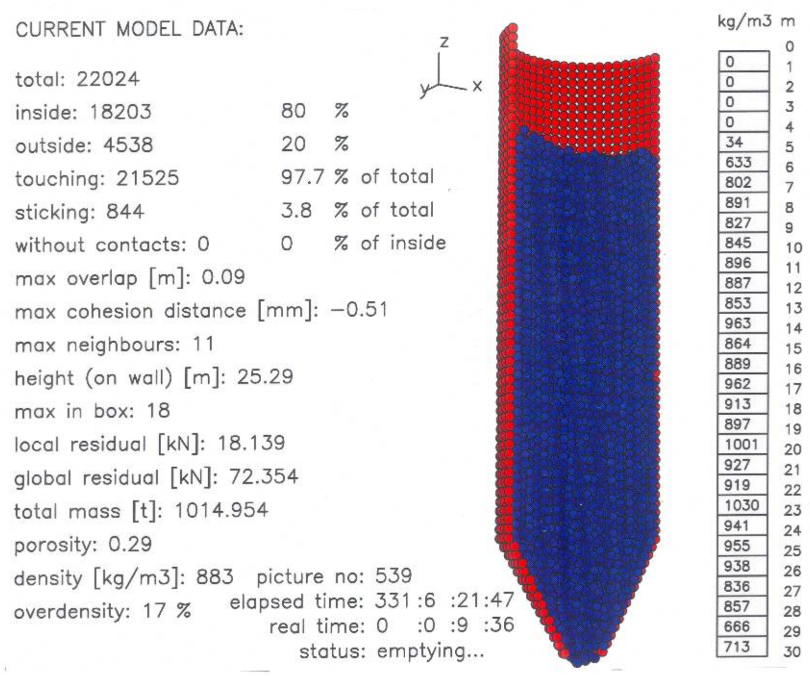


Figure 3: State of contents during the discharge stage

5.2 Contact forces

All load exerted on granular material is transmitted through the heap in form of a lattice consisting mainly of the compressive contact forces between grains. Small tensile forces exist only if cohesive phenomenon is present. Here, the lattice is presented by connections between balls centroids which, according to our rheological model, define directions of contact forces.

Force intensities are presented by various colours as illustrated in Figure 4. This figure also shows compression chains which carry most of the heap load leaving zones of the contents between them almost unloaded (green).

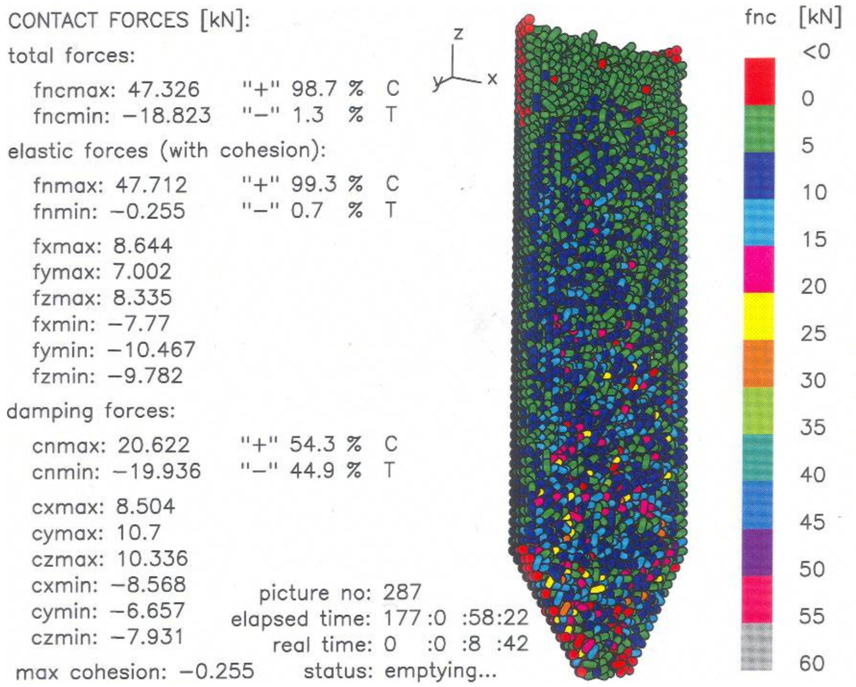
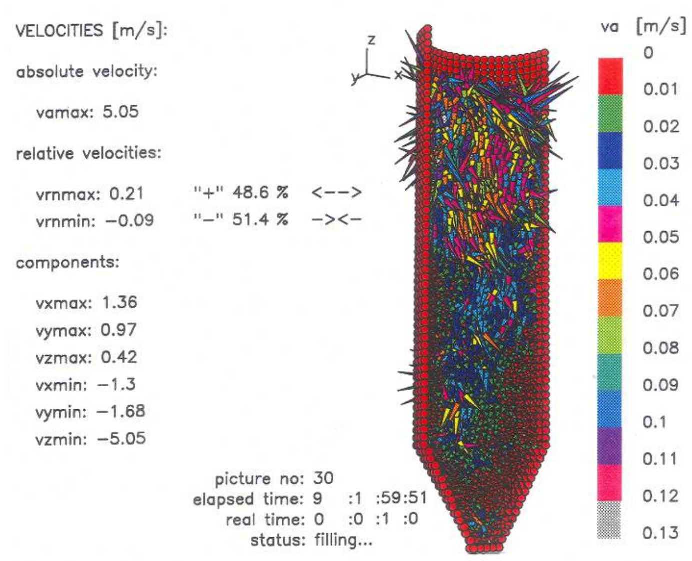


Figure 4: Contact forces between balls during the discharge stage

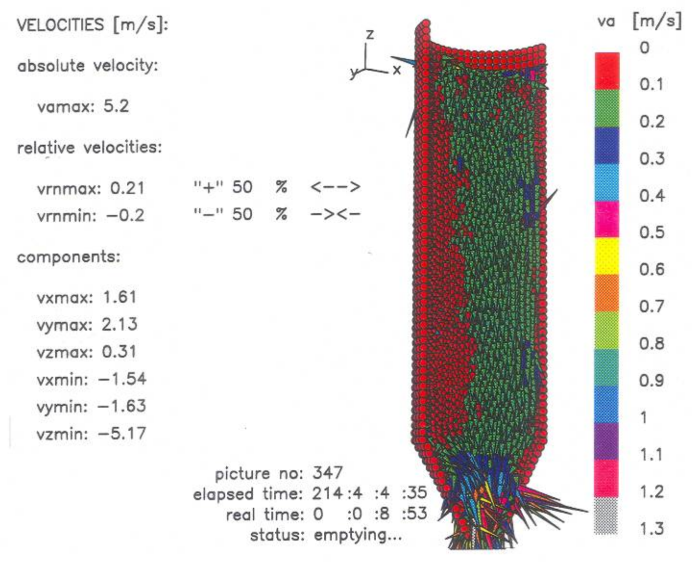
5.3 Velocity profiles

Figures 5, 6 and 7 are showing velocity vectors coloured according to their intensities. Even in the intensive motion, isolated zones of high and slow velocities can be observed.
These non-uniform states of contents do not correspond to the well defined differential equations of motion but hints at the possibility that an iteration process with different time steps could be used [9]. Thus, larger time steps are used in regions of small velocities, while shorter time steps are used only in regions of high velocities (not over the whole contents as is usually accepted, although the high velocity region may be very small).
This idea makes time stepping process faster especially in the case of small high velocity regions. Of course, because of uniqueness of the time and action-reaction law, these partial time processes must always be globally reconciled at the interfaces of mentioned regions.
To ensure better stability of the time stepping process, a relatively large viscosity and zero vertical velocity of balls at the moment of first impact was adopted, so that the model of contents at rest is obtained quickly during filling as shown in Figure 5a.
Despite symmetry in geometry or filling conditions, there is a distinct unsymmetrical shape of the discharge velocity profile. For example, in Figure 5b a little more than a half of the contents is in intensive motion.



a)

Figure 5 a): Velocity distribution, during filling



b)

Figure 5 b): Velocity distribution, unsymmetrical discharge profile

It is interesting to note that velocity fluctuations of almost the entire contents occur in very short time intervals which can cause high dynamic effects (Figures 6a and b). Arching is the main reason for that, as shown in Figure 6a.

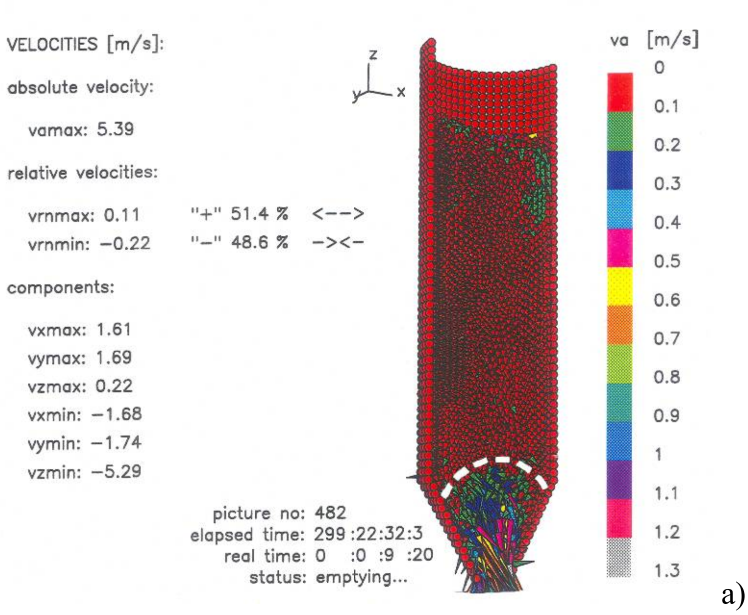


Figure 6 a): Velocity distribution, stoppage of discharge

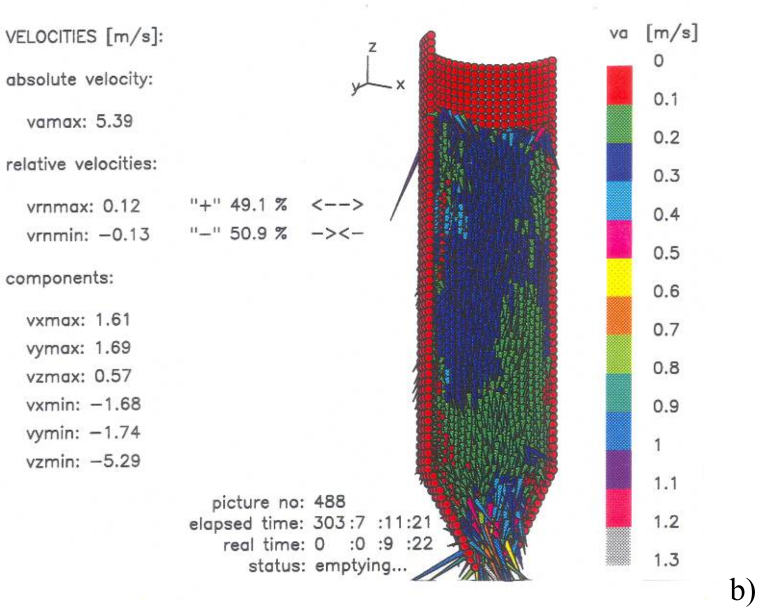


Figure 6 b): Velocity distribution, mass discharge profile after 2 s

During animation, non-stationary flow (meandering) of material through the silo was also noted, because of the irregular local arching on the silo wall. This can be seen in figure 5a during the filling stage and additionally emphasized in figures 7a and b, during the discharge stage.

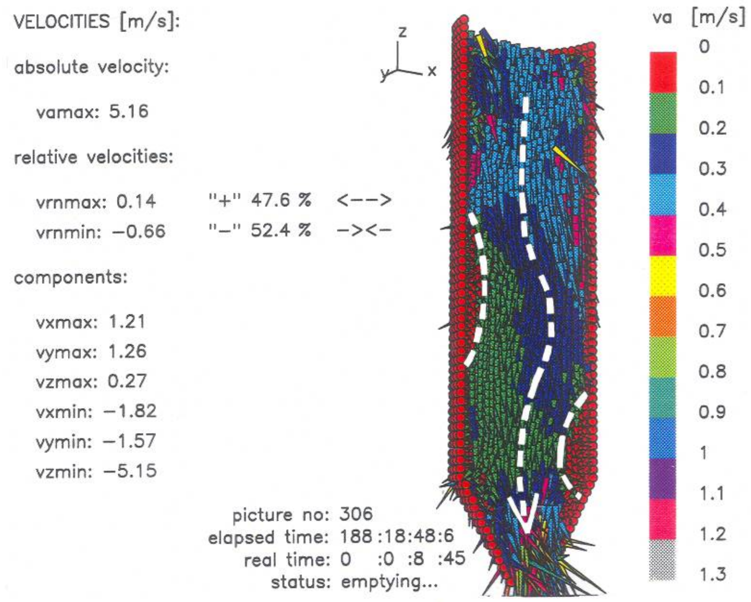


Figure 7 a): Velocity distribution, meandering of funnel at time: 8 min 45 sec

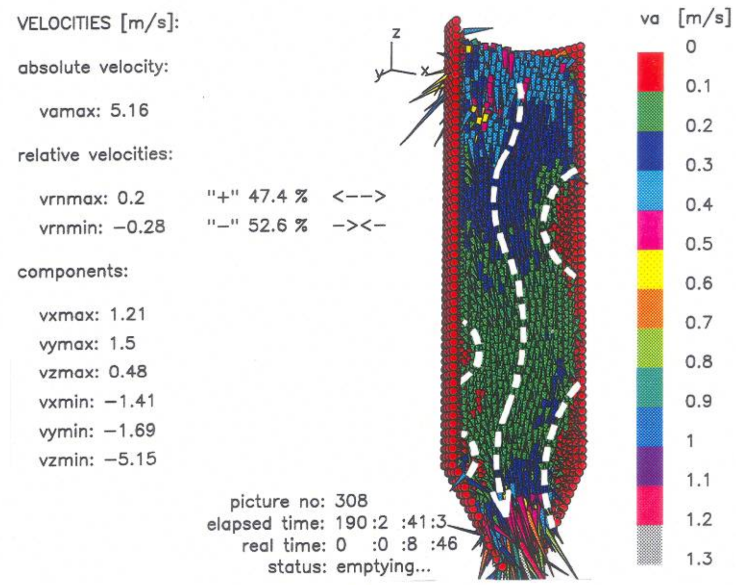


Figure 7 b): Velocity distribution, meandering of funnel at time: 8 min 46 sec

In Figures 6a, 7a and 7b, sections through the compressive forms (arches and domes) are presented schematically (intuitively) by thick dashed lines, because these phenomena always separate regions of high and small (almost zero) velocities. Special algorithms for determining and separating these shapes from the heap model are developed to ensure better understanding of these activities [10].

5.4 Horizontal pressures

A three dimensional distribution of horizontal pressures on the unfolded silo wall can be observed (Figure 8). These pressures are calculated as horizontal components of the total contact force acting on each fixed (boundary) wall ball divided by the areas of squares that circumscribe maximum cross section of that ball. Due to small overlapping between boundary balls, the sum of these areas is approximately 2% smaller than the total area of silo wall, which is negligible.
It was observed that Janssen $\mathbf{p_{hj}}$, water $\mathbf{p_w}$ and equivalent frictionless material $\mathbf{p_{ht}}$ pressure values, were highly exceeded by the averaged pressures $\mathbf{p_{ha}}$ through the horizontal layer of balls as shown in Table 2, especially in the transition zone between the cylindrical section and the hopper. Marked lines in the table of the Figure 8 are giving extreme ratios of the dynamic $\mathbf{p_{hd}}$ and static $\mathbf{p_{hs}}$ averaged pressures through the layer of balls.

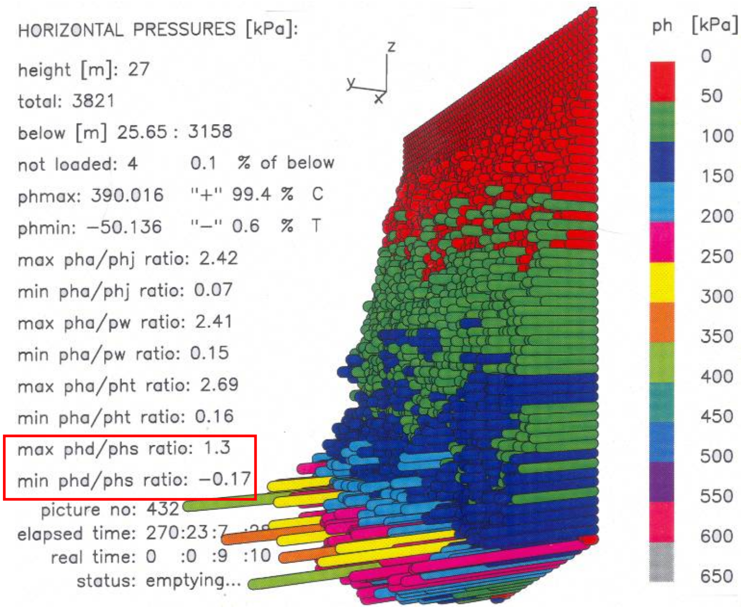


Figure 8: Horizontal pressures on the unfolded silo wall during the discharge stage

Unsymmetrical pressure diagram across the horizontal circumference of the cross section was also noted. It is interesting to note that overpressures during the material "hammer" are smaller than expected. There are three main reasons for this: (i) before closing of the outlet 22% of the content was emptied, so that the moving mass was smaller, (ii) energy dissipation during contact pressures, (iii) arching effects preserve momentary collapse of the whole material, material falls by parts between arches, which is beneficial. These effects are clearly shown on the right side of the pressure diagram in Figure 9.

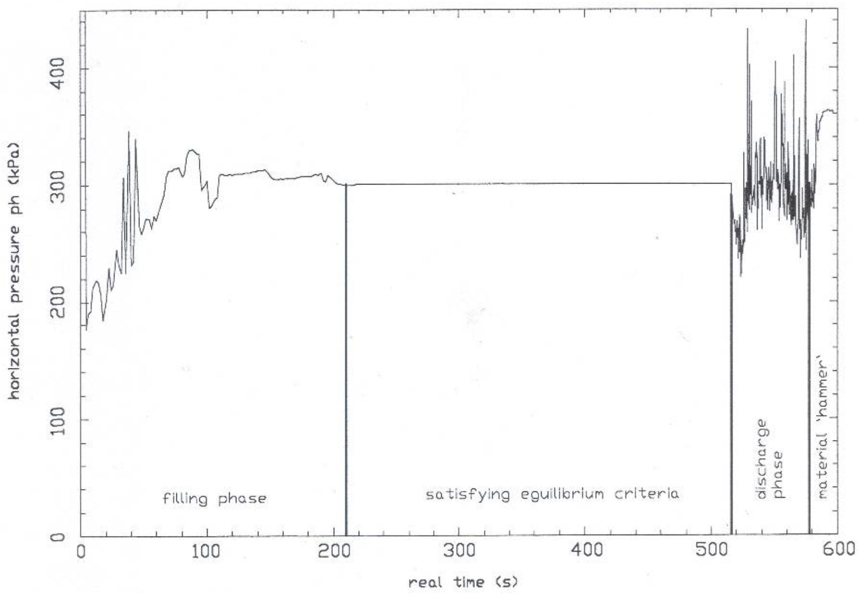


Figure 9: Horizontal pressure calculated in all phases of simulation at one point placed in the transition zone from the cylindrical section to the hopper

6 Conclusions

Some discrete properties and the diagram of horizontal pressure for a system of balls in the model of a typical silo were investigated. Important results may be summarized as follows:

1. Highly unsymmetrical discharge velocity profile were noticed although the geometry and filling conditions were symmetric
2. Arching effects in hopper may frequently stop the motion of almost entire contents and introduce beating effects on silo, which cause high overpressures,
3. Existence of narrow localized zones of intensive motion separated from material at rest make the formulation of differential equations of motion in silo impossible
4. Winding (meandering) of the large funnel during the discharge stage may cause transversal oscillations of the silo with additional dynamic effects
5. Unsymmetrical pressure diagram in symmetry conditions were observed
6. Energy dissipation, deformable balls and arching effects reduced overpressures caused by the material “hammer” scenario
7. Crystallization phenomena with discrete elements of ideally shaped balls disturb transmission of contact forces and shape of the pressure diagram

Finally, two ideas deserve special attention:

a. Depending on regions of different ball velocities, the development of time stepping methods with different durations of time increments over these regions may increase computational performance of these processes

b. Detecting and separating compressive forms from the heap give much better insight into the contents behaviour within a silo.

Results obtained by discrete element procedures are not directly useful for practical purposes at present. Qualitative results and shapes of some phenomena are well identified, but quantitative values must be considered with great caution.

This situation presently exists because of two main problems in DEM:

a) Time consuming numerical calculations of discrete systems, especially when numerous passes are required, changing different parameters of the system, finding envelope of pressures and,

b) Knowledge about the shape, fraction distribution and micromechanics of the contact behaviour (especially friction) of tiny grains in the undisturbed contents is modest.

The first problem issue was resolved here by using smaller number of larger and simpler discrete elements than they really are. The second problem is avoided here by means of simpler constitutive models during interaction between them. These simplifications are probably too crude, so that results obtained by this method may be as yet not applicable directly for silo design.

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References

[1] Knowlton, T. M.; Carson, J.W.; Klinzing, G.E.; Yang, W-C.: “The importance of Storage, Transfer and Collection”, Chemical Engineering Progress, 90 81994), pp.44-54.

[2] Nedderman, R. M.: “Statics and Kinematics of Granular Materials”, Cambridge University Press, Cambridge, New York, USA, 1992.

[3] Jaeger, H. M.: “Chicago Experiments on Convections, Compaction and Compression”, Physics of Dry Granular Media (eds. Herrmann, H. J., Hovi, J.-P.; Luding, S.), Kluwer Academic Publishers, Dordrecht, 1997, pp. 553-583.

[4] Stein, E., de Borst, R., Hughes, T.J.R.: “Encyclopedia of Computational Mechanics – Volume 1, Fundementals”, Wiley & Sons, Chichester, 2004, pp. 313-337.

[5] Li, J.; Langston, P. A.; Webb, C.; Dyakowski, T.: “Flow of sphero-disc particles in rectangular hoppers – a DEM and experimental comparison in 3D”, Chemical Engineering Science 59 (2004), 5917 – 5929.

[6] Williams, J.R., O´Connor, R.: “Discrete Element Simulation and the Contact Problem”, Archives of Computational Methods in Engineering, 6 (1999), pp. 279-304.

[7] Lazarević, D., Dvornik, J., Fresl, K.: “Diskretno numeričko modeliranje opterećenja silosa”, Građevinar, 54 (2002) pp. 135-144 (in Croatian).

[8] Lazarević, D., Dvornik, J., Fresl, K.: “Contact Detection Algorithm for Discrete Element Analysis”, KoG 6-(2002) pp. 29-40

[9] Lazarević, D., Dvornik, J.: “Selective Time Steps in Predictor-Corrector Methods Applied to Discrete Dynamic Models of Granular Assemblies”, Proceedings of ICADD-4 Fourth International Conference on Analysis of Discontinuous Deformation, 2001. pp.193-201

[10] Dvornik, J., Lazarević, D.: “Recognition of vaults and domes in granular material”, Proceedings of ICADD-4 Fourth International Conference on Analysis of Discontinuous Deformation, 2001. pp. 293-298