Plenary lectures

Circular surfaces $\mathcal{CS}(\alpha, p)$

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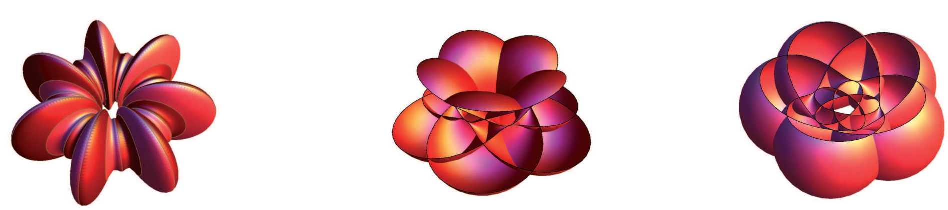
This lecture introduces a new concept of surface-construction: We consider a congruence of circles $\mathcal{C}(P_1, P_2) = \mathcal{C}(p)$ in the Euclidean space, i.e. a two-parametric set of circles which pass through the points P_1, P_2 given by the coordinates $(0, 0, \pm p)$, where $p = \sqrt{q}$, $q \in \mathbb{R}$. It is a normal curve congruence with singular points on the z axes, [3]. $\mathcal{C}(p)$ is a hyperbolic, parabolic or elliptic if q is greater, equal or less then 0, respectively. For a piecewise-differentiable curve $\alpha : I \to \mathbb{R}^3$, $I \subset \mathbb{R}$, we define a *circular surface* $\mathcal{CS}(\alpha, p)$ as the system of circles of $\mathcal{C}(p)$ which cut the curve α.

For the surfaces $\mathcal{CS}(\alpha, p)$ we derive parametric equations (which enable their visualizations in the program *Mathematica*) and investigate their properties if α is an algebraic curve. In the general case, if α is an algebraic curve of the order n, $\mathcal{CS}(\alpha, p)$ is an algebraic surface of the order $3n$ passing n times through the absolute conic and containing the n-fold straight line P_1P_2. But the order of $\mathcal{CS}(\alpha, p)$ is reduced if α passes through the absolute points or if it cuts the line P_1P_2.

The first examples of algebraic $\mathcal{CS}(\alpha, p)$ are parabolic cyclides (if α is a line), Dupin's cyclides (if α is a circle) and rose-surfaces (if α is a rose) [1], [4]. Furthermore, we consider cyclic-harmonic curves $R(a, n, d)$ lying in the plane $z = k$, $k \in \mathbb{R}$, which are given by the polar equation $\rho = \cos(\frac{n}{d}\varphi) + a$, where $\frac{n}{d}$ is a positive rational number in lowest terms and $a \in \mathbb{R}^+ \cup \{0\}$. A *generalized rose-surface* $\mathcal{R}(p, k, n, d, a)$ is the surface $\mathcal{CS}(\alpha, p)$ where the directing curve α is the cyclic harmonic curve $R(n, d, a)$ in the plane $z = k$. These surfaces have various attractive shapes, a small number of high singularities, and they are convenient for algebraic treatment and visualization in the program *Mathematica*. Some examples are shown in Fig. 1.



a b c

Figure 1: $\mathcal{R}(i, 0.75, 7, 1, 2)$ in Fig. a, and two different cuts of $\mathcal{R}(i, 0, 5, 3, 2)$ in Figs. b and c.

Since the surface-construction concept mentioned above can be applied on any curve α, numerous new forms of surfaces can be obtained. Some examples are shown in Fig. 2 and Fig. 3.

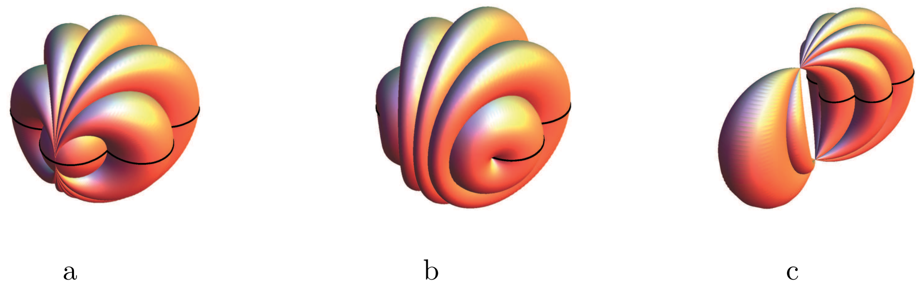


Figure 2: The surfaces directed by a parabolic, elliptic and hyperbolic congruence and one cyclic harmonic curve are shown in Figs a, b and c, respectively.

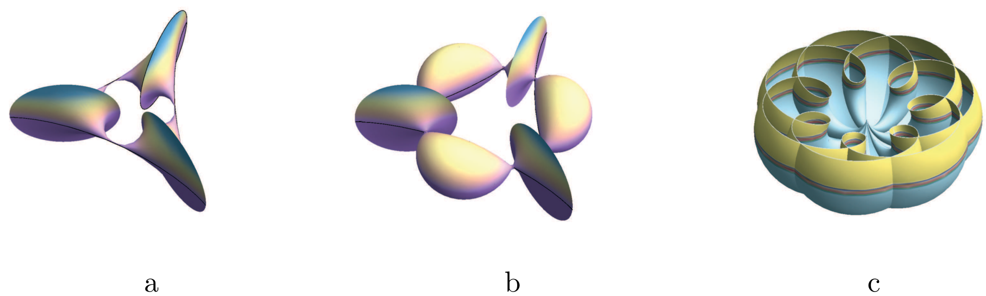


Figure 3: The surfaces directed by Steiner curve (hypocycloid) in the plane $z = 0$, and $p = i, 1.5i$ are shown in figs. a and b, respectively. The surface in fig. c is directed by $p = 0$ and one epitrochoid in the plane $z = 0$.

Key words: circular surfaces, congruence of circles, cyclic-harmonic curves, generalized rose-surfaces

MSC 2010: 51N20, 51M15

References

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[3] G. Salmon, *A Treatise on the Analytic Geometry of Three Dimensions*, Vol.II., Chelsea Publishing Company, New York, 1965 (reprint).

[4] "Special Rose Surfaces" from The Wolfram Demonstrations Project
http://demonstrations.wolfram.com/SpecialRoseSurfaces/
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