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STOCHASTIC MODELING OF PUMP-STORAGE HYDROELECTRIC POWER PLANTS, Part I

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Original scientific paper

Two parts of this paper represent a contribution to the implementation of a pump-storage hydroelectric power plant stochastic model into a power plant system reliability model. After analysis of operation modes of such type of power plants, Part I gives an overview referring to the determination of the probability distribution of variable production when natural hydraulic inflow into the upper storage is exclusively used, the probability distribution of variable production from a pumped-storage drive, as well as the probability distribution of necessary energy of the plant with natural hydraulic inflow. The inflow is thereby treated as a random variable, and stochastic modeling relies on the well-known method of constant and variable energy production. The developed model is suitable for development of an additional determination criterion when it comes to making studies related to planning strategy of production capacities in the power system. The stochastic model presented in the paper is illustrated by a simple numerical example.

Key words: *density function, distribution function, method of constant and variable energy production, probability function, reliability, stochastic inflow*

Stohastički model crpno-akumulacijskih hidroelektrana, I. dio

Izvorni znanstveni članak

Ovaj članak u dva dijela predstavlja doprinos implementiranju stohastičkog modela crpno-akumulacijskih hidroelektrana u model pouzdanosti sustava elektrana. U prvom dijelu, nakon analize pogona ove vrste elektrana, dan je prikaz određivanja razdiobe vjerojatnosti varijabilne proizvodnje pri isključivom korištenju prirodnog dotoka u gornji bazen, razdiobe vjerojatnosti varijabilne proizvodnje iz crpno-akumulacijskog pogona, te razdiobe vjerojatnosti potrebne energije crpljenja postrojenja s prirodnim dotokom. Pri tome se dotok tretira kao slučajna varijabla, a stohastičko modeliranje oslanja se na poznatu metodu konstantne i varijabilne energije. Razvijeni model je pogodan za stvaranje pomoćnog kriterija odlučivanja pri izradi studija za planiranje izgradnje proizvodnih kapaciteta u elektroenergetskom sustavu. Prikazani stohastički model ilustriran je jednostavnim brojčanim primjerom.

Ključne riječi: *funkcija gustoće, funkcija razdiobe, funkcija vjerojatnosti, metoda konstantne i varijabilne energije, pouzdanost, stohastički dotok*

1 Introduction
Uvod

For the purpose of accumulating a surplus of hydropower during off-peak periods, a pump-storage hydroelectric plants are built, which have balancing impact on the load curve. Within the pump-storage hydroelectric plants by means of low-cost off-peak electric power water is pumped from the lower storage into the upper storage, which is then in other periods transformed into peak energy of great value. Two types of pump-storage hydroelectric plants may be distinguished; i.e. the ones with and without natural hydraulic inflow into the upper storage. In both cases it is assumed that pumps on the suction side have a sufficient amount of water available for pumping at any time. The developed stochastic model can be used for reliability estimation of the electric power generating system as an auxiliary criterion by making studies related to planning strategies of generating capacities in the power system.

Prior to presentation of the insertion of a pump-storage hydroelectric plant's stochastic model into the stochastic modelling of overall system of hydro and thermal power plants it is necessary to briefly analyse the operation modes of such plants [2, 3, 4].

2 Analysis of a pump-storage hydroelectric plant operation mode
Analiza pogona crpno-akumulacijske hidroelektrane

Pump-storage hydroelectric power plants must meet the same requirements with respect to operation mode strategy as hydroelectric storage plants. Indeed, they need to produce more variable energy in the shortest time possible with maximum capacity. The method of constant and variable energy production described in [5] enables the determination of the possible, variable and constant production from hydroelectric storage plants in the week observed. In case of hydroelectric storage plants, consideration of inflow as a random variable, as described in [3], requires determination of the probability distribution of possible, variable and constant production. Dependencies used for that purpose according to the method of constant and variable energy production are shown in the Appendix, and they are also applied in the case of pump-storage hydroelectric power plants. Assuming for the purpose of simplicity that after six working days there follows one non-working day when variable energy is not needed, Fig. 1 shows the operation modes diagram of a pump-storage hydroelectric plant without (Fig. a) and with (Fig. b) natural inflow.

Symbols given in Fig. 1 mean the following: Q_A – maximum turbine flow-through (designed size), (m^3/s); $P_{h,\max}$ – maximum capacity of a pump-storage plant in turbine drive, (MW); Q – natural inflow, (m^3/s); P_h – capacity a plant would have if it used water corresponding to natural inflow, (MW); $Q_{p,\max}$ – maximum pump flow-through in the observed week, (m^3/s); $P_{p,\max}$ – pump capacity

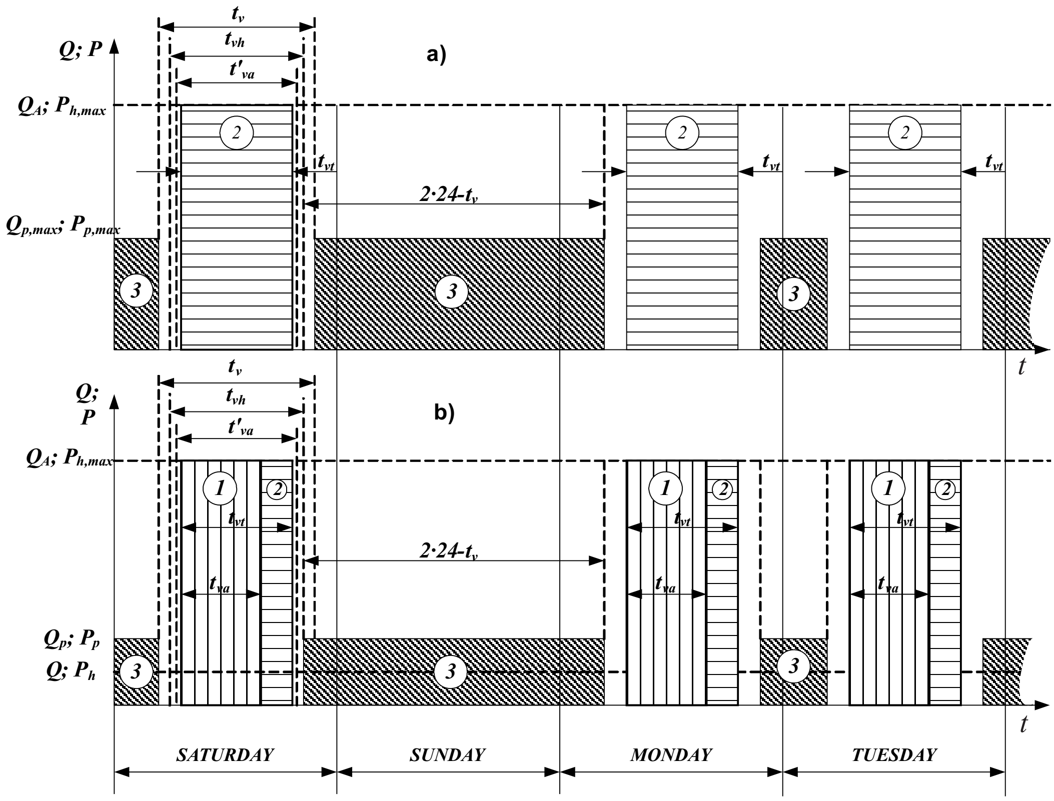

Figure 1 *Operation mode diagram of a pump-storage hydroelectric plant without (a) and with (b) natural inflow*
1 – Energy production from natural inflow; 2 – Production from pump-storage operation; 3 – Pump drive
Slika 1*. Dijagram pogona crpno-akumulacijske hidroelektrane bez (a) i sa (b) prirodnim dotokom*
1 – Energija iz prirodnog dotoka; 2 – Energija kod crpno-akumulacijskog pogona; 3 – Crpni pogon

required for flow-through $Q_{p,\max}$, (MW); Q_p – flow-through of pumps in case there is a natural inflow, (m^3/s); P_p – pump capacity required for flow-through Q_p, (MW); t_v – working day peak load duration, (h); t_{vh} – maximum duration of using variable energy of a hydroelectric plant on a working day, (h); t_{va} – duration of using variable energy of a pump-storage plant with exclusive usage of natural inflow into the upper storage, (h); t_{vt} – maximum possible duration of using maximum capacity in turbine drive (h), i.e. by energy production in the pump-storage mode of operation, regardless of possible additional production from natural inflow, (h); $t_{va'}$ – duration of using variable energy of some pump-storage plant, (*h*) with exclusive production from natural inflow corresponding to production W_{hm}' according to (D-11) – see the Appendix. This duration is calculated from (D-8) with $t_{va}=t_{va}'$. (If $t_{va}>t_{va}'$, it is also necessary to produce constant energy).

Maximum capacity of pump-storage plants in turbine drive and pump capacity needed for flow-through $Q_{p,max}$ for the observed week are given by:

$$P_{h,\max} = 9{,}81 \cdot Q_A H_n \eta_T \times 10^{-3} = \\ = 9{,}81 \cdot Q_A \alpha H_B \eta_T \times 10^{-3},\ \text{MW} \tag{1}$$

$$P_{p,\max} = 9{,}81 \cdot Q_{p,\max} H_B \frac{1}{\eta_p} \times 10^{-3},\ \text{MW} \tag{2}$$

In equations (1) and (2): H_n – net head (m); H_B – gross head (m); α – net-gross head ratio ($\alpha<1$); η_t – efficiency rate of the plant in turbine drive; η_p – efficiency rate of the plant in pump drive.

Weekly amount of energy a plant without an inflow can produce for *r* working days with the maximum capacity of turbine drive during the time t_{vt} is:

$$W_{vt,\max} = P_{h,\max} r t_{vt} \times 10^{-3},\ \text{GWh} \tag{3}$$

Energy required for the pump drive mode should be provided for by other power plants in the system. Off-peak load periods are favorable for this (see Fig. D-1).

$$T_m = T - rt_v = 24 \cdot (r+n) - rt_v\,. \tag{4}$$

The dependence of the maximum pump flow-through $Q_{p,\max}$ and the maximum possible duration of using maximum power in turbine drive t_{vt} can be drawn from the condition that in the case of a pump-storage plant without natural inflow the quantity of the pumped water must be equal to the quantity used in turbine drive:

$$Q_{p,\max} = Q_A \cdot \frac{rt_{vt}}{24(r+n) - rt_v}\,. \tag{5}$$

Equations (2) to (5) enable calculation of the weekly amount of energy by which water required for production of $W_{vt,\max}$ is pumped into the upper storage:

$$W_{pt,\max} = P_{hp,\max} T_m \times 10^{-3} = \frac{P_{hp,\max} r t_{vt} \times 10^{-3}}{\eta_t \cdot \eta_p \cdot \alpha} \\ = \frac{W_{vt,\max}}{\eta_t \cdot \eta_p \cdot \alpha},\ \text{GWh} \tag{6}$$

If there is a natural inflow into the upper storage, it will be used for energy production. How long a power plant with maximum power on working days can produce variable energy from a natural inflow is determined on the basis of the condition that the quantity of water inflowing during $24(r+n)=168$ hours with flow-through Q must be equal to the quantity of water that can be used for energy production on working days at time t_{va} at maximum flow-through Q_A (see Fig. 1-b):

$$t_{va} = \frac{Q \cdot 24(r+n)}{Q_A \cdot r}, \text{ h} \tag{7}$$

In this case energy production on a working day during t_{vt} hours is obtained partially from the natural inflow and partially from a pumped-storage drive. For $t_{va}<t_{vt}$ it is hence necessary to predict water pumping into the upper storage, i.e. pumping of those quantities which together with the natural inflow enable usage of maximum power during t_{vt} hours in a working day. During time interval t_{va} energy is produced by means of water which inflows naturally, whereas in time interval t_{vt}-t_{va} the pumped water is used.

From Fig. 1-b it can be seen that for variable energy from a natural inflow the following equation holds:

$$W_{va} = P_{h,\max} r t_{va} \times 10^{-3}, \text{ GWh} \tag{8}$$

and for variable energy from a pumped-storage operation:

$$W_{ps} = P_{h,\max} r(t_{vt} - t_{va}) \times 10^{-3} = W_{vt,\max} - W_{va}, \text{ GWh} \tag{9}$$

Pump flow-through is determined similarly to the case of a plant without a natural inflow:

$$Q_p = Q_A \cdot \frac{r(t_{vt} - t_{va})}{24(r+n) - rt_v} \tag{10}$$

Pump capacity necessary for that flow-through is:

$$P_p = 9{,}81 \cdot Q_p H_B \frac{1}{\eta_p} \times 10^{-3}; \text{ MW} \tag{11}$$

By means of equations (1), (4), (7), (10) and (11) the dependence between natural inflow Q and pump capacity P_p is established:

$$P_p = \frac{P_{h,\max} \cdot rt_{vt}}{\eta_t \eta_p T_m \alpha} - \frac{P_{h,\max} 24(r+n)}{Q_A \eta_t \eta_p T_m \alpha} \cdot Q, \text{ MW} \tag{12-a}$$

Under the assumption of a constant gross head and the efficiency rate the following linear function is obtained:

$$P_p = C_1 - C_2 Q, \text{ MW} \tag{12-b}$$

Shown in Fig. 2:

If there are $i<n$ pumps in operation, there is a family of curves which can also be seen in Fig. 2 for the case with four pumps and whose constant section can be explained in the following way: If the quantities of a natural inflow are less

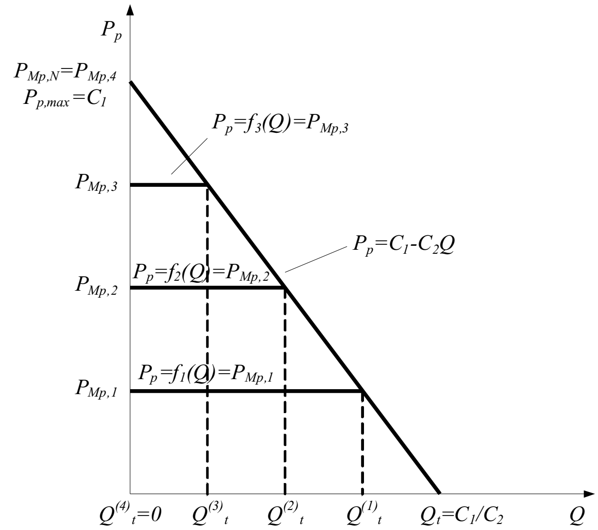

Figure 2 *Dependence Pp=f(Q)*
Slika 2. *Ovisnost Pp=f(Q)*

than $Q_t(i)$, a pumping capacity will be necessary according to function $P_p=f(Q)$ in order to provide for the requested production $W_{vt,\max}$ from water pumped additionally at time T_m. However, for $Q \leq Q_t(i)$, for every i only capacity $P_{Mp,i}$ is available.

The necessary pumping power is obtained on the basis of expressions (4), (10) and (11):

$$\begin{aligned} W_{pm} &= P_p T_m \times 10^{-3} = \\ &= 9{,}81 \cdot Q_A H_B \frac{1}{\eta_p} \times 10^{-6} r(t_{vt} - t_{va}) = \\ &= \frac{P_{h,\max} r}{\eta_t \eta_p \cdot \alpha}(W_{vt,\max} - W_{va}), \text{ GWh} \end{aligned} \tag{13}$$

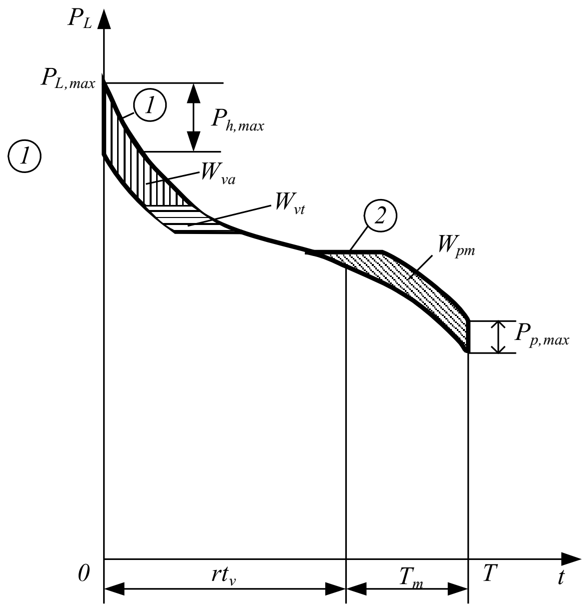

Figure 3 *Modification of the load duration curve with a pump-storage hydroelectric power plant with natural inflow*
1 – Original duration curve, 2 – Modified duration curve
Slika 3. *Modifikacija krivulje trajanja opterećenja kod crpno-akumulacijske hidroelektrane s prirodnim dotokom*
1 – originalna krivulja trajanja, 2 – Modificirana krivulja trajanja

From the aforementioned follows the way a pump-storage plant with natural inflow – by using of maximum capacity on working days during $t_{va}<t_{vt}$ hours, as well as with a 100 % reliable pump and turbine drive – modifies the load duration curve (Fig. 3).

Energy production can naturally be even greater than $W_{vt,\max}$, if it is caused by such natural inflow into the upper storage which enables usage of maximum capacity on working days during $t_{vt}<t_{va}\le t_{vh}$. In such a case pump drive is superfluous, because there is enough water from the natural inflow. Then, a pump-storage power plant becomes a normal hydroelectric storage plant which produces additional variable energy, and for higher inflows even constant energy.

3
Determining probability distributions of variable production and necessary pumping energy of a pump-storage hydroelectric plant with natural inflow
Određivanje razdioba vjerojatnosti varijabilne proizvodnje i potrebna crpna energija crpno-akumulacijske hidroelektrane s prirodnim dotokom

A stochastic character of a possibly existing natural hydraulic inflow mostly determines modeling of pump-storage hydroelectric power plants with respect to reliability calculations. For $t_{va}<t_{vt}$ the total production of such power plants is certainly equal to the deterministic value $W_{vt,\max}$ – whether that energy is produced by exclusive pump-storage operation (without natural inflow) or by partial utilization of the natural inflow with additional production from the pump-storage operation. For pumping energy that should be probability distribution can be established conditioned by stochastic natural inflows. Initial base here is also the natural inflow probability distribution described in [3]. At first from this distribution, in the way shown for storage hydro plants also in [3], as well as by means of dependence (D-5) and equality:

$$W_{va}=W_{hm}\,;\;\; 0\le W_{hm}\le W_{vt,\max} \tag{14}$$

can be created the probability density function $f(W_{va})$ of variable production of a pump-storage hydro plant using exclusively the natural inflow.

The probability of appearance of an inflow from interval $(Q';\, Q'+\Delta Q]$ equals to the probability that the corresponding variable production for exclusive use of natural inflow, determined by (D-5) and (14), lies in interval $(W'_{va};\, W'_{va}+\Delta W_{va}]$:

$$\begin{aligned}&\left[\Pr(Q'<Q\le Q'+\Delta Q)=f(Q)\Delta Q\right]=\\&=\left[\Pr(W'_{va}<W_{va}\le W'_{va}+\Delta W_{wa})=f(W_{va})\Delta W_{va}\right].\end{aligned} \tag{15}$$

The required density function follows from this:

$$f(W_{va})=\frac{f(Q)\Delta Q}{\Delta W_{va}}\cong\frac{\Pr(Q'<Q\le Q'+\Delta Q)}{\Delta W_{va}}. \tag{16}$$

In Fig. 4, containing a graph-analytical determination of function $f(W_{va})$, it can be seen that generally two cases might occur, i.e. for $Q_{\max}\le Q_t$ and $Q_{\max}>Q_t$. Here is $Q_{\max}$ the maximum inflow in the observed week and Q_t is the inflow value which during t_{vt} hours on working days in the observed week results in production $W_{vt,\max}$. Q_t is calculated from

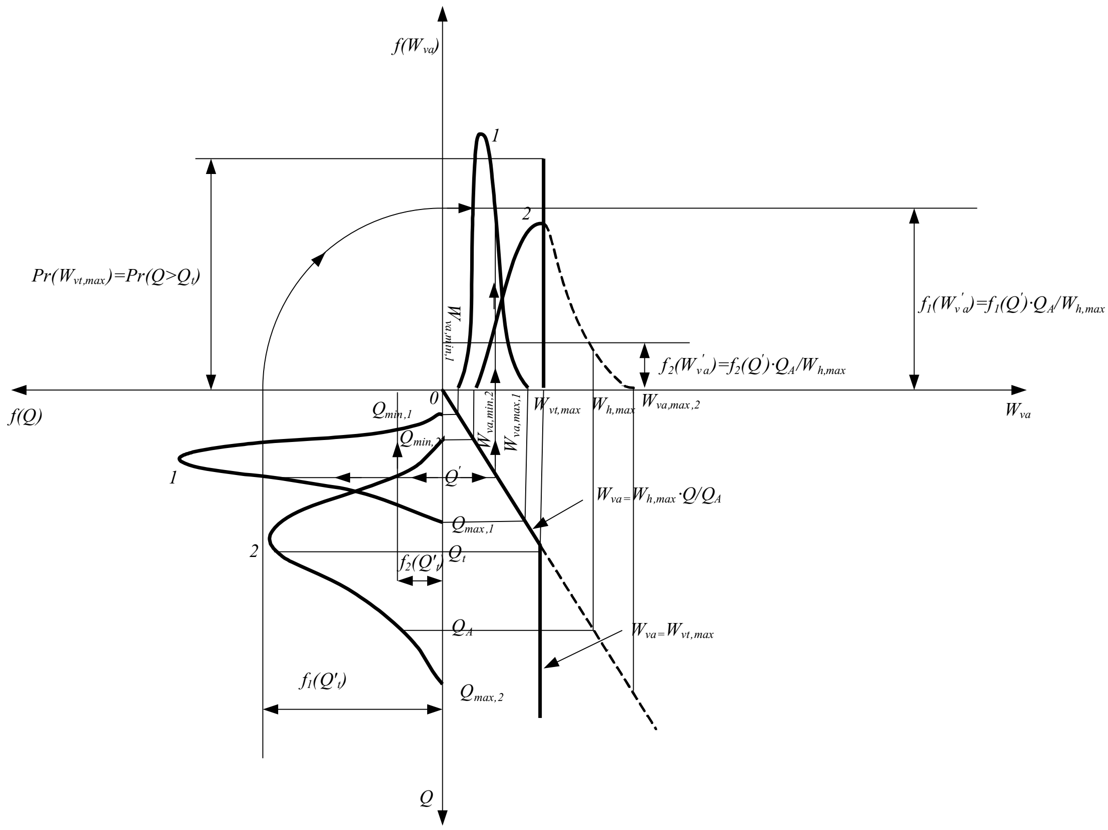

Figure 4 *Determining the variable production density function of a pump-storage hydroelectric plant with exclusive usage of natural inflow*
Slika 4. *Određivanje funkcije gustoće varijabilne proizvodnje crpno-akumulacijske hidroelektrane sa isključivo prirodnim dotokom*

equation (D-5), (D-6) and (3) with $W_{hm}=W_{vt,\max}$:

$$Q_t = Q_A \cdot \frac{rt_{vt}}{24(r+n)}, \ \mathrm{m^3/s} \tag{17}$$

In case 2 ($Q_{max}>Q_t$), the density function is limited at $W_{vt,max}$, causing thereby a discrete jump given by the following equation:

$$\Pr(W_{vt,\max}) = \Pr(W_{va} > W_{vt,\max}) = \int_{W_{vt,\max}}^{W_{vt,\max,2}} f_2(W_{va})\mathrm{d}W_{va} = \tag{18}$$
$$= \Pr(Q > Q_t) = \int_{Q_t}^{Q_{\max}} f(Q)\mathrm{d}Q.$$

The obtained probability density function $f(W_{va})$ satisfies the condition:

$$\int_{-\infty}^{\infty} f(W_{va})\mathrm{d}W_{va} = \int_{W_{va,\min,1}}^{W_{va,\max,1}} f_1(W_{va})\mathrm{d}W_{va} = \tag{19}$$
$$= \int_{W_{va,\min,2}}^{W_{va,\max}} f_2(W_{va})\mathrm{d}W_{va} = 1.$$

and it has the expected value (for case 1 and 2):

$$E_1(W_{va}) = \int_{-\infty}^{\infty} W_{va} f_1(W_{va})\mathrm{d}W_{va} = \tag{20-a}$$
$$= \int_{W_{va,\min,1}}^{W_{va,\max,1}} W_{va} f_1(W_{va})\mathrm{d}W_{va}, \ \mathrm{GWh}$$

$$E_2(W_{va}) = \int_{-\infty}^{\infty} W_{va} f_2(W_{va})\mathrm{d}W_{va} = \tag{20-b}$$
$$= \int_{W_{vt,\min,1}}^{W_{vt,\max}} W_{va} f_2(W_{va})\mathrm{d}W_{va} +$$
$$+ W_{vt,\max} \int_{Q_t}^{Q_{\max}} f(Q)\mathrm{d}Q, \ \mathrm{GWh}$$

The just determined probability density function of variable energy with exclusive usage of natural inflow, together with dependence (9), is now available for determination of variable energy density function $f(W_{ps})$, produced by the pump-storage cycle. Calculation of function $f(W_{ps})$ is based upon the following probability equality:

$$\left[\Pr(W'_{va} < W_{va} \le W'_{va} + \Delta W_{va}) \cong f(W_{va})\Delta W_{va}\right] = \tag{21}$$
$$\left[\Pr(W'_{ps} < W_{ps} \le W'_{ps} + \Delta W_{ps}) = f(W_{ps})\Delta W_{ps}\right].$$

W'_{ps} is here energy which on the basis of (9) corresponds to W'_{va} and which is obtained from the water pumped into the upper storage.

$$f(W_{ps}) = \frac{f(W_{va})\Delta W_{va}}{\Delta W_{ps}} \cong \frac{\Pr\left(W'_{va} < W_{va} \le W'_{va} + \Delta W_{va}\right)}{\Delta W_{ps}}. \tag{22}$$

The graph-analytical determination of the function $f(W_{ps})$ for both cases 1 and 2 is shown in Fig. 5.

In equation (9) which defines the dependence of W_{ps} and W_{va}, $W_{vt,\max}$ occurs as a constant given by expression (3). The jump in $f(W_{ps})$ at $W_{ps}=0$ for case 2 is determined by probability:

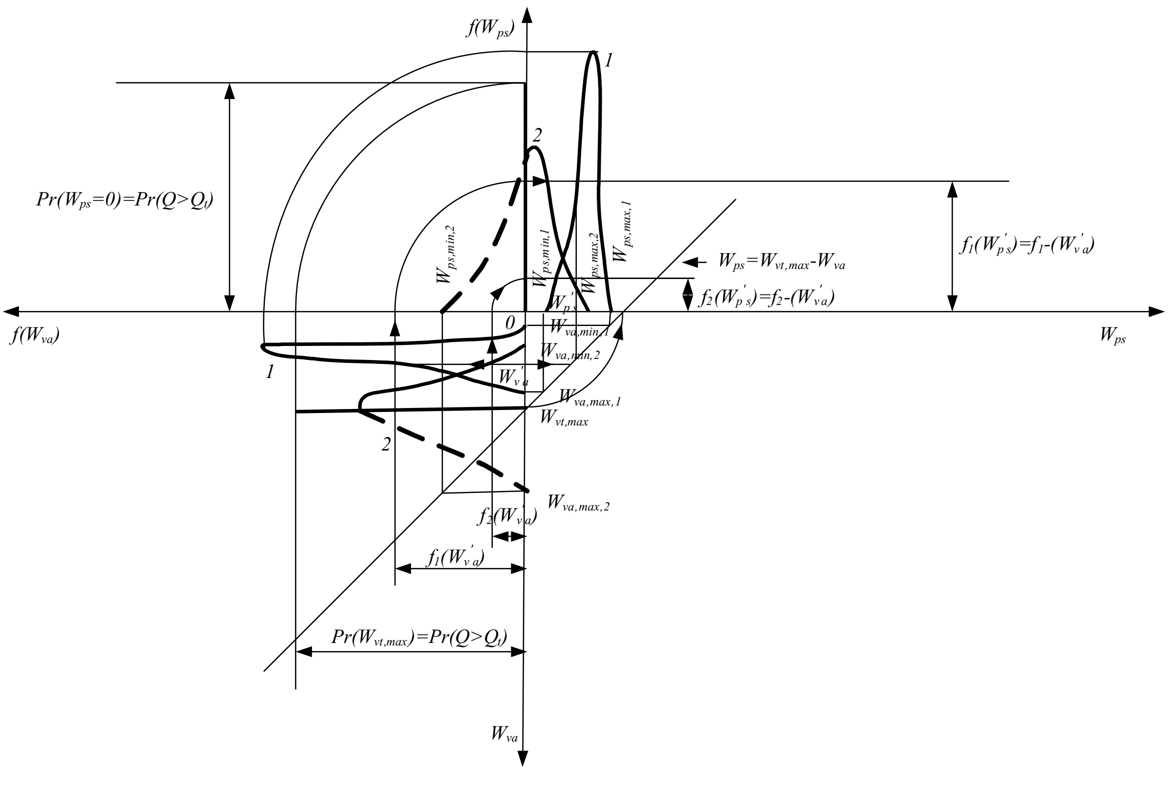

Figure 5 *Determining the probability density function of the variable production from the pump-storage operation*
Slika 5. *Određivanje funkcije gustoće vjerojatnosti varijabilne proizvodnje kod crpno-akumulacijskog pogona*

$$\begin{aligned}&\Pr(W_{ps}\le 0)=\int_{W_{ps,\min,2}}^{0} f_2(W_{ps})\mathrm{d}W_{ps}=\\&=\int_{W_{vt,\max}}^{W_{va,\max,2}} f_2(W_{va})\mathrm{d}W_{va}=\\&=\Pr(Q>Q_t)=\int_{Q_t}^{Q_{\max}} f(Q)\mathrm{d}Q.\end{aligned}\tag{23}$$

The following condition is also satisfied:

$$\begin{aligned}&\int_{-\infty}^{\infty} f(W_{ps})\mathrm{d}W_{ps}=\int_{W_{ps,\min,1}}^{W_{ps,\max,1}} f_1(W_{ps})\mathrm{d}W_{ps}=\\&=\int_{0}^{W_{ps,\max,2}} f_2(W_{ps})\mathrm{d}W_{ps}=1.\end{aligned}\tag{24}$$

The expected value (for both cases) is defined in a well-known way:

$$\begin{aligned}&E_1(W_{ps})=\int_{-\infty}^{\infty} W_{ps} f_1(W_{ps})\mathrm{d}W_{ps}=\\&=\int_{W_{ps,\min,1}}^{W_{ps,\max.1}} W_{ps} f_1(W_{ps})\mathrm{d}W_{ps}\text{, GWh}\end{aligned}\tag{25-a}$$

$$\begin{aligned}&E_2(W_{ps})=\int_{-\infty}^{\infty} W_{ps} f_2(W_{ps})\mathrm{d}W_{ps}=\\&=\int_{0}^{W_{ps,\max.2}} W_{ps} f_2(W_{ps})\mathrm{d}W_{ps}\text{, GWh}\end{aligned}\tag{25-b}$$

The sum of expected values of variable production with an exclusive usage of natural inflow and with pumped-storage drive gives maximum variable production of a pump-storage plant in turbine drive with maximum capacity on working days during the observed week:

$$W_{vt,\max}=E(W_{va})+E(W_{ps}).\tag{26}$$

The function $f(W_{va})$ will also be needed for determining the probability distribution of the required pumping energy of the plant with natural inflow. Hence the following equations can be written by applying similar logic as above:

$$\begin{aligned}&\left[\Pr(W'_{va}<W_{va}\le W'_{va}+\Delta W_{va})\cong f(W_{va})\Delta W_{va}\right]=\\&=\left[\Pr(W'_{pm}<W_{pm}\le W'_{pm}+\Delta W_{pm})=f(W_{pm})\Delta W_{pm}\right].\end{aligned}\tag{27}$$

$$f(W_{pm})=\frac{f(W_{va})\Delta W_{va}}{\Delta W_{pm}}\cong\frac{\Pr\left(W'_{va}<W_{va}\le W'_{va}+\Delta W_{va}\right)}{\Delta W_{pm}}\tag{28}$$

$$\begin{aligned}&\Pr_2(W_{pm}\le 0)=\int_{W_{pm,\min,2}}^{0} f_2(W_{pm})\mathrm{d}W_{pm}=\\&=\int_{W_{vt,\max}}^{W_{va,\max,2}} f_2(W_{va})\mathrm{d}W_{va}=\\&=\Pr(Q>Q_t)=\int_{Q_t}^{Q_{\max}} f(Q)\mathrm{d}Q.\end{aligned}\tag{29}$$

$$\begin{aligned}&\int_{-\infty}^{\infty} f(W_{pm})\mathrm{d}W_{pm}=\int_{W_{pm,\min,1}}^{W_{pm,\max,1}} f_1(W_{pm})\mathrm{d}W_{pm}=\\&=\int_{0}^{W_{pm,\max,2}} f_2(W_{pm})\mathrm{d}W_{pm}=1.\end{aligned}\tag{30}$$

$$\begin{aligned}&E_1(W_{pm})=\int_{-\infty}^{\infty} W_{pm} f_1(W_{pm})\mathrm{d}W_{pm}=\\&=\int_{W_{pm,\min,1}}^{W_{pm,\max.1}} W_{pm} f_1(W_{pm})\mathrm{d}W_{pm}\text{, GWh}\end{aligned}\tag{31-a}$$

$$\begin{aligned}&E_2(W_{pm})=\int_{-\infty}^{\infty} W_{pm} f_2(W_{pm})\mathrm{d}W_{pm}=\\&=\int_{0}^{W_{pm,\max,2}} W_{pm} f_2(W_{pm})\mathrm{d}W_{pm}\text{, GWh}\end{aligned}\tag{31-b}$$

A graph-analytical representation can be seen in Fig. 6.

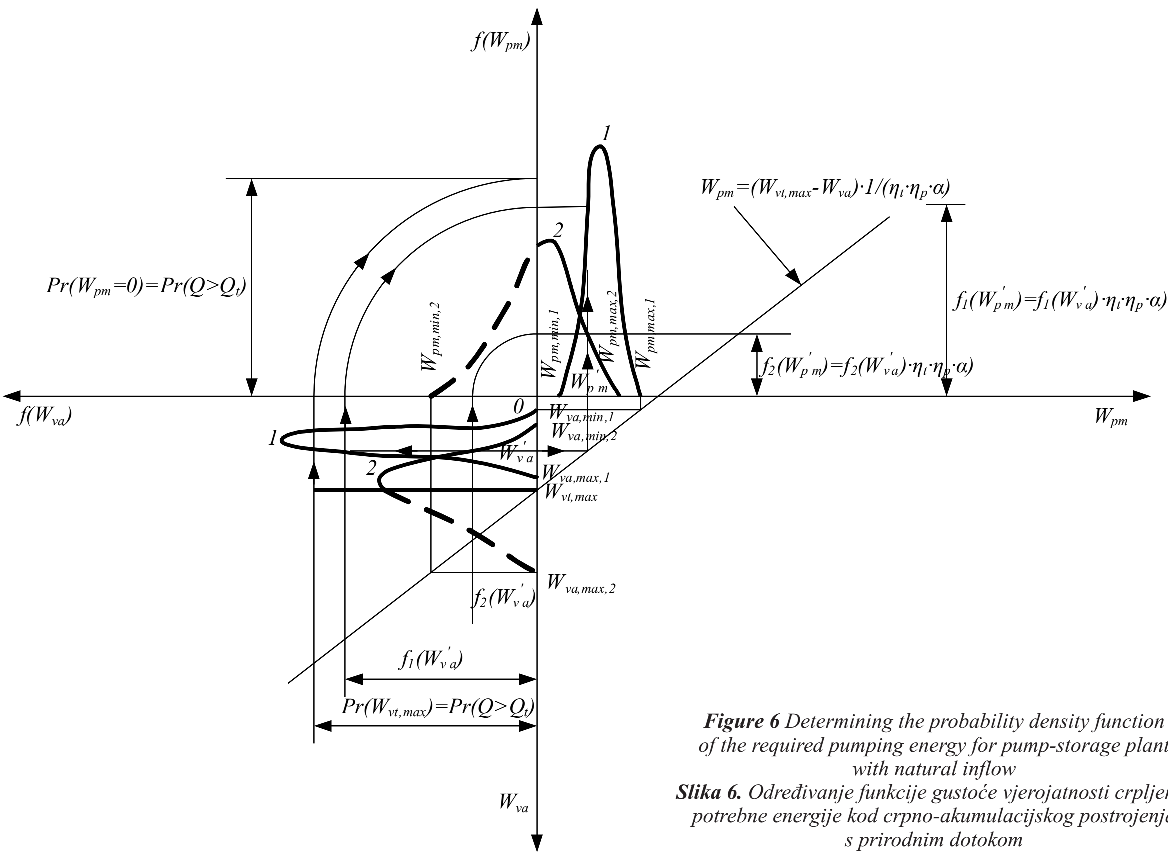

Figure 6 *Determining the probability density function of the required pumping energy for pump-storage plant with natural inflow*
Slika 6. *Određivanje funkcije gustoće vjerojatnosti crpljenja potrebne energije kod crpno-akumulacijskog postrojenja s prirodnim dotokom*

Distribution function $F(W_{pm})$, i.e. its complement; the probability function $F^*(W_{pm})$ of required pumping energy in case when natural inflow exists, is especially important for the forthcoming modeling of pump-storage power plants within the scope of a complete power plant system model. In relation to this, the following equations may be written:

$$\begin{aligned}
&F_1(W_{pm}) = 0 && W_{pm} < W_{pm,\min,1} \\
&F_1\left(W_{pm}\right) = \int_{-\infty}^{W_{pm}} f_1\left(W_{pm}\right)\mathrm{d}W_{pm} = \int_{W_{pm,\min,1}}^{W_{pm}} f_1\left(W_{pm}\right)\mathrm{d}W_{pm}; && W_{pm,\min,1} \le W_{pm} < W_{pm,\max,1} \\
&F_1(W_{pm}) = 1 && W_{pm} \ge W_{pm,\max,1}
\end{aligned} \tag{32}$$

$$\begin{aligned}
&F_1^*(W_{pm}) = 1 && W_{pm} < W_{pm,\min,1} \\
&F_1^*\left(W_{pm}\right) = \int_{W_{pm}}^{-\infty} f_1\left(W_{pm}\right)\mathrm{d}W_{pm} = \int_{W_{pm}}^{W_{pm,\max,1}} f_1\left(W_{pm}\right)\mathrm{d}W_{pm}; && W_{pm,\min,1} \le W_{pm} < W_{pm,\max,1} \\
&F_1^*(W_{pm}) = 0 && W_{pm} \ge W_{pm\ \max\ 1}
\end{aligned} \tag{33}$$

$$\begin{aligned}
&F_2(W_{pm}) = 0 && W_{pm} < 0 \\
&F_2\left(W_{pm}\right) = \int_{-\infty}^{W_{pm}} f_2\left(W_{pm}\right)\mathrm{d}W_{pm} = \int_{Q_t}^{Q_{\max}} f(Q)dQ + \int_0^{W_{pm}} f_2\left(W_{pm}\right)\mathrm{d}W_{pm}; && 0 \le W_{pm} < W_{pm,\max,2} \\
&F_2(W_{pm}) = 1 && W_{pm} \ge W_{pm\ \max\ 2}
\end{aligned} \tag{34}$$

$$\begin{aligned}
&F_2{}^*(W_{pm}) = 1; && W_{pm} < 0 \\
&F_2{}^*\left(W_{pm}\right) = \int_{W_{pm}}^{-\infty} f_2\left(W_{pm}\right)\mathrm{d}W_{pm} = \int_{W_{pm}}^{W_{pm,\max,2}} f(W_{pm})\mathrm{d}W_{pm}; && 0 \le W_{pm} < W_{pm,\max,2} \\
&F_2{}^*(W_{pm}) = 0 && W_{pm} \ge W_{pm,\max,2}
\end{aligned} \tag{35}$$

The above functions are shown in Fig. 7.

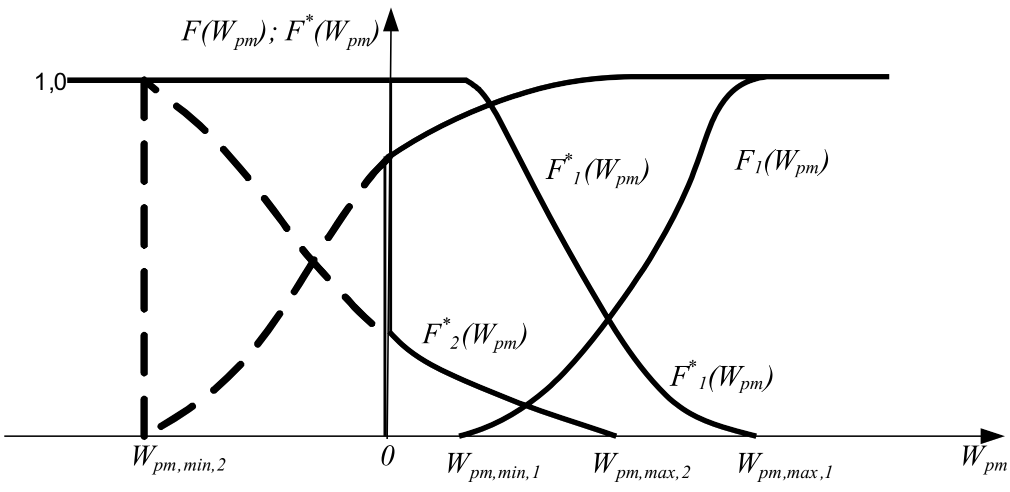

***Figure** 7 Distribution function and probability function of the required pumping energy for pump-storage plant with natural inflow*
***Slika 7.** Funkcija razdiobe i funkcija vjerojatnosti crpljenja potrebne energije kod crpno-akumulacijskog postrojenja s prirodnim dotokom*

The right hand-side marginal value of the probability function at $W_{pm}= 0$ that in both cases determines the probability according to which a pump-storage power plant with natural inflow should pump water into the upper storage in the observed week:

$$F_1^*\left(W_{pm} = 0\right) = \Pr\left(W_{pm} > 0\right) = \\ = \int_{W_{pm,\max,2}}^{W_{pm}\max,1} f_1\left(W_{pm}\right)\mathrm{d}W_{pm} = 1 \tag{36-a}$$

$$F_2^*\left(W_{pm} = 0\right) = \Pr\left(W_{pm} > 0\right) = \\ = \int_0^{W_{pm,\max,2}} f_2\left(W_{pm}\right)\mathrm{d}W_{pm} = \int_{Q_{\min}}^{Q_t} f(Q)\mathrm{d}Q. \tag{36-b}$$

Finally, it should be mentioned that - as described in [3] and mentioned in the Appendix – considerations in this Section refer to the case when the relative useful volume of the storage is equal to or greater than the relative required volume for the maximum variable energy production ($a_k \ge a_{\max}$). Thereby it is assumed that $t_{vt} \le t_{vh}$, which usually holds in practice.

As already mentioned, in pump-storage plants with natural inflow pump drive will be maintained as long as the inflowing water quantities and quantities of the water pumped into the upper storage together result in production $W_{vt,\max}$. Although the inflow is subject to the probability distribution, production is determined and it equals $W_{vt,\max}$. For higher inflows ($Q>Q_t$), as also mentioned, a pump-storage power plant becomes a common hydroelectric storage plant. Dependences from [3] that refer to determination of the constant energy probability distribution can thus be applied in this case as well.

The following simple example provides a numerical proof for the above considerations.

4
Example
Primjer

Determining the needed probability distributions of the pump-storage plant with natural inflow

Određivanje potrebnih razdioba vjerojatnosti crpno-akumulacijskog postrojenja s prirodnim dotokom

Input data required for calculation are:

- The probability density function of the inflow is shown in the third quadrant of the coordinate system in Fig. 9.
- Data for the pump-storage plant are as follows:
  Maximum turbine flow-through (designed size): Q_A=107,380189 m^3/s; gross (net) head (for the purpose of simplicity, it is assumed that gross and net head are equal, α=1): H_b=H_n=100 m*;* efficiency rate of the plant in turbine drive: η_t=0,95; efficiency rate of the plant in pump drive: η_p=0,8; useful storage volume: V_k=4 hm^3; number of equal generator units: *4;* rated power of generator units in turbine drive: $P_{t,N}$=25 MW; maximum capacity of the plant in the turbine drive during the observed week: $P_{h,max}$=100 MW; outage probability in turbine drive: $Pr_t(A)$=0,1.
- Load data are given by the load duration curve shown in Fig. 8, as well as by the following values:
  Maximum duration of using variable power plant capacity (adopted value): r_{tvh}=36 h; number of working and non-working days in the observed week: r=6*,* n=1; duration of using maximum capacity in turbine drive (adopted value): t_{vt}=5 h.

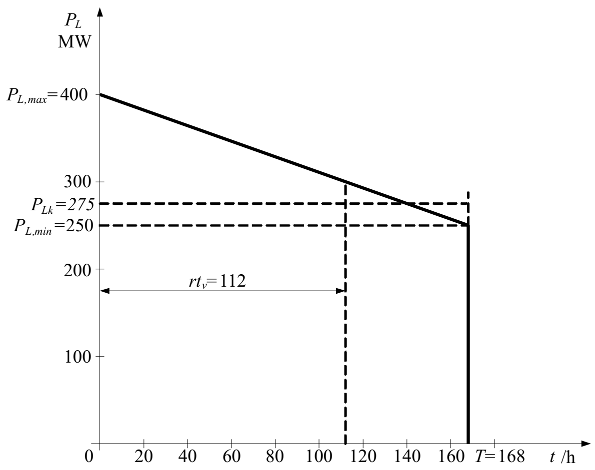

Figure 8 *Load duration curve*
Slika 8. *Krivulja trajanja opterećenja*

Shortened calculation procedure:

Dependence W_{va}=$f(Q)$ is determined by (D-5) and (14), where $W_{h,\max}$ is given by (D-6), and $P_{h,\max}$ is given by (1): W_{va}=0,1565676057$\cdot Q$ (GWh).

Equation (3) holds for $W_{vt,\max}$: $W_{vt,\max}$=3 GWh.

Q_t is obtained from function W_{va}=$f(Q)$ after replacing W_{va}= $W_{vt,\max}$: Q_t=19,16105178 m^3/s.

Determination of the probability density function for variable generation $f(W_{va})$ by using natural inflow only is shown in Fig. 9.

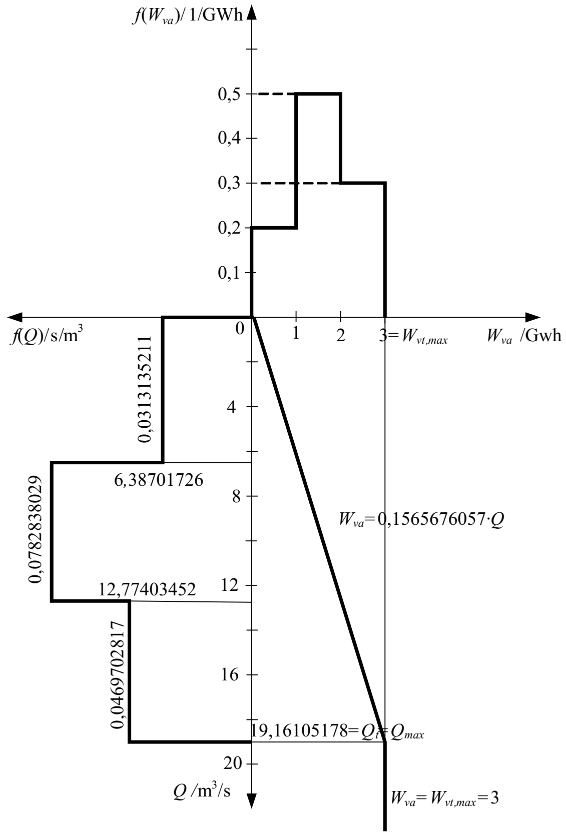

Figure 9 *Determination of the probability density function for variable generation by using natural inflow only*
Slika 9. *Određivanje funkcije gustoće vjerojatnosti kod varijabilnog generiranja energije uz samo prirodan dotok*

Fig. 10 represents the determination of probability density function $f(W_{ps})$ of energy provided by the pump-storage drive. The required dependence W_{ps}= $f(W_{va})$ is obtained by (9):

$$W_{ps} = 3 - W_{va}\ ,\ \mathrm{GWh}$$

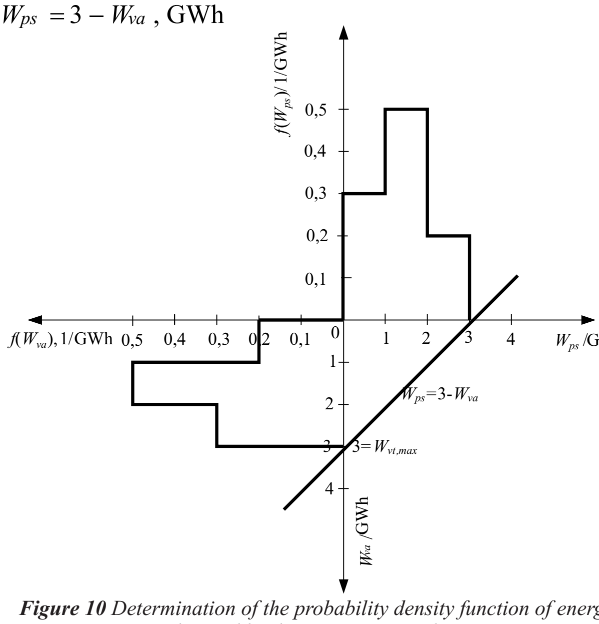

Figure 10 *Determination of the probability density function of energy obtained by the pump-storage drive*
Slika 10. *Određivanje funkcije gustoće vjerojatnosti energije dobivene crpno akumulacijskim pogonom*

In order to determine the probability distribution of the required pumping energy (Fig. 11), dependence $W_{pm}=f(W_{va})$ is calculated from (13):

$$W_{pm} = 3{,}947368421 - 1{,}315789474 \cdot W_{va}\text{, GWh}$$

The sum of the expected values $E_1(W_{va})$=1,6 GWh and $E_1(W_{ps})$=1,4 GWh, calculated according to (20) and (25), meets the condition (26).

The probability of pump drive commitment can be determined from the function $F^*(W_{pm})$ for W_{pm}=0 according to expression (33), therefore: $\Pr_1(W_{pm}>0)=1$.

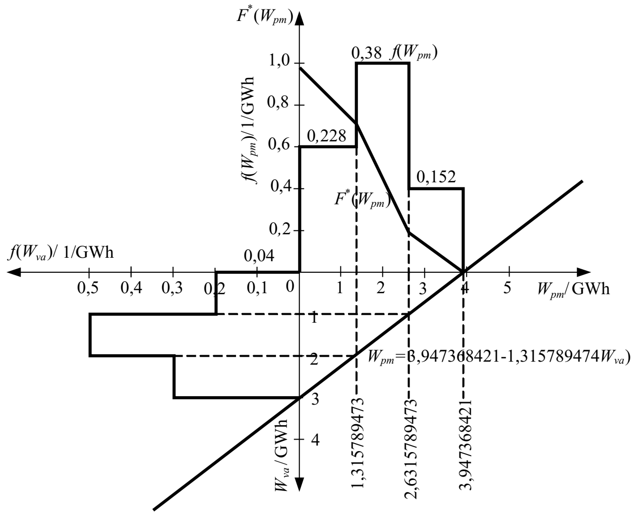

Figure 11 Determination of the density function and probability function of the required pumping energy
Slika 11. Određivanje funkcije gustoće i vjerojatnosti potrebne energije crpljenja

5 Appendix
Dodatak

The method of constant and variable energy for the case of deterministic approach for determining available amounts of water used for energy generation is described in [5] in more detail. In the stochastic approach to storage hydro plant and pump-storage hydro plant modeling the following dependencies of this method are used:

Weekly load duration curve:

It is approximated so that the loading during off-peak load is substituted by horizontal according to Fig. D-1, as a result of which the demanded weekly energy, represented by the area under the curve, is divided into constant and variable.

In Fig. D-1 are: t_v – daily peak load duration, r – number of working days in a week, T_m – weekly off-peak load duration.

Relative useful storage volume:

$$a_k = \frac{V_k \cdot 10^3}{3{,}6 \cdot 24 \cdot Q_A} = \frac{W_k}{24 \cdot P_{h,\max} \times 10^{-3}} \tag{D-1}$$

There are: V_k – useful storage volume in hm³, Q_A – designed size of hydroelectric storage plant in m³/s, W_k – energy value of useful storage volume in GWh, $P_{h,\max}$ – maximum capacity of the storage plant in the observed period in MW.

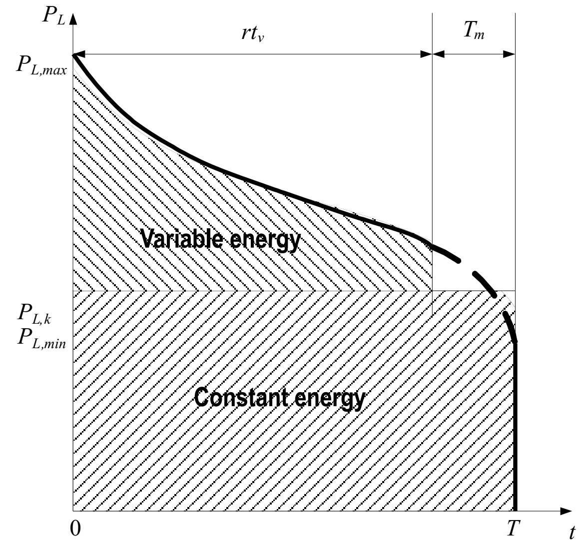

***Figure D-1** Partition of the demanded energy into variable and constant*
***Slika D-1.** Podjela potrebne energije u varijabilnu i stalnu*

Relative required storage volume for maximum variable energy generation:

$$a_{\max} = \frac{rt_{vh}}{24} \cdot \frac{2 \cdot 24 - t_{vh}}{24(r+n)} \tag{D-2}$$

Here are: t_{vh} ($t_{vh}<t_v$) – longest daily duration of using variable capacity of the hydro power plant, given in hours, r – number of working days, n – number of non-working days in the observed period.

Considering the size of the storage reservoir, two scenarios are possible: $a_k \geq a_{\max}$ and $a_k < a_{\max}$, but only the first scenario will be considered in this article ($a_k \geq a_{\max}$).

- **Use of water by the storage plant:**

Depending on inflow two cases are distinguished as well as shown in the diagram in Fig. D-2 representing the use of water in the storage hydro plant.

The first case is when the natural inflow is low (i.e. the hydro plant capacity P_h corresponding the inflow is low), the entire amount of water can be used to generate only variable energy W_{hv} during the time t_{vh} on working days, which equals the possible generation W_{hm}.

$$W_{hv} = W_{hm}; \qquad 0 \leq W_{hm} \leq W'_{hm} \tag{D-3}$$

$$W_{hk} = 0; \qquad 0 \leq W_{hm} \leq W'_{hm} \tag{D-4}$$

The possible generation is calculated from the inflow Q, designed size Q_A and maximum generation $W_{h,\max}$:

$$W_{hm} = \frac{W_{h,\max}}{Q_A} \cdot Q \tag{D-5}$$

$$W_{h,\max} = P_{h,\max} \cdot 24(r+n) \times 10^{-3} \tag{D-6}$$

$$P_{h,max} = 9{,}81 \cdot H_n \cdot Q_A \times 10^{-3} \tag{D-7}$$

The duration of using variable capacity of the plant t_{va} is given by the following equation:

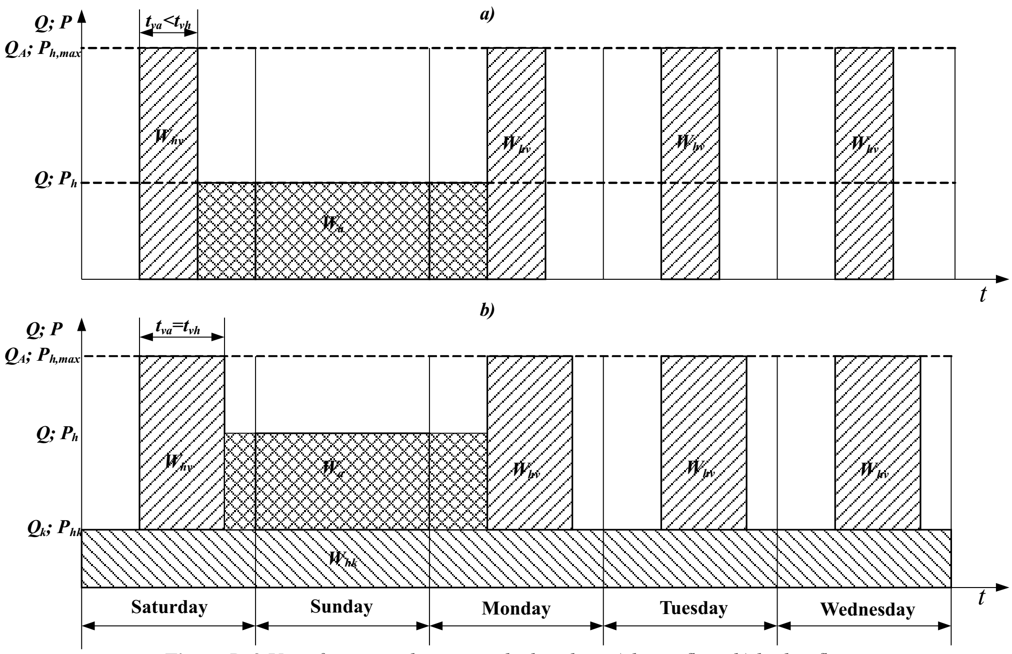

Figure D-2 *Use of water in the storage hydro plant a) low inflow; b) high inflow*
Slika D-2. *Korištenje vode kod akumulacijske hidroelektrane a) slab dotok; b)visok dotok*

$$t_{va} = \frac{Q}{Q_A} \cdot \frac{24(r+n)}{r} = \frac{P_h}{P_{h,\max}} \cdot \frac{24(r+n)}{r} \qquad \text{(D-8)}$$

If there are higher amounts of inflow, constant energy must be generated to, where t_{va} just equals t_{vh}. Those amounts of energy are determined from the following expressions:

$$W_{hk} = \frac{24(r+n)}{24(r+n) - rt_{vh}} \cdot W_{hm} - \frac{rt_{vh} \cdot W_{h,\max}}{\left[24(r+n) - rt_{vh}\right]} \qquad \text{(D-9)}$$
$$W'_{hm} \le W_{hm} \le W_{hm,\max}$$

$$W_{hv} = \frac{W_{h,\max} \cdot rt_{vh}}{24(r+n) - rt_{vh}} - \frac{rt_{vh}}{\left[24(r+n) - rt_{vh}\right]} \cdot W_{hm} \qquad \text{(D-10)}$$
$$W'_{hm} \le W_{hm} \le W_{hm,\max}$$

The limit value between these two cases is at the value of possible generation:

$$W'_{hm} = \frac{W_{h,\max} \cdot rt_{vh}}{24(r+n)}. \qquad \text{(D-11)}$$

6 References
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